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The University of Manchester

PHYS20762 Computational Physics

Project 2: Numerical Integration of Differential Equations
The Damped Harmonic Oscillator

2021/2022 Session, Semester 2

Project Description

In this project, you will study numerically a spring-mass system, subjected to an external force $F(t)$:

$$mx''(t)+bx'(t)+kx(t) = F(t),$$

where $x(t)$ is the position, and the rest of the notation follows the usual conventions. Each one of *you* will be given a value of mass m and spring constant k (see the Class List).

- Start by assuming no external force (F=0). Write a program to calculate the solutions ($x(t)$ *etc)* for each of the four proposed numerical methods (Euler, improved Euler, Verlet and Euler-Cromer). Assume some initial conditions, say x = 0 m and x' = -1 m/s at t = 0 s. All four methods should be coded as functions. The code should be well laid out, structured and commented (*week 1 of project*).
- Plot *results* for a sufficient number of calculation steps to clearly show the differences between the four methods (to achieve this, choose suitable values of the damping term, b, and the time step, h); and with the analytical solution. *Note: you will need to derive or look up the analytical solution for a damped simple harmonic oscillator. The analytical solution will change depending on the values of your physical parameters and this needs to be coded up properly* (*week 1 of project*).
- Investigate the effect of the choice of time step h (how small it is) on the accuracy of different methods. Choose a measure to demonstrate the effect of the time step variation on the accuracy of the numerical methods, e.g. look at the energy. Comment on the results in your notebook and label graphs appropriately (*week 2 of project*).
- For the best method, plot solutions for the damping term, b, equal to (a) half of the critical value, (b) its critical value and (c) double the critical value (where the critical value, b_{cr}, can be found as $b_{cr}^2=4km$). Plot the results and add comments to your notebook that explain them (*week 2 of project*).
- For the best method show the effect of:
  1. sudden application of an external force after a few oscillation periods (a 'push'). Explore different situations where the force has the same or opposite sign to the instantaneous velocity and is applied in different parts of a cycle. Comment on your findings in the notebook (*week 3 of project*).
  2. forced oscillations with a sinusoidal external force with frequency different from the undamped natural frequency. Make sure your graph shows steady oscillations after the transient period. Compare with unforced oscillations (use the appropriate comments in the notebook or plots) (*week 3 of project*).
- Stretch yourself: Using the best numerical method, investigate the resonance, i.e. calculate and plot the amplitude of oscillations as a function of frequency for a range of frequencies above and below the natural frequency of the system (*week 3 of project*).

- You will also score points in this project by showing *initiative*, but this time it is up to you to come up with a way to demonstrate the initiative. Make sure that your Jupyter Notebook reads like an interactive report (*week 4 of project*).

23/01/2022 Dr Draga Pihler-Puzovic (adapted from previous course material)