

There Is No Largest Prime Number

Euclid of Alexandria

euclid@alexandria.edu

January 12, 2019

Outline

1. Motivation

1.1. The Basic Problem That We Studied

1. Motivation

1.1 The Basic Problem That We Studied

What Are Prime Numbers? I

Definition: Prime number

A *prime number* is a number that has exactly two divisors.

What Are Prime Numbers? II

Example:

- 2 is prime (two divisors: 1 and 2).
- 3 is prime (two divisors: 1 and 3).
- 4 is not prime (**three** divisors: 1, 2, and 4).

There Is No Largest Prime Number

Theorem: Prime numbers

There is no largest prime number.

There Is No Largest Prime Number

Proof:

1. Suppose p were the largest prime number.

There Is No Largest Prime Number

Proof:

1. Suppose p were the largest prime number.
2. Let q be the product of the first p numbers.

There Is No Largest Prime Number

Proof:

1. Suppose p were the largest prime number.
2. Let q be the product of the first p numbers.
3. Then $q + 1$ is not divisible by any of them.

There Is No Largest Prime Number

Proof:

1. Suppose p were the largest prime number.
2. Let q be the product of the first p numbers.
3. Then $q + 1$ is not divisible by any of them.
4. But $q + 1$ is greater than 1, thus divisible by some prime number not in the first p numbers.

The proof used *reductio ad absurdum*.

What's Still To Do?

- Answered Questions
 - How many primes are there?
- Open Questions
 - Is every even number the sum of two primes?

An Algorithm For Finding Prime Numbers. I

An Algorithm For Finding Prime Numbers. II

FindPrimeNumbers

Input: an integer $n > 1$.

Let A be an array of Boolean values, indexed by integers 2 to n , initially all set to true.

```
for  $i = 2, 3, 4, \dots$ , not exceeding  $\sqrt{n}$ :  
  if  $A[i]$  is true:  
    for  $j = i^2, i^2+i, i^2+2i, i^2+3i, \dots$ , not  
      exceeding  $n$ :  
       $A[j] := \text{false}$ .
```

Output: all i such that $A[i]$ is true.

It's me, Euclid



Figure: *It's me, Euclid* [1]

References I

- [1] URL: https://upload.wikimedia.org/wikipedia/commons/3/30/Euklid-von-Alexandria_1.jpg (visited on 10/22/2018).
- [2] URL: https://en.wikipedia.org/wiki/Sieve_of_Eratosthenes (visited on 10/29/2018).
- [3] [Noam Chomsky](#). *Syntactic Structures*. The Hague: Mouton, 1957.
- [4] [William Labov](#). *Sociolinguistic Patterns*. Philadelphia: University of Pennsylvania Press, 1972.