

There Is No Largest Prime Number

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Outline

1. Motivation

1.1. The Basic Problem That We Studied

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What Are Prime Numbers?

Definition: Prime number

A *prime number* is a number that has exactly two divisors.

Example:

- 2 is prime (two divisors: 1 and 2).
- 3 is prime (two divisors: 1 and 3).
- 4 is not prime (**three** divisors: 1, 2, and 4).

There Is No Largest Prime Number

Theorem: Prime numbers

There is no largest prime number.

There Is No Largest Prime Number

Proof:

1. Suppose p were the largest prime number.

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2. Let q be the product of the first p numbers.
3. Then $q + 1$ is not divisible by any of them.
4. But $q + 1$ is greater than 1, thus divisible by some prime number not in the first p numbers.

The proof used *reductio ad absurdum*.

What's Still To Do?

- Answered Questions
 - How many primes are there?
- Open Questions
 - Is every even number the sum of two primes?

An Algorithm For Finding Prime Numbers.

FindPrimeNumbers

```
int main (void)
{
    std::vector<bool> is_prime (100, true);
    for (int i = 2; i < 100; i++)
        if (is_prime[i])
        {
            std::cout << i << "_";
            for (int j = i; j < 100; is_prime [j] = false, j
                +=i);
        }
    return 0;
}
```

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It's me, Euclid



Figure: *It's me, Euclid* [1]

References I (1)

- [1] URL: https://upload.wikimedia.org/wikipedia/commons/3/30/Euklid-von-Alexandria_1.jpg (visited on 10/22/2018).
- [2] [Noam Chomsky](#). *Syntactic Structures*. The Hague: Mouton, 1957.
- [3] [William Labov](#). *Sociolinguistic Patterns*. Philadelphia: University of Pennsylvania Press, 1972.