Assignment 1: Three- & Four-Sum Problem

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2022-09-20

1 Introduction

The Three-Sum and Four-Sum problem is a classic computer science challenge in the area of algorithm design and implementation. The objective of the Three-Sum challenge is to determine if three distinctive elements exist within a data structure that sum up to zero when added together. This is extended to one more distinctive element in the Four-Sum Challenge. The Three-Sum problem satisfies the following mathematical properties:

$$i, j, k\,(i \neq j,\, i \neq k,\, j \neq k),\;\; x_i + x_j + x_k = 0$$

This academic paper aims to report on three different implementations that solve the Three-Sum and Four-Sum problem. Accordingly, the comparison should highlight their respective performance complexity characteristics with various tables and figures.

2 Implementation

The implementation of the solutions to the Three-Sum and Four-Sum problems can be found in the code repository here: Git Repository. The time and space complexity can be seen for all implementations in the below table:

Algorithm	Implementation	Time Complexity	Space Complexity
Three-Sum	Cubic	$O(n^3)$	$O(1)$
	Quadratic	$O(n^2)$	$O(1)$
	HashMap	$O(n^2)$	$O(n)$
Four-Sum	Quartic	$O(n^4)$	$O(1)$
	Cubic	$O(n^3)$	$O(1)$
	HashMap	$O(n^2)$	$O(n^2)$

Table 1: The time and space complexity denoted for each algorithm.

2.1 Hardware and Software Specification

All of the experiments have been executed on the following equipment:

- Computer: MacBook Pro, 14-inch 2021, Apple M1 Pro, 16 GB RAM.
- OS: Mac OS X Ventura 14.0
- Software: Java: 17.0.5 2022-10-18 LTS, JUnit: 4.13.2, Python: 3.11.5, Gradle: 7.6 & Kotlin: 1.7.10

The computer performing the tests has been connected to a wall power outlet during the benchmarking of the algorithms to ensure maximum power output from the CPU.

2.2 Correctness Tests

The implementation of the four different Three-Sum and Four-Sum algorithms has been tested with various JUnit tests to verify their correctness. This includes multiple variable test cases of 0, 1 and more triplets summing to null or not.
It is also important to note, that the tests for the ThreeSumHashMapMissingComparison() method are to show the importance of the `&& j < k` condition in the HashMap implementation as it ensures x_i, x_j and x_k are at distinctive positions in the input array.

2.3 Input Data

The data generated for the tests of the different solutions of the Three-Sum and Four-Sum problem has used the following mathematical expression:

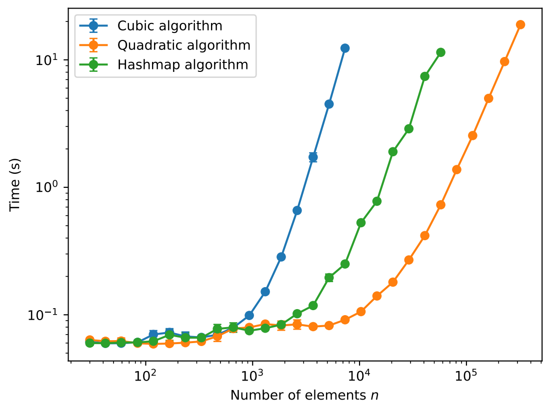
$$n_i = 30 \cdot 1.41^i \text{ , } i \text{ was set to 30.}$$

This is in order to produce an approximate increasing factor of $\sqrt{2}$ for every element in the input data. Hereby, providing a sufficient incremental scale and range of data.
Please note the minimum heap size for Java has been increased to 8 GB in the Python experiments script in order to accommodate the OutOfMemoryError exception. This error occurred around the $30 \cdot 1.41^{25}$ which typically gave an OutOfMemoryError exception on the larger input data on the HashMap implementation for the Four-Sum problem and random OutOfMemoryError exception side-effects on some arbitrary input data.

3 Experiments

3.1 Three-Sum Experiments

The three solutions to the Three-Sum problem have an increasing average execution time as the input size increases. This is the expected behaviour for each of the respective solutions which can be seen in Figure 1. The Cubic solution has the slowest execution time compared to the two other solutions in Figure 1a which also matches the time complexity in table 1 in chapter 2 "Implementation" of $O(n^3)$. This can also be deducted from the table 1b as it increases drastically once it reaches the interval of 931 to 7321 elements where after it hits the 30-second time limit. The HashMap solution performs an intermediate between the two other solutions as seen in Figure 1a. Notably, even though it has a running time of $O(n^2)$ like the Quadratic solution, it also has to create the actual HashMap data structure which takes $O(n)$ space complexity. Therefore, the Quadratic solution performs the best compared to the other solutions as it only takes space complexity of $O(1)$ enabling it to handle a bigger input of values. This can also be analysed in the table 1b.



(a) Three-Sum running time visualized

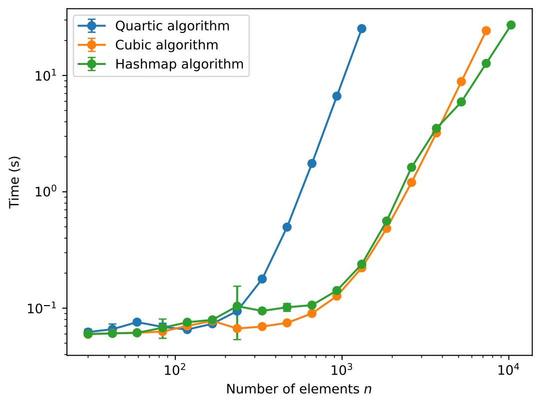
	Quadratic	HashMap	Cubic
n	Average (s)	Average (s)	Average (s)
30	0.063505	0.059988	0.061507
42	0.061963	0.059843	0.059706
59	0.062234	0.060554	0.059845
84	0.059868	0.060749	0.060798
118	0.059056	0.061999	0.070067
167	0.059508	0.0696	0.072669
235	0.060454	0.066129	0.067609
332	0.061804	0.066078	0.066479
468	0.067846	0.077522	0.069739
660	0.077953	0.079692	0.07929
931	0.079583	0.075003	0.098821
1313	0.084588	0.078429	0.15172
1852	0.082476	0.083722	0.284409
2611	0.084126	0.101965	0.658235
3682	0.080772	0.118165	1.724355
5192	0.082349	0.195614	4.496418
7321	0.091411	0.25034	12.346113
10323	0.105925	0.527608	-
14556	0.140508	0.777544	-
20525	0.18033	1.902266	-
28940	0.269289	2.878875	-
40805	0.417973	7.414434	-
57536	0.729295	11.447228	-
81126	1.377281	-	-
114387	2.546927	-	-
161286	4.983893	-	-
227414	9.691578	-	-
320654	18.88899	-	-

(b) Average running time for each Three-Sum implementation.

Figure 1: Figure and table for Three-Sum implementations

3.2 Four-Sum Experiments

The Four-Sum problem increases the time complexity across all of the solutions as it adds one more element to the challenge. This can be seen in the smaller amount of elements which has been tested against each solution in figure 2. The Quartic solution performs the worst as it uses a time complexity of $O(n^4)$ as also highlighted in table 1 in chapter 2 "Implementation". The average running time table 2b also highlights the larger running time for the Quartic solution as it only reaches 1313 elements before it encounters the 30-second time limit. Interestingly, the HashMap and Cubic solutions perform rather similarly when looking at the figure 2a even though they have different time complexity of $O(n^2)$ and $O(n^3)$, respectively. The HashMap solution manages to perform a benchmark with the largest input of elements for the Four-Sum problem of 10323 elements. However, as mentioned in Chapter 2.3 "Input Data" the minimum heap size for Java was increased to handle the larger input data because it threw an OutOfMemoryError exception. This is due to the space complexity that the HashMap solution utilises of $O(n^2)$ to create the actual HashMap data structure. Thus, presents the important argument that time complexity cannot solely be the determinator to analyse any algorithm as the space complexity also has to be considered.
The Cubic solution performs rather similarly to the HashMap solution on the smaller data input as seen in table 2b until it reaches 5192 elements where the impact of its time complexity of $O(n^3)$ takes effect. Accordingly, the Cubic solution would arguably become slower intensively if the time limit was increased hereby diverging further from the HashMap solution.



(a) Four-Sum running time visualised

	HashMap	Cubic	Quartic
n	Average (s)	Average (s)	Average (s)
30	0.06003	0.059621	0.062323
42	0.060648	0.060944	0.065905
59	0.061434	0.061416	0.07585
84	0.067853	0.062963	0.068966
118	0.075652	0.070409	0.065506
167	0.079377	0.077767	0.07342
235	0.104045	0.06685	0.094405
332	0.094721	0.069491	0.178064
468	0.101786	0.074724	0.497435
660	0.106156	0.090133	1.752576
931	0.141698	0.12693	6.648049
1313	0.238788	0.222236	25.285927
1852	0.561357	0.484803	-
2611	1.623853	1.204709	-
3682	3.511084	3.209423	-
5192	5.92609	8.863057	-
7321	12.712385	24.261229	-
10323	27.181545	-	-

(b) Average running time for each Four-Sum implementation.

Figure 2: Figure and table for Four-Sum implementations