University of Stuttgart
Institute of Geodesy

Satellite Geodesy

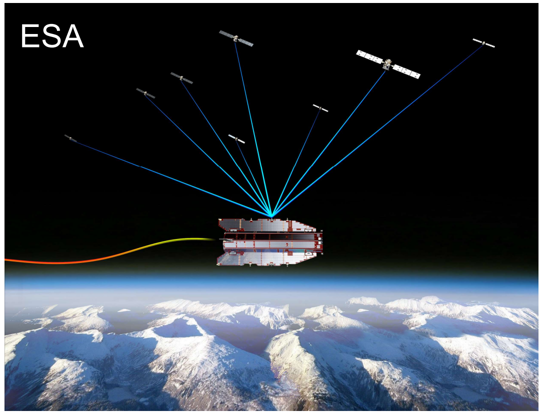
Atmospheric drag

GIS

Atmospheric drag

Exercise

- GOCE (last days)
  - ✓ Semi major axis $a = R + 225\ \text{km}$
  - ✓ Inclination $\mathrm{I} = 96.6^{\circ}$
  - ✓ Eccentricity $\mathrm{e} = 0$
  - ✓ Right Ascension of the ascending node $\Omega = 335^{\circ}$
  - ✓ Argument of perigee $\omega = 273^{\circ}$
  - ✓ Mean anomaly $\mathrm{M} = 5^{\circ}$



- Tasks
  - ✓ Compute & plot Keplerian orbit
  - ✓ Create a code, e.g., drag.m, to compute the atmospheric drag
  - ✓ Plot variations of Kepler elements in a 24-hour time interval

Atmospheric drag

Drag

- Force

$$\boldsymbol{f} = -\frac{1}{2} C_D \rho(\boldsymbol{r}, t) \frac{A}{m} (\dot{\boldsymbol{r}} - \dot{\boldsymbol{r}}_a) |\dot{\boldsymbol{r}} - \dot{\boldsymbol{r}}_a|$$

  - ✓ m: mass of the satellite
  - ✓ A: cross-sectional area of the satellite
  - ✓ C_D: drag coeffitient (satellite specific)
  - ✓ $\rho(\boldsymbol{r}, t)$: density of atmosphere near the satellite
  - ✓ $\boldsymbol{r}$: position vector of the satellite
  - ✓ $\dot{\boldsymbol{r}}$: velocity vector of the satellite
  - ✓ $\dot{\boldsymbol{r}}_a$: velocity of the atmosphere near the satellite

- Simplified

$$\boldsymbol{f} = -\frac{1}{2} \rho(r) \frac{A}{m} \dot{\boldsymbol{r}} |\dot{\boldsymbol{r}}|$$

  - ✓ $\rho(\boldsymbol{r})$: Harris & Priester radial density model
  - ✓ $r = |\boldsymbol{r}|$
  - ✓ $A/m = 0.001 \, \mathrm{m}^2/\mathrm{kg}$

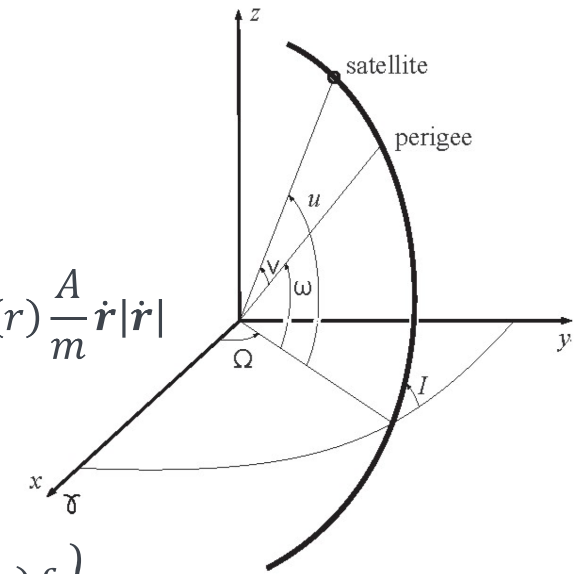
	min	max		min	max
h [km]	ρ_m [g/km^3]	ρ_M [g/km^3]	h [km]	ρ_m [g/km^3]	ρ_M [g/km^3]
100	497400.0	497400.0	420	1.558	5.684
120	24900.0	24900.0	440	1.091	4.355
130	8377.0	8710.0	460	0.7701	3.362
140	3899.0	4059.0	480	0.5474	2.612
150	2122.0	2215.0	500	0.3916	2.042
160	1263.0	1344.0	520	0.2819	1.605
170	800.8	875.8	540	0.2042	1.267
180	528.3	601.0	560	0.1488	1.005
190	361.7	429.7	580	0.1092	0.7997
200	255.7	316.2	600	0.08070	0.6390
210	183.9	239.6	620	0.06012	0.5123
220	134.1	185.3	640	0.04519	0.4121
230	99.49	145.5	660	0.03430	0.3325
240	74.88	115.7	680	0.02632	0.2691
250	57.09	93.08	700	0.02043	0.2185
260	44.03	75.55	720	0.01607	0.1779
270	34.30	61.82	740	0.01281	0.1452
280	26.97	50.95	760	0.01036	0.1190
290	21.39	42.26	780	0.008496	0.09776
300	17.08	35.26	800	0.007069	0.08059
320	10.99	25.11	840	0.004680	0.05741
340	7.214	18.19	880	0.003200	0.04210
360	4.824	13.37	920	0.002210	0.03130
380	3.274	9.955	960	0.001560	0.02360
400	2.249	7.492	1000	0.001150	0.01810

Atmospheric drag

Kepler element variations

- Force

$$\boldsymbol{f} = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \end{bmatrix} = \begin{bmatrix} f_{\text{along-trac}} \\ f_{\text{cross-track}} \\ f_{\text{radial}} \end{bmatrix} = -\frac{1}{2}\rho(r)\frac{A}{m}\dot{\boldsymbol{r}}|\dot{\boldsymbol{r}}|$$



- Variations

$$\dot{a} = \frac{2a^2 v}{GM} f_1$$

$$\dot{e} = \frac{1}{v}\left\{\frac{r}{a}\sin v\, f_3 + 2(e + \cos v) f_1\right\}$$

$$\dot{I} = \frac{r}{L}\cos u\, f_2$$

$$\dot{\Omega} = \frac{r}{L \sin I}\sin u\, f_2$$

$$\dot{\omega} = \frac{1}{ev}\left\{-\left(2e + \frac{r}{a}\right)\cos v\, f_3 + 2\sin v\, f_1\right\} - \frac{r}{L}\cot I \sin u\, f_2$$

$$\dot{M} = n + \frac{b}{a}\frac{1}{ev}\left\{\frac{r}{a}\cos v\, f_3 - 2\left(1 + e^2\frac{r}{p}\right)\sin v\, f_1\right\}$$

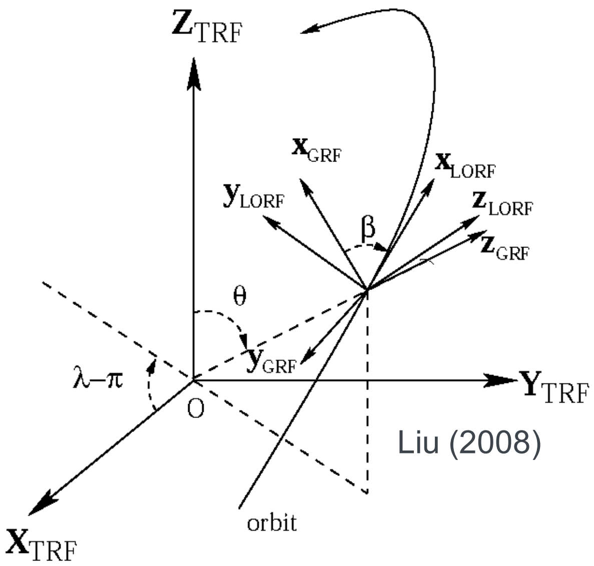
$L = na^2\sqrt{1 - e^2}$ $\quad p = r(1 + e\cos v)$ $\quad v = \frac{L}{p}\sqrt{1 + e^2 + 2e\cos v}$ $\quad u =$ Argument of latitude

$n^2 a^3 = GM$ $\quad u = \omega + v$ $\quad v =$ True anomaly $\quad n =$ Angular velocity

Atmospheric drag

Frame rotation

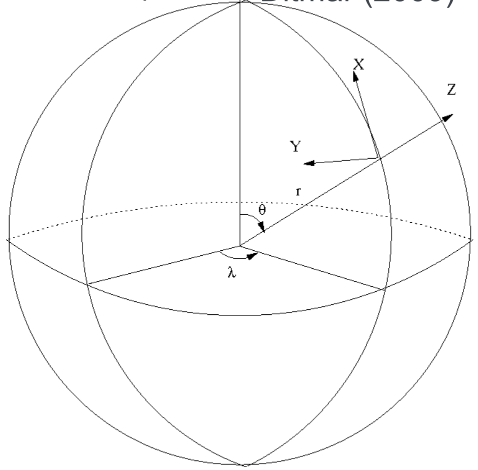
- Terrestrial Reference Frame (TRF)
  - ✓ X-axis: Greenwich
  - ✓ Z-axis: Pole
  - ✓ Y-axis: Right-handed



Liu (2008)

- Local Orbital Reference Frame (LORF)
  - ✓ X-axis: along-track (direction of velocity vector)
  - ✓ Y-axis: cross-track (direction of the orbital angular momentum)
  - ✓ Z-axis: radial

Ditmar (2009)



- Geographical Reference Frame (GRF)
  - ✓ X-axis: North
  - ✓ Y-axis: West
  - ✓ Z-axis: upwards (right-handed)

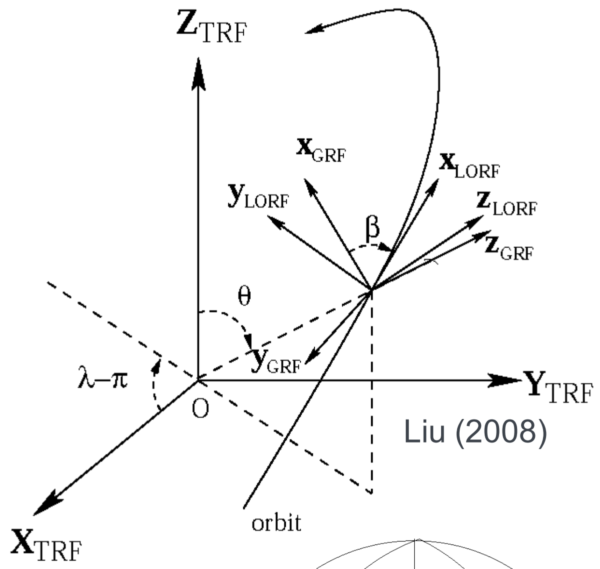
Atmospheric drag

Frame rotation

- Terrestrial Reference Frame (TRF)
  - ✓ X-axis: Greenwich
  - ✓ Z-axis: Pole
  - ✓ Y-axis: Right-handed
- Local Orbital Reference Frame (LORF)
  - ✓ X-axis: along-track
  - ✓ Y-axis: cross-track
  - ✓ Z-axis: radial
- TRF → LORF

$$\boldsymbol{r}^{\mathrm{LORF}} = R_y(-\alpha)R_z(-\beta)R_y(-\theta)R_z(\lambda-\pi)\boldsymbol{r}^{\mathrm{TRF}}$$

  - ✓ θ: co-latitude
  - ✓ λ : longitude
  - ✓ α: elevation (angle between velocity vector & horizon measured upwards from horizon)
  - ✓ β: azimuth of the velocity vector (measured clockwise from the North)



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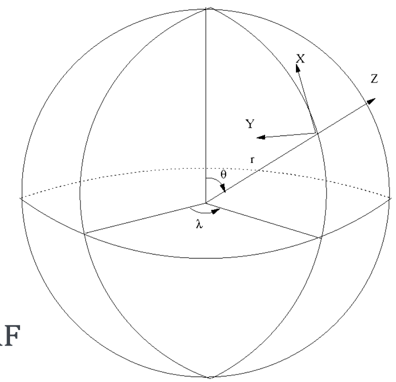

Ditmar (2009)