Table 3.7. Solar flux values and geomagnetic indices

	$F_{10.7}$	$\bar{F}_{10.7}$	K_p
Minimum	70	70	0o
Typical	200	155	4o
Maximum	330	240	8o

A lag of 6.7 hours in the response of temperature changes to geomagnetic storms, indicated by K_p values from 6 to 9, has been observed. Along with the solar flux values, geomagnetic data are also available in the above-mentioned "Solar-Geophysical Data prompt reports".

Semi-annual variations in the atmospheric density show a strong height dependence and periodic variations throughout the year. However, these variations seem not to be connected with the solar activity, and the geophysical mechanisms behind these variations are not well understood.

At lower altitudes of 90–120 km, *latitudinal density fluctuations* have been observed in the thermosphere related to seasonal variations. The amplitude of these variations attains a maximum at about 110 km height, and is assumed to decrease rapidly with increasing height.

Seasonal-latitudinal variations of the helium density in the upper atmosphere have been observed, resulting from helium migration towards the winter pole. No major height-dependency seems to exist.

Additionally, there are a number of further atmospheric processes that affect the density, as e.g. variations in the hydrogen density and pressure waves in the atmosphere. However, as accurate modeling of global atmospheric properties is lacking, these smaller effects are neglected in most cases.

3.5.2 The Harris–Priester Density Model

Although the dynamics of the upper atmosphere shows a significant temporal and spatial variation, there exist relatively simple atmospheric models that already allow for a reasonable atmospheric density computation. Thus, prior to a description of elaborate and complex models, we consider the algorithm of Harris–Priester (Harris & Priester 1962, see also Long et al. 1989), which is still widely used as a standard atmosphere and may be adequate for many applications.

The Harris–Priester model is based on the properties of the upper atmosphere as determined from the solution of the heat conduction equation under quasi-hydrostatic conditions. While neglecting the explicit dependence of semi-annual and seasonal latitude variations, it has been extended to consider the diurnal density bulge. As the atmospheric heating due to the solar radiation leads to a gradual increase of the atmospheric density, the apex of this bulge is delayed by approximately 2 hours, equivalent to a location 30° to the east of the subsolar point (Long et al. 1989). The antapex and apex density $\rho_m(h)$ and $\rho_M(h)$ at a given altitude h is computed through the exponential interpolation between tabulated minimum and maximum density values $\rho_m(h_i)$ and $\rho_M(h_i)$ according to

$$
\begin{aligned}
\rho_m(h) &= \rho_m(h_i)\exp\left(\frac{h_i - h}{H_m}\right) \qquad (h_i \le h \le h_{i+1}) \\
\rho_M(h) &= \rho_M(h_i)\exp\left(\frac{h_i - h}{H_M}\right) \quad ,
\end{aligned}
\tag{3.101}
$$

where h is the height above the Earth's reference ellipsoid. The corresponding scale heights are given as

$$
\begin{aligned}
H_m(h) &= \frac{h_i - h_{i+1}}{\ln(\rho_m(h_{i+1})/\rho_m(h_i))} \\
H_M(h) &= \frac{h_i - h_{i+1}}{\ln(\rho_M(h_{i+1})/\rho_M(h_i))} \quad .
\end{aligned}
\tag{3.102}
$$

The diurnal density variation from the apex to the antapex due to the solar radiation is accomplished through a cosine variation according to

$$
\rho(h) = \rho_m(h) + (\rho_M(h) - \rho_m(h)) \cdot \cos^n\left(\frac{\Psi}{2}\right) \quad , \tag{3.103}
$$

where Ψ is the angle between the satellite position vector and the apex of the diurnal bulge. In practice, the latitudinal density variations are roughly taken into account by the declination-dependent angle Ψ and by the exponent n, which has a numerical value of 2 for low-inclination orbits, and 6 for polar orbits. Using trigonometric calculus and the definition of Ψ we derive

$$
\cos^n\left(\frac{\Psi}{2}\right) = \left(\frac{1+\cos\Psi}{2}\right)^{\frac{n}{2}} = \left(\frac{1}{2} + \frac{\boldsymbol{e}_r \cdot \boldsymbol{e}_b}{2}\right)^{\frac{n}{2}} \quad , \tag{3.104}
$$

with the unit satellite position vector $\boldsymbol{e}_r$. The unit vector $\boldsymbol{e}_b$ to the apex of the diurnal bulge is given as

$$
\boldsymbol{e}_b == \begin{pmatrix} \cos\delta_\odot \cos(\alpha_\odot + \lambda_l) \\ \cos\delta_\odot \sin(\alpha_\odot + \lambda_l) \\ \sin\delta_\odot \end{pmatrix} \tag{3.105}
$$

with the Sun's right ascension $\alpha_\odot$, declination $\delta_\odot$ and the lag angle in longitude $\lambda_l \approx 30°$.

In Table 3.8 the minimum and maximum density values are given for an altitude regime of 100 km to 1000 km and mean solar activity. In addition to its computational simplicity, the benefit of the Harris–Priester density model is that it can easily be tailored or extended to other altitude regimes or to other solar flux conditions. A multi-parametric comparison with the Jacchia 1971 model shows a mean deviation in density of about 40% for mean solar flux conditions, which increases to 60% for maximum solar activity. Since considerably higher deviations have been observed for minimum solar flux conditions, the tabular coefficients should be modified suitably for low solar activity phases.

Table 3.8. Harris–Priester atmospheric density coefficients valid for mean solar activity (Long et al. 1989)

h [km]	ρ_m [g/km^3]	ρ_M [g/km^3]	h [km]	ρ_m [g/km^3]	ρ_M [g/km^3]
100	497400.0	497400.0	420	1.558	5.684
120	24900.0	24900.0	440	1.091	4.355
130	8377.0	8710.0	460	0.7701	3.362
140	3899.0	4059.0	480	0.5474	2.612
150	2122.0	2215.0	500	0.3916	2.042
160	1263.0	1344.0	520	0.2819	1.605
170	800.8	875.8	540	0.2042	1.267
180	528.3	601.0	560	0.1488	1.005
190	361.7	429.7	580	0.1092	0.7997
200	255.7	316.2	600	0.08070	0.6390
210	183.9	239.6	620	0.06012	0.5123
220	134.1	185.3	640	0.04519	0.4121
230	99.49	145.5	660	0.03430	0.3325
240	74.88	115.7	680	0.02632	0.2691
250	57.09	93.08	700	0.02043	0.2185
260	44.03	75.55	720	0.01607	0.1779
270	34.30	61.82	740	0.01281	0.1452
280	26.97	50.95	760	0.01036	0.1190
290	21.39	42.26	780	0.008496	0.09776
300	17.08	35.26	800	0.007069	0.08059
320	10.99	25.11	840	0.004680	0.05741
340	7.214	18.19	880	0.003200	0.04210
360	4.824	13.37	920	0.002210	0.03130
380	3.274	9.955	960	0.001560	0.02360
400	2.249	7.492	1000	0.001150	0.01810

3.5.3 The Jacchia 1971 Density Model

A number of different atmospheric density models have been published since 1965 by L. G. Jacchia (1965, 1970, 1971, 1977) and Jacchia & Slowey (1981). The first model, called J65, was solely based upon the primary parameters geodetic height and temperature, with the latter determining the atmospheric conditions. When further density-related data became available from the analysis of satellite accelerations due to drag, an improved atmospheric model was established (Jacchia 1971). The J71 model includes density variations as a function of time, and covers the altitude interval from 90 km to 2500 km. It was adopted by the COSPAR (Committee on Space Research) working group as the International Reference Atmosphere in 1972, for heights ranging from 110 km to 2000 km (see CIRA 1972).

In 1977 Jacchia published the atmospheric model J77 (Jacchia 1977), which was based upon measurements of the acceleration of satellites, and additionally upon analyses of mass spectrometer data. The J77 model was revised once more in 1981 (Jacchia et al. 1981).