digits of $F_n = \lceil \log_{10}(F_n) \rceil$

Binet $\Rightarrow$ $F_n = \lfloor \frac{\phi^n}{\sqrt{5}} \rceil \equiv k$

$$\Rightarrow \quad \left\lceil \log_{10}\left(\left\lfloor \frac{\phi^n}{\sqrt{5}} \right\rceil\right) \right\rceil \equiv k$$

$$\Rightarrow \quad k-1 \leq \log_{10} \left\lfloor \frac{\phi^n}{\sqrt{5}} \right\rceil < k$$

$$\Rightarrow \quad 10^{k-1} \leq \left\lfloor \frac{\phi^n}{\sqrt{5}} \right\rceil < 10^k$$

$$\Rightarrow \quad 10^{k-1} \leq \frac{\phi^n}{\sqrt{5}} < 10^k$$

$$\Rightarrow \quad k-1 \leq \log \frac{\phi^n}{\sqrt{5}} < k$$

$$\Rightarrow \quad \left\lceil \log \frac{\phi^n}{\sqrt{5}} \right\rceil = k$$

$$\Rightarrow \quad \left\lceil \log\left(\left\lfloor \frac{\phi^n}{\sqrt{5}} \right\rceil\right) \right\rceil = \left\lceil \log \frac{\phi^n}{\sqrt{5}} \right\rceil$$

desired digits $\equiv N = \left\lceil \log \frac{\phi^n}{\sqrt{5}} \right\rceil$

$$\Rightarrow \quad N-1 \leq \log \frac{\phi^n}{\sqrt{5}} < N$$

$$\Rightarrow \quad \frac{N-1+\frac{1}{2}\log(5)}{\log \phi} \leq n < \frac{N+\frac{1}{2}\log(5)}{\log \phi}$$

we want first integer n in this range

$$\therefore \quad \boxed{n = \left\lceil \frac{N-1+\frac{1}{2}\log(5)}{\log \phi} \right\rceil}$$