

[My Courses](#) / [My courses](#) / [Algorithms and Data Structures, MSc \(Spring 2023\)](#) / [Old exams](#) / [Exam 13 May 2020](#)

Started on Thursday, 20 April 2023, 15:55

State Finished

Completed on Thursday, 20 April 2023, 15:58

Time taken 2 mins 57 secs

Grade Not yet graded

Question 1

Correct

Mark 1.00 out of 1.00

Which pair of functions satisfies $f(N) \sim g(N)$?

Select one:

- ☐ a. $f(N) = 2N \log_2 N$ and $g(N) = 2N + \log_2 N$
- ☐ b. $f(N) = N \log_2 N$ and $g(N) = N + \log_2 N$
- ☐ c. $f(N) = 2N \log_2 N$ and $g(N) = 2N + N$
- ☒ d. $f(N) = 2N \log_2 N + \log_2 N$ and $g(N) = 2N \log_2 N$



Question 2

Correct

Mark 1.00 out of 1.00

For which pair of functions does it hold that $f(N)$ is $O(g(N))$?

Select one:

- ☐ a. $f(N) = N^3$ and $g(N) = (\log N)^3$
- ☐ b. $f(N) = N^3$ and $g(N) = 3N$
- ☒ c. $f(N) = 3N$ and $g(N) = N^3 \log N$
- ☐ d. $f(N) = N^3$ and $g(N) = N^2 \log N$



Question 3

Correct

Mark 1.00 out of 1.00

How many stars are printed?

```
# python
for i in range(n):
    if i % 2 == 0: # i is even
        print('*')
    else:
        print('*')
```

```
// java
for (int i = 0; i < n; i++) {
    if (i % 2 == 0) // i is even
        System.out.println("**");
    else
        System.out.println("*");
}
```

Select one:

- ☐ a. $\sim n$
- ☒ b. $\sim \frac{3}{2}n$
- ☐ c. $\sim 2n$
- ☐ d. $\sim n \log_2 n$



Question 4

Correct

Mark 1.00 out of 1.00

What is the asymptotic running time of the following piece of code?

(Give the smallest correct estimate.)

```
# python
i = 0
while i < n:
    print('*')
    i = i + 1
i = 1
while i < n:
    print('*')
    i = i * 2
```

```
// java
int i = 0;
while (i < n) {
    System.out.println("*");
    i = i + 1;
}
i = 1;
while (i < n) {
    System.out.println("*");
    i = i * 2;
}
```

Select one:

- ☐ a. $O(n \log n)$
- ☐ b. $O(n^2)$
- ☒ c. $O(n)$
- ☐ d. $O(\log n)$



Question 5

Correct

Mark 1.00 out of 1.00

Find a recurrence relation for the number A of arithmetic operations (additions, subtractions, multiplications, and divisions) performed by the following recursive method. The base case is $A(0) = 0$.

```
# python
def c(N):
    if N == 0:
        return 42
    else:
        return 2 * c(N - 1) + 2
```

```
// java
static int c(int N) {
    if (N == 0)
        return 42;
    else
        return 2 * c(N - 1) + 2;
}
```

Select one:

- ☒ a. $A(N) = A(N - 1) + 3$
- ☐ b. $A(N) = 2 * A(N - 1) + 2$
- ☐ c. $A(N) = 2 * A(N - 1) + 3$
- ☐ d. $A(N) = A(N - 1) + 2$



Question 6

Correct

Mark 1.00 out of 1.00

(hard)

Recall that $f(n)$ is $O(g(n))$ if there exists $n_0 \geq 0$ and $C > 0$ such that $f(n) \leq C \cdot g(n)$ holds for all $n \geq n_0$. I claim the following:

”

Claim. Let h be a non-decreasing real function and assume that f and g are functions so that $f(n)$ is $O(g(n))$. Then $h(f(n))$ is $O(h(g(n)))$.

Select one:

- ☐ a. False, unless h is continuous.
- ☐ b. True. Consider, for instance, $f(n) = 2 \log_2 n$, $g(n) = \log_2 n$, and $h(x) = 2^x$.
- ☐ c. True, as seen from the definition, with $n_0 = \min\{n \mid f(n) \leq g(n)\}$ and $C = f(0)$.
- ☐ d. The claim makes no sense.
- ☐ e. True, as seen from the definition, with $n_0 = 1$ and $C = 1$.
- ☐ f. Always true, no matter which function h is.
- ☒ g. False. Consider, for instance, $f(n) = 2n$, $g(n) = n$, and $h(x) = 2^x$.



Class S

The following questions are about the data structure defined by the following code:

```
# python
class S:
    def __init__(self):
        self.a = [None]
        self.n = 0

    def size(self):
        return self.n

    def push(self, value):
        if self.n == len(self.a):
            self._resize(2 * len(self.a))
        self.a[self.n] = value
        self.n += 1

    def _resize(self, capacity):
        temp = [None] * capacity
        for i in range(self.n):
            temp[i] = self.a[i]
        self.a = temp

    def peek(self):
        return self.a[self.n - 1]

    def decimate(self):
        temp = [None] * (len(self.a) // 2)
        for i in range(self.n // 2):
            temp[i] = self.a[2 * i]
        self.a = temp
        self.n = self.n // 2
```

```
public class S {
    int[] a = new int[1];
    int n = 0;

    public int size() { return n; }

    public void push(int value) {
        if (n == a.length) resize(2 * a.length);
        a[n] = value;
        n += 1;
    }

    private void resize(int capacity){
        int[] temp = new int[capacity];
        for (int i = 0; i < n; i++) temp[i] = a[i];
        a = temp;
    }

    public int peek() { return a[n - 1]; }

    public void decimate() {
        int[] temp = new int[a.length / 2];
        for (int i = 0; i < n / 2; i++) temp[i] = a[2 * i];
        a = temp;
        n = n / 2;
    }
}
```

Question 7

Correct

Mark 1.00 out of 1.00

What is the result of the following operations on a newly created object `s` of class `S`? Your answer must be exactly the string that is printed (not including the newline symbol) and not contain any additional explanation.

```
# python
s.push(3); s.push(4); s.push(5)
print(s.peak())
```

```
// java
s.push(3); s.push(4); s.push(5);
System.out.println(s.peak());
```

Answer: 5



Question 8

Correct

Mark 1.00 out of 1.00

What is the result of the following operations on a newly created object `s` of class `S`? Your answer must be exactly the string that is printed (not including the newline symbol) and not contain any additional explanation.

```
# python
s.push(3); s.push(4); s.push(5); s.push(6); s.push(7); s.push(8)
s.decimate()
print(s.peak())
```

```
// java
s.push(3); s.push(4); s.push(5); s.push(6); s.push(7); s.push(8);
s.decimate();
System.out.println(s.peak());
```

Answer: 7



Question 9

Complete

Marked out of 1.00

Draw the data structure at the end of the operations from the previous question. "Draw" here means "show in such a way that it is clear what the values of the variables are". It's up to you whether you want to draw, e.g., an array of characters as

```
['a', 'b']
```

or

```
+---+---+
| 'a' | 'b' |
+---+---+
```

or even

```
0 | 1
---+---
'a' | 'b'
```

What is important is that the contents of each array entry, their type, and its total length, are clear.

```
[3, 5, 7, null]
```

Question 10

Correct

Mark 1.00 out of 1.00

What is the running time of **decimate** on a data structure of size n ?

Select the smallest correct estimate.

Select one:

- ☐ a. $O(\log n)$
- ☐ b. Constant
- ☒ c. $O(n)$
- ☐ d. $O(n \log n)$



Question 11

Complete

Marked out of 2.00

(hard) I claim the following:

”

Claim. *The amortized cost of the operations `S.push(..)`, `S.peek()`, `S.size()`, and `S.decimate()` is constant.*

Recall that the amortized cost is the average number of array accesses that any sequence of these operations causes in the worst case, starting from an empty data structure.

If the claim is true, write “True.” followed by a concise argument.

If the claim is false, write “False.” followed by a concise argument.

True.

This is due to the worst case of the methods "resize" and "decimate" are linear running time.



Question 12

Complete

Marked out of 1.00

Add a method `determine_sum()` that returns the sum of the elements in the data structure. You can assume that all the elements are integers. You can use linear time. Don't change any other methods. Provide only the code for `determine_sum`:

```
public int determine_sum(){
    int sum = 0;
    for(i =0; i<a.length; i++){
        sum = sum + a[i];
    }
    return sum;
}
```

Question 13

Complete

Marked out of 1.00

Add a method `fast_sum()` that returns the sum of the elements in the data structure in *constant* worst-case time. You can assume that all the elements are integers. You are allowed to change other methods and add new variables, but (of course) without changing the class's original functionality or efficiency.

Provide the complete class; try to closely follow the style of the original class.

```
public class S {
    int[] a = new int[1];
    int n = 0;
    int sum = 0;

    public int size() { return n; }

    public void push(int value) {
        if (n == a.length) resize(2 * a.length);
        a[n] = value;
        n += 1;
        sum += value;
    }

    private void resize(int capacity){
        int[] temp = new int[capacity];
        for (int i = 0; i < n; i++) temp[i] = a[i];
        a = temp;
    }

    public int peek() {
        return a[n - 1];
    }

    public void decimate() {
        int[] temp = new int[a.length / 2];
        sum = 0;
        for (int i = 0; i < n / 2; i++) {
            temp[i] = a[2 * i];
            sum += a[2 * i];
        }
        a = temp;
        n = n / 2;
    }

    public int fast_sum(){
        return sum;
    }
}
```

Information

The following questions all concern the operation of standard algorithms and data structures.

The names refer to the definition and implementation found in the course book, Sedgewick and Wayne's *Algorithms* (4th ed.).

Question 14

Correct

Mark 1.00 out of 1.00

Using terminology from Union–Find, initialize 8 sites with integer names $0, \dots, 7$. For each of the following lines $p \ q$, add a connection between p and q :

```
0 1
0 1
2 3
4 5
6 5
7 6
```

What is the resulting number of connected components?

Answer: ✓

Question 15

Complete

Marked out of 1.00

Consider the key-value pairs

A 0
B 1
C 2

inserted into a search tree-based symbol table, and let's agree to draw the result like this:

```
A:0
  \
   B:1
    \
     C:2
```

Starting from an empty data structure, insert the key-value pairs

C 1
O 2
R 3
O 4
N 5
A 6

in this order and draw the result.

```
  C:1
 /   \
A:6   O:4
 /     \
N:5     R:3
```

Question 16

Complete

Marked out of 1.00

Here's a heap of the letters in **VIRUS**:

```
  V
 / \
U   R
 / \
I   S
```

Call **delMax()** and **insert('T')** in that order and draw the result in the same style.

```
  U
 / \
T   R
 / \
I   S
```

Question 17

Complete

Marked out of 1.00

Let's draw 2-3-Trees using $, | \backslash$ like this:

```

  B,D
 / | \
A  C E,F

```

Draw the 2-3 Tree resulting from inserting the letters of **VIRUS** in that order into an empty data structure.

```

  R,U
 / | \
I  S  V

```

Question 18

Correct

Mark 1.00 out of 1.00

I want to find the median of a sequence of n integers. To make things simple for this exercise, the integers are distinct and there is an odd number of them. For instance, the median of

10, 4, 1, 3, 8, 21, 7

is 7.

Of the suggestions below, what's a correct way of doing this?

Select one:

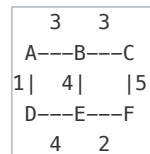
- ☐ a. Insert into a heap pq and return the element at position $pq[(n-1)/2]$.
- ☒ b. Mergesort the input and report the element in the middle of the resulting sequence. ✓
- ☐ c. Shuffle the input in linear time, partition once by the first element in the shuffled list, and report the maximum of the left part.
- ☐ d. Hash the elements (assuming constant lookup time) and report the key with the most collisions.
- ☐ e. Use two stacks for odd and even elements, respectively. Report the stacktop of the even stack.
- ☐ f. Compute the average (add the elements using a linear scan, divide by their number) and round to the nearest integer.

Question 19

Correct

Mark 1.00 out of 1.00

We are running some graph algorithm on the following undirected, weighted, 6-vertex graph:



The algorithm already added the edges **A-D**, **A-B**, and **B-C** to a tree, in that order. Now it adds **B-E**. Which algorithm are we observing?

Select one:

- ☐ a. DFS
- ☐ b. Dijkstra's
- ☐ c. Kruskal's
- ☒ d. Prim's



Island Infection

Description

The world consists of R rows, each of length C . Each position is 0 ("water"), 1 ("land"), 2 ("virus"), or 3 ("human"). The virus spreads in the obvious fashion to non-water positions with a shared border. For instance, here is the development in a small world with $R=1$ and $C=10$:

```
0101211030 -> 0102221030 -> 0102222030
```

Note that the process stops here, and the human will never be infected.

Here are a few rounds of development in a world with $R=4$ and $C=6$:

```
111001 112001 122001 222001
112000 122000 222000 222000
011103 -> 012103 -> 022203 -> 022203
101111 101111 102111 102211
```

The process will continue beyond these 4 rounds, and you can convince yourself that the human will eventually ("før eller senere") be infected.

More precisely, a tile position marked 1 or 3 turns into a 2 in round i (we call that "getting infected") exactly if any of the at most 4 adjacent tiles (to the north, south, east, or west) contains a 2 in round $i-1$. Note that infections don't spread "diagonally across corners", as shown in the bottom left position in the larger example. No position ever changes back from 2 , and no water position ever changes.

To fix notation, there are R rows, C columns, and K ones.

The goal is to determine *if* the human gets infected.

Kattis problem

There are test cases on itu.islandinfection for you to run against. The algs4 libraries are available for both Python 3 and Java.

Input

Input begins with R and C on the first line, followed by R lines of C symbols describing the starting world. There is exactly one 2 and exactly one 3 in the input. Example input for the larger example above:

```
4 6
111001
112000
011103
101111
```

Output

The output is a single integer: 1 if the human will get infected, 0 otherwise.

Test groups

There are test groups for various parameters for internal use; all you need to know is that test groups 1 and 2 have $R=1$.

Question 20

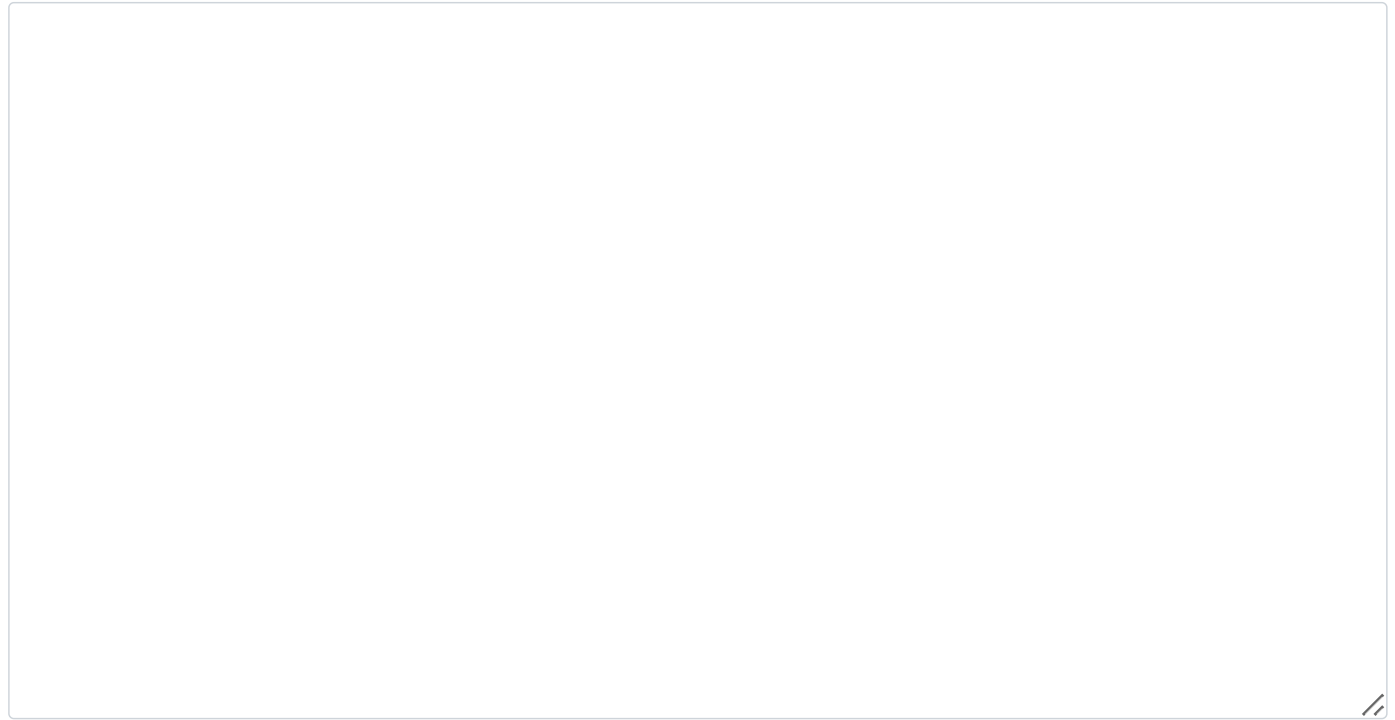
Not answered

Marked out of 2.00

Describe briefly and precisely, in prose, how to efficiently solve island infection when $R=1$.

On a separate line, state the asymptotic worst-case running time in terms of the parameters given in the problem description. Use the formulation "The running time is ..." followed by a big-Oh expression. If you need, use \wedge and $_$ for superscripts and subscripts, such as x^2 for x^2 and $\log_2 n$ for $\log_2 n$.

On a separate line, state the submission ID of an implementation of your solution on itu.kattis.com. Use the formulation "My Kattis implementation is ..." followed by an integer. *Important:* The Kattis user of the submission ID must use your ITU email.



Question 21

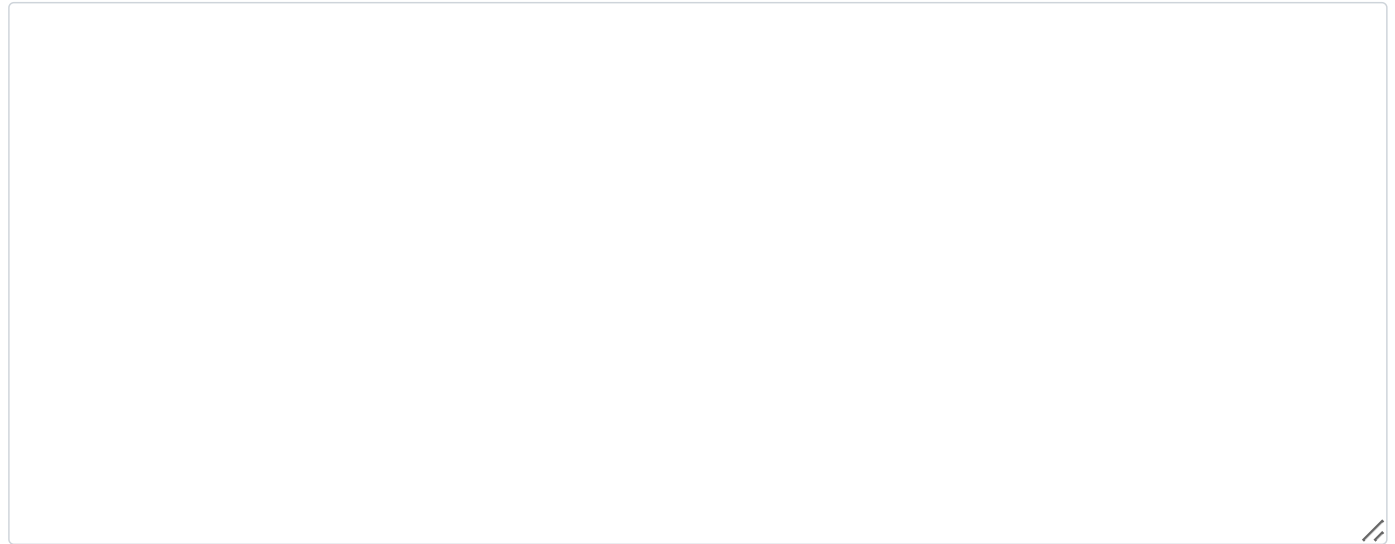
Not answered

Marked out of 3.00

Describe briefly and precisely, in prose, how to efficiently solve island infection in general.

On a separate line, state the asymptotic worst-case running time in terms of the parameters given in the problem description. Use the formulation "The running time is ..." followed by a big-Oh expression. If you need, use [^] and _{_} for superscripts and subscripts, such as x^2 for x^2 and $\log_2 n$ for $\log_2 n$.

On a separate line, state the submission ID of an implementation of your solution on itu.kattis.com. Use the formulation "My Kattis implementation is ..." followed by an integer. *Important:* The Kattis user of the submission ID must use your ITU email.



Information

This is the end of the Exam Quiz.

Remember:

1. Click "Finish Attempt ..." and click "Submit all and finish" at the bottom of the following page.
2. Then click "Finish Exam" up in the right corner after you submitted the quiz.
3. Finally uninstall the ProctorExam extension when you're done.