Regulations

- Each exercise has 100 point.
- After the deadline, three points per day is reduced in the case of late submission.
- The calculations and programs must be submitted together with the report
- Please write comments on your code and prepare a runnable MATLAB file at the end
- MATLAB codes must be submitted separately and also inside the report
- For admission to the exam, you must achieve more than 60% of each lab and in total, more than 70% of the points must be achieved
- Working in a group of two persons is permitted (from the same semester)
- Please submit your exercise in ilias system

If we consider a Keplerian orbit, The basic equation is

$$\ddot{r} = -\frac{GM}{|r^3|} \cdot r$$

In reality a certain number of additional forces act on the near-earth satellite

$$\ddot{r} = -\frac{GM}{|r^3|} \cdot r \ + \mathrm{f} \longrightarrow \text{Perturbing forces}$$

Perturbing forces:

Gravitational

Force due to the
- Non-spherically and inhomogeneous mass distribution of the earth
- Other celestial bodies (sun, moon,…)
- Earth and ocean tides

non gravitational

Force due to the
- Earth-reflected and direct solar radiation pressure
- Reaction to transmitting signal
- **Atmospheric drag**

Atmospheric drag

The largest non-gravitational perturbing force on LEO

Accurate modeling of the atmospheric drag is hard because

1. Physical properties of the atmosphere are not accurately known.
2. Detailed information about the incretion of neutral gases with aircraft structure.
3. The varying attitude of the non-spherical satellites

$$\mathbf{f}_{drag} = -\frac{1}{2} \cdot C_D \cdot \rho(\mathbf{r}, t) \frac{A}{m} \left(\mathbf{V} - \mathbf{V}_{atm}\right) \left|\mathbf{V} - \mathbf{V}_{atm}\right|$$

Atmospheric drag depends on:

- Satellite geometry
- Satellite velocity
- Atmosphere velocity
- Atmosphere density and temperature

Lab 1: Atmospheric drag

Exercise 1

In order to display the trajectory of the orbits for the two scenarios, first the initial and position and velocity must be determined by using the Kepler elements

Kepler elements for both scenarios:

GOCE

$$a = R + h$$
$$I = 96,6^\circ$$
$$e = 0$$
$$\Omega = 335^\circ$$
$$\omega = 273^\circ$$
$$M = 5^\circ$$

Aerobreaking

$$a = \frac{h_{per}+R+h_{apo}+R}{2}$$
$$I = 96,6^\circ$$
$$e = \frac{a_{max}-a_{min}}{a_{max}+a_{min}}$$
$$\Omega = 335^\circ$$
$$\omega = 273^\circ$$
$$M = 5^\circ$$

Lab 1: Atmospheric drag

Exercise 1

In order to display the trajectory of the orbits for the two scenarios, first the initial and position and velocity must be determined by using the Kepler elements

$$r_f = \begin{pmatrix} a \cdot (cosE - e) \\ a \cdot \sqrt{1-e^2} \cdot sinE \\ 0 \end{pmatrix}$$

$$\dot{r}_f = \frac{n_a}{1 - a \cdot cosE} \begin{pmatrix} -sinE \\ \sqrt{1-e^2} \cdot cosE \\ 0 \end{pmatrix}$$

$$r_i = R_3(-\Omega)R_1(-I)R_3(-\omega)r_f$$

$$\dot{r}_i = R_3(-\Omega)R_1(-I)R_3(-\omega)\dot{r}_f$$

You need to develop the function *[r, v]=kep2cart(a, e, I, omega, Omega, M, n)*

Step 2: developing the drag function

Now we want to calculate the influence of the atmosphere drag

$$\mathbf{f}_{drag} = -\frac{1}{2} \cdot C_D \cdot \rho(\mathbf{r}, t) \frac{A}{m} (\mathbf{V} - \mathbf{V}_{atm}) |\mathbf{V} - \mathbf{V}_{atm}|$$

$C_D = 1$ $\quad$ $v_{atm} = 0$ $\quad$ $\rho_{atm} = \rho(r)$

We need to determine the density based on the height of the satellite. To do this, we use the Harris Priester model (read attached file)

$$\rho(h) = \frac{\rho_m(h) + \rho_M(h)}{2}$$

Step 3: numerical integration

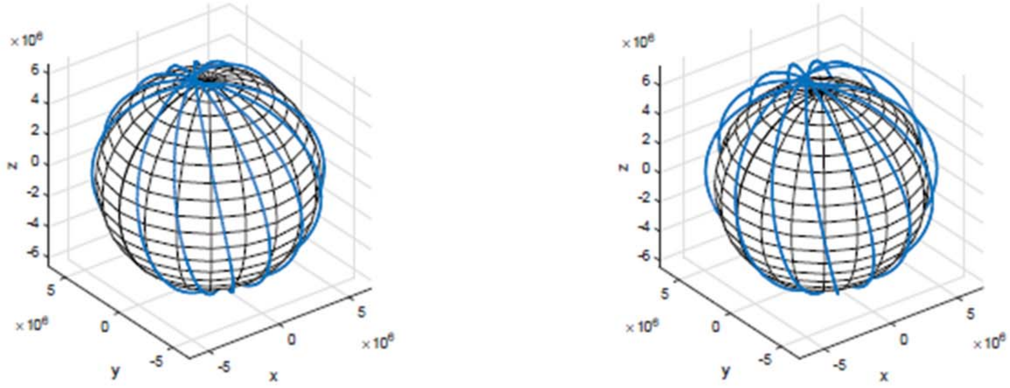
In order to now calculate the satellite orbits (position and velocity of the satellite) with the help of the numerical integration of Newton's equation of motion for the Kepler problem, the following system of differential equations should be solved

$$\ddot{\mathbf{r}} = -\frac{GM}{r^3}\mathbf{r} + \mathbf{f} \qquad \mathbf{f} = \begin{pmatrix} f_1 \\ f_2 \\ f_3 \end{pmatrix}$$

You have already done it for considering the effect of C20

$$\mathbf{res} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ -\frac{GM}{r^3}\cdot x \\ -\frac{GM}{r^3}\cdot y \\ -\frac{GM}{r^3}\cdot z \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ f_1 \\ f_2 \\ f_3 \end{bmatrix}$$

```
option = odeset('RelTol',1e-15,'AbsTol',1e-15);
[T1,Y1]=ode45(@odefun, t_1_sec, r11(:,1), option);
```



(a) GOCE (b) Aerobraking

For plotting the variation of the Kepler elements during the time, consider their difference from the initial values.

Step 3: Discuss the results in terms of LPE Gauss

3.3 Gauss form of LPE

The LPE describe perturbed motion as long as the force can be written as the gradient of a potential. For non-gravitational forces, e.g. solar pressure or air drag Gauss found the following form:

$$\text{Force } \boldsymbol{f} \;=\; \begin{pmatrix} f_1 \\ f_2 \\ f_3 \end{pmatrix} \begin{array}{ll} \text{- quasi along-track} & \text{- complementary in RHS sense} \\ \text{- cross-track} & \text{- } \boldsymbol{L} \\ \text{- radial} & \text{- } \boldsymbol{r} \end{array}$$

For near-circular orbits ($e \approx 0$) the Gauss LPE reduce to:

$$\begin{aligned} \dot{a} &= \frac{2}{n} f_1 \\ \dot{e} &= \frac{1}{na} (\sin\nu f_3 + 2\cos\nu f_1) \\ \dot{I} &= \frac{1}{na} \cos u f_2 \\ \dot{\Omega} &= \frac{1}{na\sin I} \sin u f_2 \\ \dot{\omega} + \dot{M} &= n - \frac{e}{na} f_3 - \cos I \dot{\Omega} \end{aligned}$$

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```
[ri, v]=kep2cart(a,e,I,omega,Omega,M,n);
```

Compare the Keplerian elements of both solutions!!

Calculate k (between Hill and tangential triad): numerical and analytical

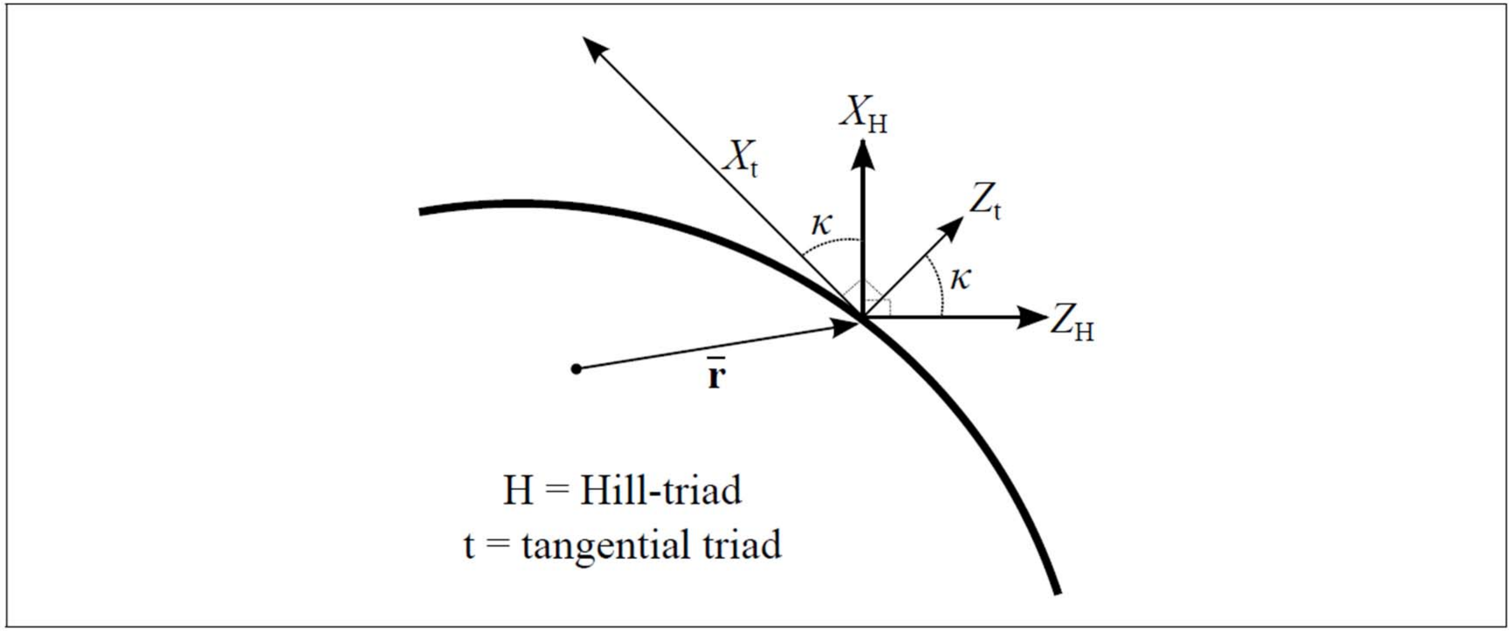

Figure 3.2: Hill-triad H, tangential triad t