AU311, Pattern Recognition Tutorial (Fall 2020) **Homework: 2. Classification**

2. Classification

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Problem 1

There have been many variants of SVM for different purpose. The following is called ν-SVM which can controls the ratio of support vectors. The primal formulation of ν-SVM is given as

$$\begin{aligned} \min_{w,\rho,\xi} \quad & \frac{1}{2}\|w\|_2^2 - \nu\rho + \frac{1}{m}\sum_{i=1}^{m}\xi_i \\ \text{s.t.} \quad & y_i(w^T x_i + b) \geq \rho - \xi_i \\ & \rho \geq 0, \xi_i \geq 0, \forall i = 1, 2, \dots m. \end{aligned} \tag{1}$$

Please derive its dual problem and discuss the meaning of ν.

Answer

The ν-SVM in primal space :

$$\begin{aligned} \min_{w,\rho,\xi} \quad & \frac{1}{2}\|w\|_2^2 - \nu\rho + \frac{1}{m}\sum_{i=1}^{m}\xi_i \\ \text{s.t.} \quad & y_i(w^T x_i + b) \geq \rho - \xi_i \\ & \rho \geq 0, \xi_i \geq 0, \forall i = 1, 2, \ldots m. \end{aligned} \tag{2}$$

The corresponding Lagrange:

$$L = \frac{1}{2}\|w\|_2^2 - \nu\rho + \frac{1}{m}\sum_{i=1}^{m}\xi_i + \sum_{i=1}^{m}\alpha_i(\rho - \xi_i - y_i(w^T x_i + b)) - \sum_{i=1}^{m}\mu_i\xi_i - \lambda\rho \tag{3}$$

The KKT condition:

$$\frac{\partial L}{\partial w} = 0 \Rightarrow w - \sum_{i=1}^{m}\alpha_i y_i x_i = 0 \tag{4}$$

$$\frac{\partial L}{\partial \xi_i} = 0 \Rightarrow \frac{1}{m} - \alpha_i - \mu_i = 0 \tag{5}$$

$$\frac{\partial L}{\partial \rho} = 0 \Rightarrow \sum_{i=1}^{m}\alpha_i - v = \lambda \tag{6}$$

Then we can substitute (4)(5)(6) into Lagrange and get the SVM in dual space:

$$\min_{\alpha} \quad \frac{1}{2}\sum_i\sum_j \alpha_i\alpha_j x_i^T x_i y_i y_j \tag{7}$$

$$\text{s.t.} \quad 0 \leq \alpha_i \leq \frac{1}{m} \tag{8}$$

$$\sum_i \alpha_i \geq \nu \tag{9}$$

For the ν in ν-SVM,I thick we can consider it as the substitute of C in normal SVM.In a sense, the main different between the two method is the parameterization of ν.

What's more, ν is the upper bound of the proportion of sample points that are misclassified, and ν is the lower bound of the proportion of support vectors in the sample points.