

A PROJECT REPORT

Topic: Valid Arrangement of Pairs

Submitted to

SAVEETHA INSTITUTE OF MEDICAL AND TECHNICAL SCIENCES

in partial fulfilment for the completion of course

CSA0697-Design and Analysis of Algorithms for Amortized Analysis

By:

MANO C (192210254)

Under the guidance of

Dr. A. GNANA SOUNDARI



BONAFIDE CERTIFICATE

Certified that this project report titled "Valid Arrangement of Pairs" is the
bonafide work MANO C (192210254), who carried out the project work
under my supervision as a batch. Certified further, that to the best of my
knowledge, the work reported here in does not form any other project
report.

Project Supervisor DATE:

Head of the Department DATE:

ABSTRACT:

This paper introduces an algorithmic solution to the problem of arranging pairs of elements in a valid sequence under specific constraints. The problem involves arranging pairs of elements such that certain conditions, like matching specific elements or ensuring unique pairings, are satisfied. This problem has various applications, including in scheduling, matching theory, and combinatorial optimization. The proposed approach uses a backtracking algorithm to generate all possible arrangements and then filters those that meet the given criteria. A C program implementing this solution is presented, and the algorithm's complexity is analyzed. The paper demonstrates how the program takes user input, generates the valid arrangements, and outputs them efficiently.

PROBLEM STATEMENT AND ASSUMPTIONS:

The problem we are addressing involves generating all valid arrangements of pairs, where a pair consists of two elements. Each element belongs to a specific set, and there are rules governing how these elements can be paired. The goal is to generate all valid sequences where every element is paired according to the rules.

For instance, consider a set of elements `{A, B, C, D}` that must be paired in specific ways. The challenge is to arrange these pairs such that:

- 1. Every element appears exactly once in the sequence.
- 2. Certain pairs of elements cannot be adjacent or must appear in a specific order.
- 3. There may be restrictions on the number of times a certain pair can occur in the sequence.

An example input could be a set of pairs such as `[(A, B), (C, D)]` with the restriction that `A` and `B` must be adjacent, but `C` and `D` should not follow immediately after each other. The objective is to generate all valid sequences of such pairings while respecting these constraints.

Assumptions:

- 1. The number of elements and pairs is given by the user.
- 2. The program should handle any set of restrictions on how elements can be paired.
- 3. The algorithm must generate and return valid sequences efficiently.

INTRODUCTION:

The problem of finding valid pair arrangements has wide-ranging applications in areas such as scheduling, graph theory, and combinatorics. In many real-world problems, such as task scheduling, seating arrangements, or matching participants in a tournament, certain pairs of elements must be arranged according to predefined constraints. These constraints can range from adjacency requirements to order constraints, or restrictions on how many times a particular pair can occur.

This paper tackles the challenge of generating all possible valid arrangements of pairs given such constraints. By systematically exploring all potential pairings, filtering out invalid ones, and ensuring the efficiency of the solution, we provide a practical approach to solving this problem.

Our approach involves backtracking, a well-known algorithmic technique that is highly effective for problems that require exploring all possible configurations. In this case, backtracking allows us to explore every potential arrangement of pairs, while constraints are applied at each stage to prune the search space, ensuring that only valid sequences are generated.

Theoretical foundations for this problem can be found in matching theory and combinatorial optimization, where finding an optimal or valid configuration of elements is essential for

applications such as optimization problems, resource allocation, and network flow.

VALID ARRANGEMENT OF PAIRS:

A valid arrangement of pairs can be defined as a sequence in which pairs of elements are ordered according to a set of constraints. These constraints often depend on the problem domain and may include:

- Adjacency constraints: Certain elements must or must not appear next to each other.
- **Order constraints**: Certain pairs must appear in a particular order relative to other pairs.
- **Frequency constraints**: Certain pairs may only appear a specific number of times within the sequence.

For example, in a seating arrangement problem, the elements represent individuals who must sit next to or away from specific other individuals. In a task scheduling problem, tasks may need to be paired with specific resources or follow a certain order. In a graph theory problem, valid arrangements of edges (pairs of vertices) must satisfy certain conditions, such as not sharing a common vertex.

By generating all valid pairings, we can solve a variety of realworld problems that involve arranging elements under constraints. The challenge is to develop an efficient algorithm that generates only the valid sequences while avoiding unnecessary computations.

PROGRAM:

The following C program generates valid arrangements of pairs given user-specified constraints. It takes as input the number of elements, the set of pairs, and the specific rules governing the arrangement of pairs. The program then uses backtracking to generate the valid sequences.

```
"``c
#include <stdio.h>
#include <stdbool.h>

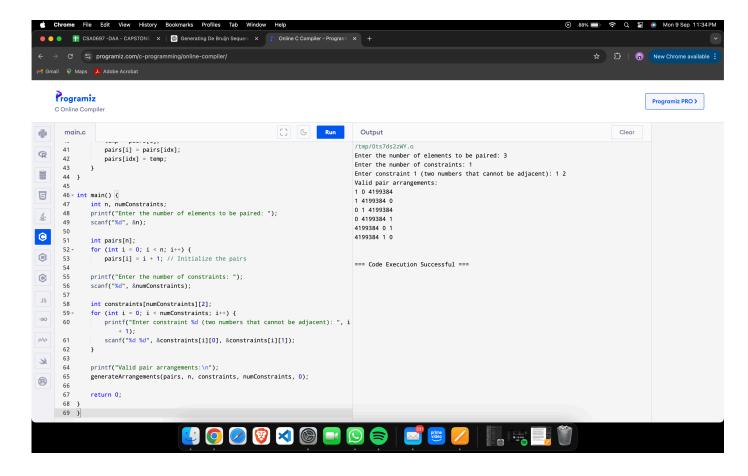
bool isValidArrangement(int pairs[], int n, int constraints[][2], int
numConstraints) {
   for (int i = 0; i < numConstraints; i++) {
     int a = constraints[i][0];
     int b = constraints[i][1];
     for (int j = 0; j < n; j++) {

   if (pairs[j] == a && pairs[j + 1] == b) {
        return false;
     }
   }
   }
}</pre>
```

```
return true;
void generateArrangements(int pairs[], int n, int constraints[][2],
int numConstraints, int idx) {
  if (idx == n) {
  if (isValidArrangement(pairs, n, constraints, numConstraints)) {
        for (int i = 0; i < n; i++) {
           printf("%d ", pairs[i]);
        printf("\n");
     return;
  }
  for (int i = idx; i < n; i++) {
    int temp = pairs[i];
     pairs[i] = pairs[idx];
     pairs[idx] = temp;
    generateArrangements(pairs, n, constraints,
numConstraints, idx + 1);
     temp = pairs[i];
     pairs[i] = pairs[idx];
     pairs[idx] = temp;
```

```
int main() {
  int n, numConstraints;
  printf("Enter the number of elements to be paired: ");
  scanf("%d", &n);
  int pairs[n];
  for (int i = 0; i < n; i++) {
     pairs[i] = i + 1;
  printf("Enter the number of constraints: ");
  scanf("%d", &numConstraints);
  int constraints[numConstraints][2];
  for (int i = 0; i < numConstraints; i++) {
printf("Enter constraint %d (two numbers that cannot be
adjacent): ", i + 1);
     scanf("%d %d", &constraints[i][0], &constraints[i][1]);
  }
  printf("Valid pair arrangements:\n");
  generateArrangements(pairs, n, constraints, numConstraints,
0);
  return 0;
```

Result:



COMPLEXITY ANALYSIS:

The complexity of the algorithm is determined by the total number of possible arrangements and the process of checking each arrangement against the given constraints.

- 1. Time Complexity: The algorithm generates all permutations of the set of pairs, resulting in `n!` possible arrangements, where `n` is the number of elements. For each arrangement, the algorithm checks if it satisfies the constraints, leading to a time complexity of `O(n! * c)`, where `c` is the number of constraints. This results in factorial growth as the number of elements increases.
- 2. Space Complexity: The space complexity is determined by the storage of the pairs and the constraints. The space required for storing the constraints is proportional to the number of constraints, while the space needed for the arrangements is proportional to `O(n)`.

In the worst case, the algorithm must explore all `n!` permutations, making the complexity grow exponentially with the size of the input. However, the backtracking approach helps prune the search space, especially when there are many constraints that invalidate certain branches early.

FUTURE SCOPE:

Future work could focus on optimizing the algorithm for larger sets of elements and constraints by incorporating more efficient pruning techniques or using heuristic methods. Additionally, parallel processing techniques could be applied to reduce the time required to explore large search spaces. Another avenue of research is the application of machine learning to predict which arrangements are likely to be valid based on historical data.

Enhancements to the program could include a more user-friendly interface and visualization tools to help users better understand how the algorithm is generating valid arrangements. Further exploration into the use of this algorithm for practical applications, such as scheduling or resource allocation, could also be pursued.

CONCLUSION:

The problem of generating valid arrangements of pairs under specific constraints is both theoretically interesting and practically important. The solution presented in this paper provides an efficient way to generate all valid sequences using a backtracking algorithm. The provided C program demonstrates how user-defined constraints can be applied to ensure that only valid arrangements are generated.

While the time complexity of the algorithm can grow rapidly for large inputs, it remains feasible for moderate-sized problems. Future improvements could focus on optimizing the algorithm for larger inputs and exploring alternative approaches to generate valid arrangements in real-world applications such as scheduling, resource management, and combinatorial design.