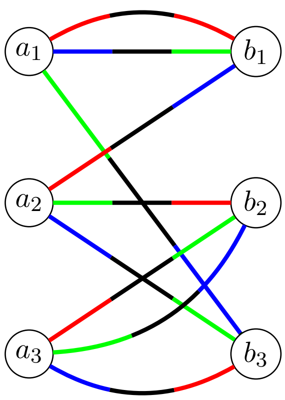
graphs to many other problems, be it finding the fewest colours required to colour a map of the counties of Britain without two bordering counties having the same colour, or determining the relationships of people at a party. When we talk about colouring a graph, we are usually referring to labelling each of the vertices or edges with a number, which can be thought as representing a colour. There are many different ways a graph can be coloured. One of the most common graph colourings is *Proper Vertex Colouring*, which is when we assign each vertex a colour such that no two vertices with the same colours are adjacent. Another type of colouring is *Proper Edge Colouring*, where we colour each edge such that no vertex has two edges of the same colour incident to it. We'll be looking at an extension to edge colouring.

2.3.1 Distortion Colouring

In distortion colourings, we have a bipartite graph $G = (\{A, B\}, E)$, and a set of colours we can use, $Col = \{1, ..., n\}$. Each edge, $e = ab$ (with $a \in A, b \in B$), in our graph has a bijection $r_e : Col \to Col$ associated to it. For each edge $e = ab$ we assign some colour, $c \in Col$, to the A-end of e, and then assign $r_e(c)$ to the B-end of e. A distortion colouring of $G = (\{A, B\}, E)$ with permutations on Col r_e $\forall e \in E$ can be considered as a function $f : E \to Col$ where we have the A-end of each edge, e, being coloured $f(e)$, and the B-end of e being coloured $r_e(f(e))$. We call f a *n-proper distortion colouring* if

$$\forall a \in A,\ \ \forall\, e, e' \in E,\ e \neq e',\ \text{incident to } a,\ \ f(e) \neq f(e')$$
$$\forall b \in B,\ \ \forall\, e, e' \in E,\ e \neq e',\ \text{incident to } b,\ \ r_e(f(e)) \neq r_{e'}(f(e'))$$

The following figure demonstrates a distortion colouring.



Edge	0	1	2
a_1b_1 Top	0	1	2
a_1b_1 Bottom	1	2	0
a_1b_3	1	0	2
a_2b_1	2	1	0
a_2b_2	0	2	1
a_2b_3	1	2	0
a_3b_2 Top	2	0	1
a_3b_2 Bottom	1	0	2
a_3b_3	2	1	0

Figure 3: A 3-proper distortion colouring of a bipartite graph. To demonstrate the colouring in this example, we colour the graph with blue where the edge is assigned 0, green for 1, and red for 2.

In the above diagram, each vertex in G has degree 3, and we're able to colour G with just 3 colours, we will see in the Section 3 that this is not always the case.