Using ML for Propensity Scores

- Propensity score: $\Pr(W_i = w_i | X_i = x_i) = p_w(x)$
- Propensity score weighting and matching is common in treatment effects literature
  - "Selection on observables assumption": $Y_i(w) \perp W_i | X_i$
  - See Imbens and Rubin (2015) for extensive review
- Propensity score estimation is a pure prediction problem
  - Machine learning literature applies propensity score weighting: e.g. Beygelzimer and Langford (2009), Dudick, Langford and Li (2011)
  - Properties or tradeoffs in selection among ML approaches
    - Estimated propensity scores work better than true propensity score (Hirano, Imbens and Ridder (2003)), so optimizing for out of sample prediction is not the best path
    - Various papers consider tradeoffs, no clear answer, but classification trees and random forests do well

Using ML for Model Specification under Selection on Observables

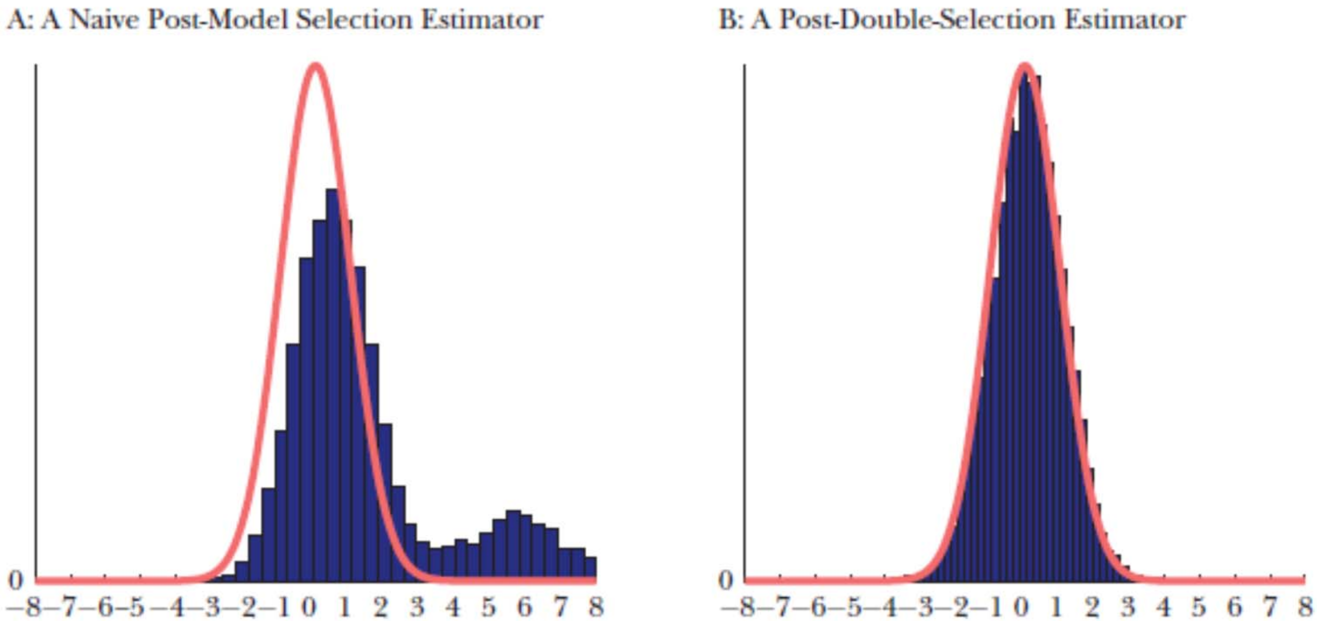
- A heuristic:
  - If you control richly for covariates, can estimate treatment effect ignoring endogeneity
  - This motivates regressions with rich specification
- Naïve approach motivated by heuristic using LASSO
  - Keep the treatment variable out of the model selection by not penalizing it
  - Use LASSO to select the rest of the model specification
  - Problem:
    - Treatment variable is forced in, and some covariates will have coefficients forced to zero.
    - Treatment effect coefficient will pick up those effects and will thus be biased.
    - See Belloni, Chernozhukov and Hansen JEP 2014 for an accessible discussion of this
- Better approach:
  - Need to do variable selection via LASSO for the selection equation and outcome equation separately
  - Use LASSO with the union of variables selected
  - Belloni, Chernozhukov & Hansen (2013) show that this works under some assumptions, including constant treatment effects

Single Equation versus Two-Equation LASSO estimation

Figure 1

The "Double Selection" Approach to Estimation and Inference versus a Naive Approach: A Simulation from Belloni, Chernozhukov, and Hansen (forthcoming)

(distributions of estimators from each approach)



Source: Belloni, Chernozhukov, and Hansen (forthcoming).

Notes: The left panel shows the sampling distribution of the estimator of α based on the first naive procedure described in this section: applying LASSO to the equation $y_i = d_i + x_i' \theta_y + r_{yi} + \zeta_i$ while forcing the treatment variable to remain in the model by excluding α from the LASSO penalty. The right panel shows the sampling distribution of the "double selection" estimator (see text for details) as in Belloni, Chernozhukov, and Hansen (forthcoming). The distributions are given for centered and studentized quantities.

Using ML for the first stage of IV

- IV assumptions
  - $Y_i(w) \perp Z_i | X_i$
  - See Imbens and Rubin (2015) for extensive review
- First stage estimation: instrument selection and functional form
  - $E[W_i | Z_i, X_i]$ This is a prediction problem where interpretability is less important
  - Variety of methods available
  - Belloni, Chernozhukov, and Hansen (2010); Belloni, Chen, Chernozhukov and Hansen (2012) proposed LASSO
    - Under some conditions, second stage inference occurs as usual
    - Key: second-stage is immune to misspecification in the first stage

Open Questions and Future Directions

- Heterogeneous treatment effects in LASSO
- Beyond LASSO
  - What can be learned from statistics literature and treatment effect literature about best possible methods for selection on observables and IV cases?
  - Are there methods that avoid biases in LASSO, that preserve interpretability and ability to do inference?
    - Only very recently are there any results about normality of random forest estimators (e.g. Wager 2014)
  - What are best-performing methods?
  - More general conditions where standard inference is valid, or corrections to standard inference

Machine Learning Methods for Estimating Heterogeneous Causal Effects

Athey and Imbens, 2015
http://arxiv.org/abs/1504.01132

Motivation I: Experiments and Data-Mining

- Concerns about ex-post "data-mining"
  - In medicine, scholars required to pre-specify analysis plan
  - In economic field experiments, calls for similar protocols
- But how is researcher to predict all forms of heterogeneity in an environment with many covariates?
- Goal:
  - Allow researcher to specify set of potential covariates
  - Data-driven search for heterogeneity in causal effects with valid standard errors

Motivation II: Treatment Effect Heterogeneity for Policy

- Estimate of treatment effect heterogeneity needed for optimal decision-making
- This paper focuses on estimating treatment effect as function of attributes directly, not optimized for choosing optimal policy in a given setting
- This "structural" function can be used in future decision-making by policy-makers without the need for customized analysis

Preview

- Distinguish between causal effects and attributes
- Estimate treatment effect heterogeneity:
  - Introduce estimation approaches that combine ML prediction & causal inference tools
- Introduce and analyze new cross-validation approaches for causal inference
- Inference on estimated treatment effects in subpopulations
  - Enabling post-experiment data-mining

Regression Trees for Prediction

Data

- Outcomes Y_i, attributes X_i.
- Support of X_i is $\mathcal{X}$.
- Have training sample with independent obs.
- Want to predict on new sample
- Ex: Predict how many clicks a link will receive if placed in the first position on a particular search query

Build a "tree":

- Partition of $\mathcal{X}$ into "leaves" $\mathcal{X}_j$
- Predict Y conditional on realization of X in each region $\mathcal{X}_j$ using the sample mean in that region
- Go through variables and leaves and decide whether and where to split leaves (creating a finer partition) using in-sample goodness of fit criterion
- Select tree complexity using cross-validation based on prediction quality

Regression Trees for Prediction: Components

1. Model and Estimation
   A. Model type: Tree structure
   B. Estimator $\hat{Y}_i$: sample mean of Y_i within leaf
   C. Set of candidate estimators C: correspond to different specifications of how tree is split
2. Criterion function (for fixed tuning parameter λ)
   A. In-sample Goodness-of-fit function:

      $$Q^{is} = \text{-MSE (Mean Squared Error)} = -\frac{1}{N}\sum_{i=1}^{N}\left(\hat{Y}_i - Y_i\right)^2$$

   A. Structure and use of criterion
      i. Criterion: $Q^{crit} = Q^{is} - \lambda \times \#$ leaves
      ii. Select member of set of candidate estimators that maximizes Q^{crit}, given λ
3. Cross-validation approach
   A. Approach: Cross-validation on grid of tuning parameters. Select tuning parameter λ with highest Out-of-sample Goodness-of-Fit Q^{os}.
   B. Out-of-sample Goodness-of-fit function: Q^{os} = -MSE

Using Trees to Estimate Causal Effects

Model:

$$Y_i = Y_i(W_i) = \begin{cases} Y_i(1) & if \;\; W_i = 1, \\ Y_i(0) & otherwise. \end{cases}$$

- Suppose random assignment of W_i
- Want to predict individual *i*'s treatment effect
  - $\tau_i = Y_i(1) - Y_i(0)$
  - This is not observed for any individual
  - Not clear how to apply standard machine learning tools
- Let

$$\mu(w, x) = \mathbb{E}[Y_i | W_i = w, X_i = x]$$
$$\tau(x) = \mu(1, x) - \mu(0, x)$$

Using Trees to Estimate Causal Effects

$$\mu(w,x) = \mathbb{E}[Y_i | W_i = w, X_i = x]$$
$$\tau(x) = \mu(1,x) - \mu(0,x)$$

- Approach 1: Analyze two groups separately
  - Estimate $\hat{\mu}(1,x)$ using dataset where $W_i = 1$
  - Estimate $\hat{\mu}(0,x)$ using dataset where $W_i = 0$
  - Use propensity score weighting (PSW) if needed
  - Do within-group cross-validation to choose tuning parameters
  - Construct prediction using
    $$\hat{\mu}(1,x) - \hat{\mu}(0,x)$$
- Approach 2: Estimate $\mu(w,x)$ using tree including both covariates
  - Include PS as attribute if needed
  - Choose tuning parameters as usual
  - Construct prediction using
    $$\hat{\mu}(1,x) - \hat{\mu}(0,x)$$
  - Estimate is zero for *x* where tree does not split on *w*

- Observations
  - Estimation and cross-validation not optimized for goal
  - Lots of segments in Approach 1: combining two distinct ways to partition the data

- Problems with these approaches
  1. Approaches not tailored to the goal of estimating treatment effects
  2. How do you evaluate goodness of fit for tree splitting and cross-validation?
     - $\tau_i = Y_i(1) - Y_i(0)$ is not observed and thus you don't have ground truth for any unit

Proposed Approach 3: Transform the Outcome

- Suppose we have 50-50 randomization of treatment/control
  - Let $Y_i^* = \begin{cases} 2Y_i & if\ W_i = 1 \\ -2Y_i & if\ W_i = 0 \end{cases}$
  - Then $E[Y_i^*] = 2 \cdot \left(\frac{1}{2}E[Y_i(1)] - \frac{1}{2}E[Y_i(0)]\right) = E[\tau_i]$

- Suppose treatment with probability p_i
  - Let $Y_i^* = \frac{W_i - p}{p(1-p)} Y_i = \begin{cases} \frac{1}{p}Y_i & if\ W_i = 1 \\ -\frac{1}{1-p}Y_i & if\ W_i = 0 \end{cases}$
  - Then $E[Y_i^*] = \left(p\frac{1}{p}E[Y_i(1)] - (1-p)\frac{1}{1-p}E[Y_i(0)]\right) = E[\tau_i]$

- Selection on observables or stratified experiment
  - Let $Y_i^* = \frac{W_i - p(X_i)}{p(X_i)(1-p(X_i))} Y_i$
  - Estimate $\hat{p}(x)$ using traditional methods

Causal Trees: Approach 3 (Conventional Tree, Transformed Outcome)

1. Model and Estimation
   A. Model type: Tree structure
   B. Estimator $\hat{\tau}_i^*$: sample mean of Y_i^* within leaf
   C. Set of candidate estimators *C*: correspond to different specifications of how tree is split
2. Criterion function (for fixed tuning parameter λ)
   A. In-sample Goodness-of-fit function:

      Q^{is} = -MSE (Mean Squared Error) $= -\frac{1}{N}\sum_{i=1}^{N}(\hat{\tau}_i^* - Y_i^*)^2$

   A. Structure and use of criterion
      i. Criterion: $Q^{crit} = Q^{is} - \lambda \times$ # leaves
      ii. Select member of set of candidate estimators that maximizes Q^{crit}, given λ
3. Cross-validation approach
   A. Approach: Cross-validation on grid of tuning parameters. Select tuning parameter λ with highest Out-of-sample Goodness-of-Fit Q^{os}.
   B. Out-of-sample Goodness-of-fit function: Q^{os} = -MSE

Critique of Proposed Approach 3: Transform the Outcome

$$Y_i^* = \frac{W_i - p}{p(1-p)} Y_i = \begin{cases} \frac{1}{p} Y_i & if\ W_i = 1 \\ -\frac{1}{1-p} Y_i & if\ W_i = 0 \end{cases}$$

- Within a leaf, sample average of Y_i^* is not most efficient estimator of treatment effect
  - The proportion of treated units within the leaf is not the same as the overall sample proportion
  - This weights treatment group mean by p, not by actual fraction of treated in leaf
- This motivates modification:
  - Use sample average treatment effect in the leaf (average of treated less average of control)

Critique of Proposed Approach 3: Transform the Outcome

- Use of transformed outcome creates noise in tree splitting and in out-of-sample cross-validation
  - In-sample:
    - Use variance of prediction for in-sample goodness of fit
    - For an estimator guaranteed to be unbiased in sample (such as sample average treatment effect), the variance of the estimator measures predictive power
  - Out of sample:
    - Use a matching estimator to construct estimate of ground truth treatment effect out of sample. Single match minimizes bias. $\hat{\tau}_i^{m,os}$
    - Matching estimator and transformed outcome both unbiased in large sample when perfect matching can be found. But transformed outcome introduces variance due to weighting factor; matching estimator controls for predictable component of variance
    - $Var[Y_i^*] \approx \frac{1}{p} Var[Y_i(1)] + \frac{1}{1-p} Var[Y_i(0)]$
    - $Var[\hat{\tau}_i^m] \approx Var[Y_i(1)|X] + Var[Y_i(0)|X]$

Causal Trees

1. **Model and Estimation**
   A. Model type: Tree structure
   B. Estimator $\hat{\tau}_i^{CT}$: sample average treatment effect within leaf
   C. Set of candidate estimators C: correspond to different specifications of how tree is split
2. **Criterion function (for fixed tuning parameter λ)**
   A. In-sample Goodness-of-fit function:

$$Q^{is} = -\frac{1}{N}\sum_{i=1}^{N}(\hat{\tau}_i^{CT})^2$$

   A. Structure and use of criterion
      i. Criterion: $Q^{crit} = Q^{is} - \lambda \times$ # leaves
      ii. Select member of set of candidate estimators that maximizes Q^{crit}, given λ
3. **Cross-validation approach**
   A. Approach: Cross-validation on grid of tuning parameters. Select tuning parameter λ with highest Out-of-sample Goodness-of-Fit Q^{os}.
   B. Out-of-sample Goodness-of-fit function: $Q^{os} = -\frac{1}{N}\sum_{i=1}^{N}(\hat{\tau}_i^{CT} - \hat{\tau}_i^{m,os})^2$

Comparing "Standard" and Causal Approaches

- They will be more similar
  - If treatment effects and levels are highly correlated
- Two-tree approach
  - Will do poorly if there is a lot of heterogeneity in levels that is unrelated to treatment effects; trees are much too complex without predicting treatment effects
  - Will do well in certain specific circumstances, e.g.
    - Control outcomes constant in covariates
    - Treatment outcomes vary with covariates
- Transformed outcome
  - Will do badly if there is a lot of observable heterogeneity in outcomes, and if treatment probabilities are unbalanced or have high variance
  - Variance in criterion functions leads to trees that are too simple as they criterion erroneously finds a lack of fit

- How to compare approaches?

1. Oracle (simulations)
2. Transformed outcome goodness of fit
3. Matching goodness of fit

Inference

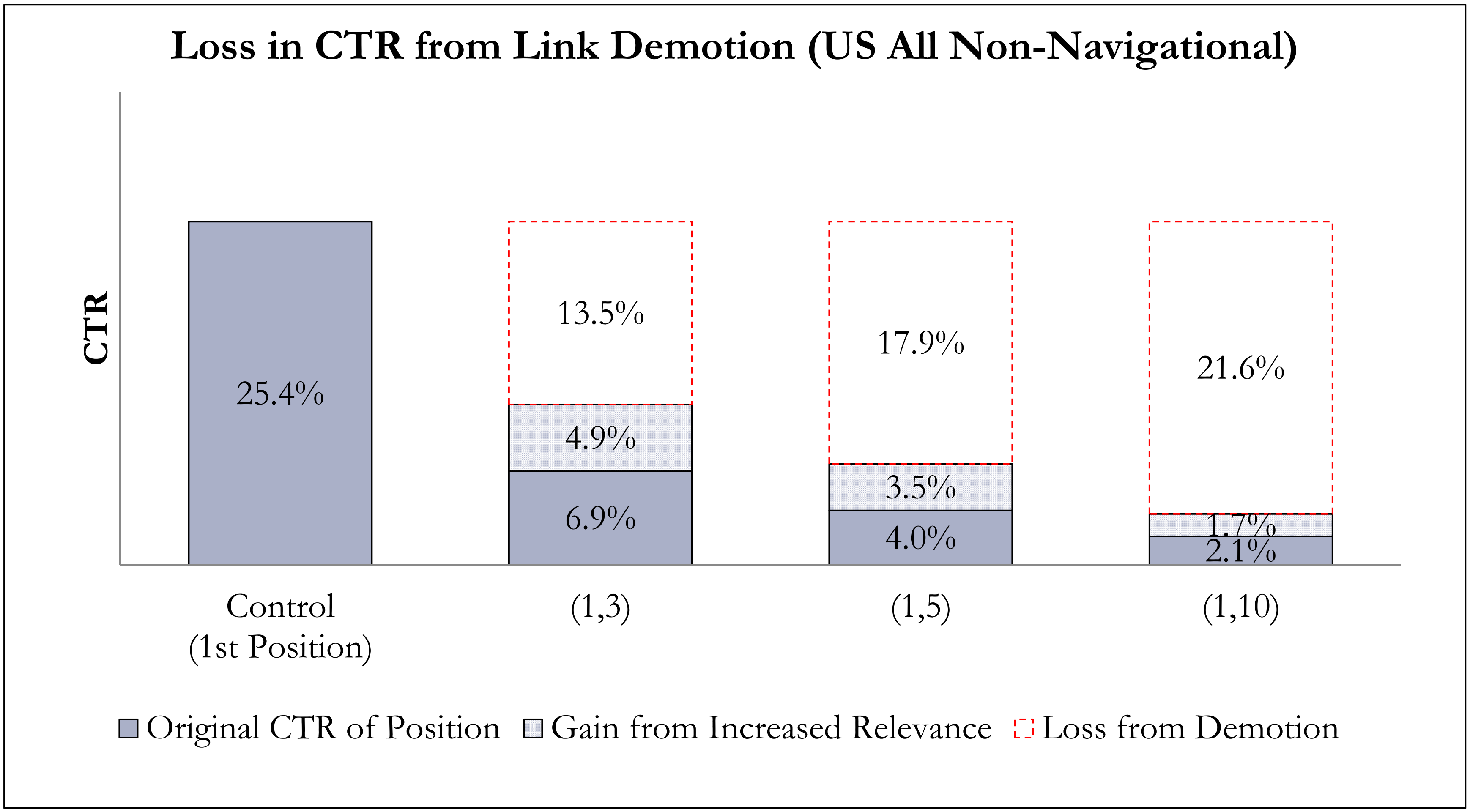
- Conventional wisdom is that trees are bad for inference
  - The *predictions* are discontinuous in tree and not normally distributed. But we are not interested in inference on tree structure.

- Attractive feature of trees:
  - Can easily separate tree construction from treatment effect estimation
  - Tree constructed on training sample is independent of sampling variation in the test sample
  - Holding tree from training sample fixed, can use standard methods to conduct inference within each leaf of the tree on test sample
    - Can use any valid method for treatment effect estimation, not just the methods used in training
  - For observational studies, literature (e.g. Hirano, Imbens and Ridder (2003)) requires additional conditions for inference
    - E.g. leaf size must grow with population
- Future research: extend ideas beyond trees
  - Bias arises in LASSO as well in the absence of strong sparsity conditions.
  - Expand on insight: separate datasets used for model selection and estimation of prediction for a given model yields valid inference. See, e.g., Denil, Matheson, and de Freitas (2014) on random forests.

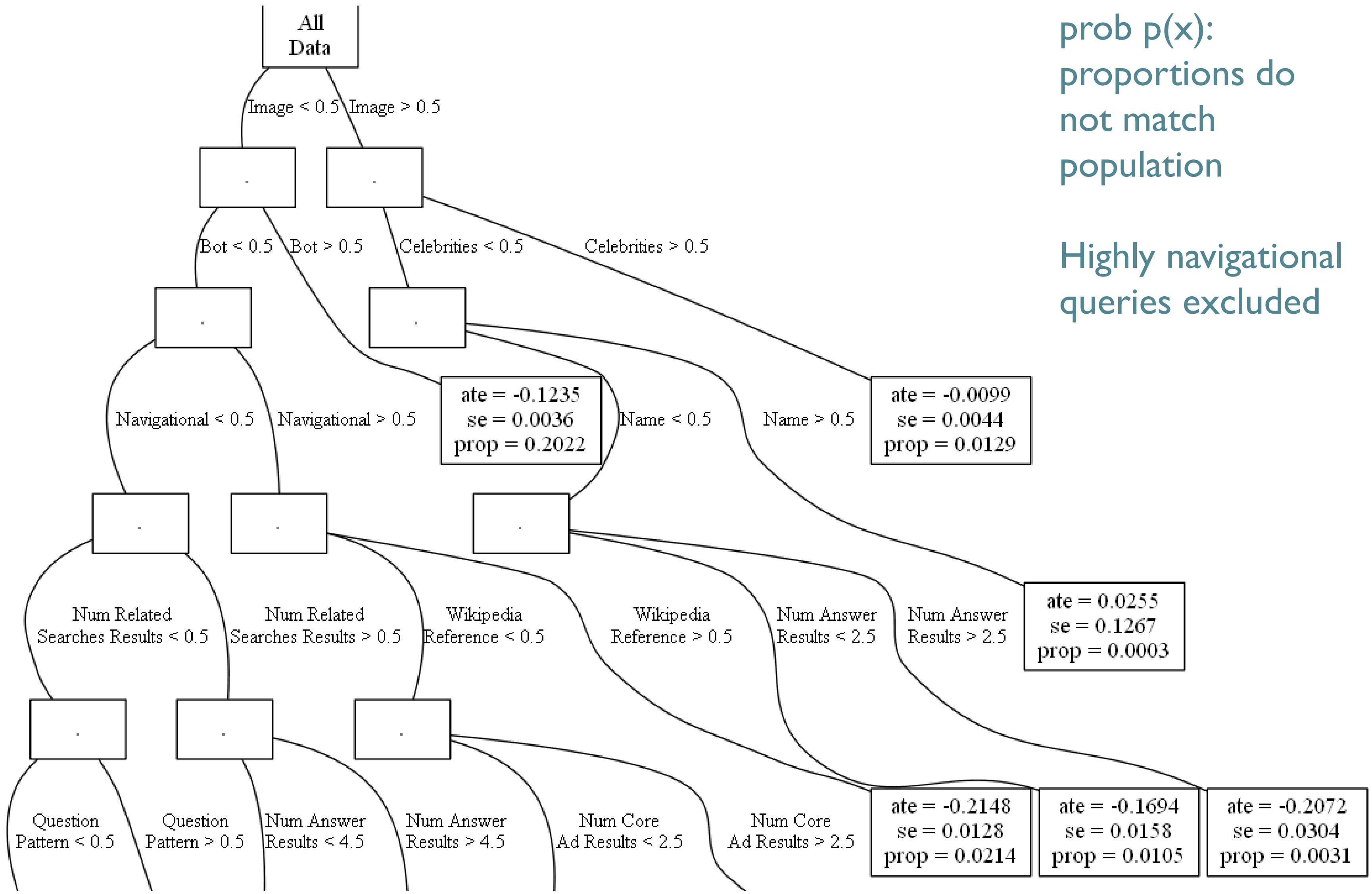
Problem: Treatment Effect Heterogeneity in Estimating Position Effects in Search

- Queries highly heterogeneous
  - Tens of millions of unique search phrases each month
  - Query mix changes month to month for a variety of reasons
  - Behavior conditional on query is fairly stable
- Desire for segments.
  - Want to understand heterogeneity and make decisions based on it
  - "Tune" algorithms separately by segment
  - Want to predict outcomes if query mix changes
    - For example, bring on new syndication partner with more queries of a certain type

Relevance v. Position

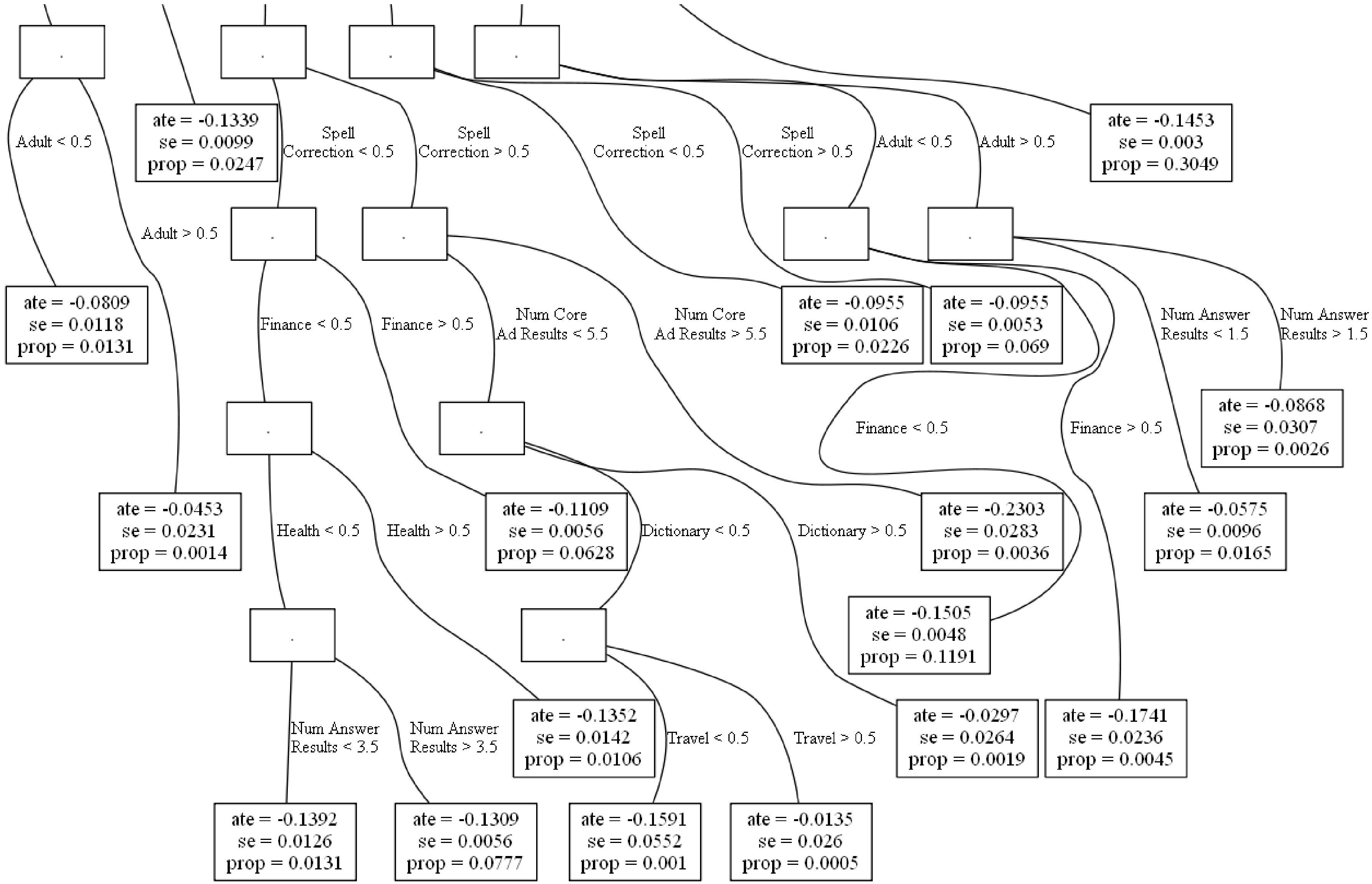


Search Experiment Tree: Effect of Demoting Top Link (Test Sample Effects)



Some data excluded with prob p(x): proportions do not match population

Highly navigational queries excluded



Adult < 0.5
Adult > 0.5
ate = -0.1339
se = 0.0099
prop = 0.0247
Spell Correction < 0.5
Spell Correction > 0.5
Spell Correction < 0.5
Spell Correction > 0.5
Adult < 0.5
Adult > 0.5
ate = -0.1453
se = 0.003
prop = 0.3049
ate = -0.0809
se = 0.0118
prop = 0.0131
Finance < 0.5
Finance > 0.5
Num Core Ad Results < 5.5
Num Core Ad Results > 5.5
ate = -0.0955
se = 0.0106
prop = 0.0226
ate = -0.0955
se = 0.0053
prop = 0.069
Num Answer Results < 1.5
Num Answer Results > 1.5
ate = -0.0868
se = 0.0307
prop = 0.0026
Finance < 0.5
Finance > 0.5
ate = -0.0453
se = 0.0231
prop = 0.0014
Health < 0.5
Health > 0.5
ate = -0.1109
se = 0.0056
prop = 0.0628
Dictionary < 0.5
Dictionary > 0.5
ate = -0.2303
se = 0.0283
prop = 0.0036
ate = -0.0575
se = 0.0096
prop = 0.0165
ate = -0.1505
se = 0.0048
prop = 0.1191
Num Answer Results < 3.5
Num Answer Results > 3.5
ate = -0.1352
se = 0.0142
prop = 0.0106
Travel < 0.5
Travel > 0.5
ate = -0.0297
se = 0.0264
prop = 0.0019
ate = -0.1741
se = 0.0236
prop = 0.0045
ate = -0.1392
se = 0.0126
prop = 0.0131
ate = -0.1309
se = 0.0056
prop = 0.0777
ate = -0.1591
se = 0.0552
prop = 0.001
ate = -0.0135
se = 0.026
prop = 0.0005

Use Test Sample for Segment Means & Std Errors to Avoid Bias

Variance of estimated treatment effects in training sample 2.5 times that in test sample

Test Sample			Training Sample		
Treatment Effect	Standard Error	Proportion	Treatment Effect	Standard Error	Proportion
-0.124	0.004	0.202	-0.124	0.004	0.202
-0.134	0.010	0.025	-0.135	0.010	0.024
-0.010	0.004	0.013	-0.007	0.004	0.013
-0.215	0.013	0.021	-0.247	0.013	0.022
-0.145	0.003	0.305	-0.148	0.003	0.304
-0.111	0.006	0.063	-0.110	0.006	0.064
-0.230	0.028	0.004	-0.268	0.028	0.004
-0.058	0.010	0.017	-0.032	0.010	0.017
-0.087	0.031	0.003	-0.056	0.029	0.003
-0.151	0.005	0.119	-0.169	0.005	0.119
-0.174	0.024	0.005	-0.168	0.024	0.005
0.026	0.127	0.000	0.286	0.124	0.000
-0.030	0.026	0.002	-0.009	0.025	0.002
-0.135	0.014	0.011	-0.114	0.015	0.010
-0.159	0.055	0.001	-0.143	0.053	0.001
-0.014	0.026	0.001	0.008	0.050	0.000
-0.081	0.012	0.013	-0.050	0.012	0.013
-0.045	0.023	0.001	-0.045	0.021	0.001
-0.169	0.016	0.011	-0.200	0.016	0.011
-0.207	0.030	0.003	-0.279	0.031	0.003
-0.096	0.011	0.023	-0.083	0.011	0.022
-0.096	0.005	0.069	-0.096	0.005	0.070
-0.139	0.013	0.013	-0.159	0.013	0.013
-0.131	0.006	0.078	-0.128	0.006	0.078

Conclusions

- Key to approach
  - Distinguish between causal and predictive parts of model
- "Best of Both Worlds"
  - Combining very well established tools from different literatures
  - Systematic model selection with many covariates
  - Optimized for problem of causal effects
    - In terms of tradeoff between granular prediction and overfitting
  - With valid inference
  - Easy to communicate method and interpret results
    - Output is a partition of sample, treatment effects and standard errors
- Important application
  - Data-mining for heterogeneous effects in randomized experiments

Literature

Approaches in the spirit of single tree/2 trees

- Beygelzimer and Langford (2009)
  - Analogous to "two trees" approach with multiple treatments; construct optimal policy
- Foster, Tailor, Ruberg(2011)
  - Estimate $\mu(w,x)$ using random forests, define $\hat{\tau}_i = \hat{\mu}(1, X_i) - \hat{\mu}(0, X_i)$, and do trees on $\hat{\tau}_i$.
- Imai and Ratkovic (2013)
  - In context of randomized experiment, estimate $\mu(w,x)$ using lasso type methods, and then $\hat{\tau}(x) = \hat{\mu}(1,x) - \hat{\mu}(0,x)$.

Transformed outcomes or covariates

- Tibshirani et al (2014); Weisberg and Pontes (2015) in regression/LASSO setting
- Dudick, Langford, and Li (2011) and Beygelzimer and Langford (2009) for optimal policy
- Don't highlight or address limitations of transformed outcomes in estimation & criteria

Estimating treatment effects directly at leaves of trees

- Su, Tsai, Wang, Nickerson, Li (2009)
  - Do regular tree, but split if the t-stat for the treatment effect difference is large, rather than when the change in prediction error is large.
- Zeileis, Hothorn, and Hornick (2005)
  - "Model-based recursive partitioning": estimate a model at the leaves of a tree. In-sample splits based on prediction error, do not focus on out of sample cross-validation for tuning.

None of these explore cross-validation based on treatment effect.

Extensions (Work in Progress)

- Alternatives for cross-validation criteria
- Optimizing selection on observables case
  - What is the best way to estimate propensity score for this application?
  - Alternatives to propensity score weighting

Heterogeneity: Instrumental Variables

Setup

- Binary treatment, binary instrument case
  - Instrument Z_i

$$\Delta^{Y_i}(S) = E[Y_i|Z_i = 1, X_i \in S] - E[Y_i|Z_i = 0, X_i \in S]$$

$$\Delta^{W_i}(S) = \Pr(W_i = 1|Z_i = 1, X_i \in S) - \Pr(W_i = 1|Z_i = 0, X_i \in S)$$

  - LATE estimator for $x_i \in S$ is:

$$LATE(S) = \frac{\Delta^{Y_i}(S)}{\Delta^{W_i}(S)}$$

- LATE heterogeneity issues
  - Tree model: want numerator and denominator on same set S, to get LATE for units w/ x_i in S.
    - Set of units shifted by instrument varies with x
  - Average of LATE estimators over all regions is NOT equal to the LATE for the population

Proposed Method

- Estimation & Inference:
  - Estimate numerator and denominator simultaneously with a single tree model
  - Inference on a distinct sample, can do separately within each leaf
- In-sample goodness of fit:
  - Prediction accuracy for both components separately
  - Weight two components
  - In-sample criterion penalizes complexity, as usual
- Cross-validation
  - Bias paramount: is my tree overfit?
  - Criterion: For each unit, find closest neighbors and estimate LATE (e.g. kernel)
  - Two parameters instead of one: complexity and relative weight to numerator
    - Can also estimate an approximation for optimal weights

Next Steps

- Application to demand elasticities (Amazon, eBay, advertisers)
  - Can apply methods with regression at the bottom of the tree—some modifications needed

- More broadly, richer demand models at scale

- Lessons for economists
  - ML methods work better in economic problems when customized for economic goals
  - Not hard to customize methods and modify them
  - Not just possible but fairly easy to be systematic about model selection

Optimal Decision Rules as a Classification Problem

Optimal Decision Policies

Decision Policies v. Treatment Effects

- In some applications, the goal is directly to estimate an optimal decision policy
- There may be a large number of alternatives
- Decisions are made immediately

Examples

- Offers or marketing to users
  - Advertisements
  - Mailings or emails
- Online web page optimization
- Customized prices

Model

- Outcome Y_i incorporates both costs and benefits
  - If cost is known, e.g. a mailing, define outcome to include cost
- Treatment W_i is multi-valued
- Attributes X_i observed
- Maintain selection on observables assumption:

$$Y_i(w) \perp W_i | X_i$$

- Propensity score:

$$\Pr(W_i = w_i | X_i = x_i) = p_w(x)$$

- Optimal policy:

$$\pi^*(x) = \text{argmax}_w E[Y_i(w)|X_i=x]$$

Examples/interpretation

- Marketing/web site design
  - Outcome is voting, purchase, a click, etc.
  - Treatment is the offer
  - Past user behavior used to define attributes
  - Selection on observables justified by past experimentation (or real-time experimentation)
- Personalized medicine
  - Treatment plan as a function of individual characteristics

Learning Policy Functions

- ML Literature:
  - Contextual bandits (e.g., John Langford), associative reinforcement learning, associative bandits, learning with partial feedback, bandits with side information, partial label problem
- Cost-sensitive classification
  - Classifiers (e.g. logit, CART, SVM) = discrete choice models
  - Weight observations by observation-specific weight
  - Objective function: minimize classification error
- The policy problem
  - Minimize regret from suboptimal policy ("policy regret")

$$R(\pi) = E[Y_i(\pi^*(X_i)) - Y_i(\pi(X_i))]$$

- For 2-choice case:
  - Procedure with transformed outcome:
    - Train classifier as if obs. treatment is optimal:

      (features, choice, weight)= $(X_i, W_i, \frac{Y_i}{\hat{p}(x)})$.
    - Estimated classifier is a possible policy
  - Result:
    - The loss from the cost-weighted classifier (misclassification error minimization)

      is **the same** in expectation
    - as the policy regret
  - Intuition
    - The expected value of the weights conditional on x_i,w_i is $E[Y_i(w_i)|X_i=x_i]$
- Implication
  - Use off-the-shelf classifier to learn optimal policies, e.g. logit, CART, SVM
  - Literature considers extensions to multi-valued treatments (tree of binary classifiers)

Weighted Classification Trees: Transformed Outcome Approach

Interpretation

- This is analog of using the transformed outcome approach for heterogeneous treatment effects, but for learning optimal policies
- The in-sample and out-of-sample criteria have high variance, and don't adjust for actual sample proportions or predictable variation in outcomes as function of X

Comparing two policies: Loss A – Loss B with N^T units

Policy A & B Rec's	Treated Units	Control Units	Expected value of sum
Region S_{10} A: Treat B: No Treat	$-\frac{1}{N^T}\sum\frac{Y_i}{p}$	$\frac{1}{N^T}\sum\frac{Y_i}{1-p}$	$-\Pr(X_i \in S_{10})E[\tau_i \mid X_i \in S_{10}]$
Region S_{01} A: No Treat B: Treat	$\frac{1}{N^T}\sum\frac{Y_i}{p}$	$-\frac{1}{N^T}\sum\frac{Y_i}{1-p}$	$\Pr(X_i \in S_{01})E[\tau_i \mid X_i \in S_{01}]$

Alternative: Causal Policy Tree (Athey-Imbens 2015-WIP)

- Improve by same logic that causal trees improved on transformed outcome
- In sample
  - Estimate treatment effect $\hat{\tau}(x)$within leaves using actual proportion treated in leaves (average treated outcomes – average control outcomes)
  - Split based on classification costs, using sample average treatment effect to estimate cost within a leaf. Equivalent to modifying TO to adjust for correct leaf treatment proportions.

Comparing two policies: Loss A – Loss B with N^T units

Policy A & B Rec's	Treated Units	Control Units	Expected value of sum
Region S_{10} A: Treat B: No Treat	$-\frac{1}{N^T}\sum\hat{\tau}(S_{10})$	$-\frac{1}{N^T}\sum\hat{\tau}(S_{10})$	$-\Pr(X_i \in S_{10})E[\tau_i \| X_i \in S_{10}]$
Region S_{01} A: No Treat B: Treat	$\frac{1}{N^T}\sum\hat{\tau}(S_{01})$	$\frac{1}{N^T}\sum\hat{\tau}(S_{10})$	$\Pr(X_i \in S_{01})E[\tau_i \| X_i \in S_{01}]$

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Region S_{10} A: Treat B: No Treat	$-\frac{1}{N^T}\sum\hat{\tau}(S_{10})$	$-\frac{1}{N^T}\sum\hat{\tau}(S_{10})$	$-\Pr(X_i \in S_{10})E[\tau_i \mid X_i \in S_{10}]$
Region S_{01} A: No Treat B: Treat	$\frac{1}{N^T}\sum\hat{\tau}(S_{01})$	$\frac{1}{N^T}\sum\hat{\tau}(S_{01})$	$\Pr(X_i \in S_{01})E[\tau_i \mid X_i \in S_{01}]$

Alternative: Causal Policy Tree (Athey-Imbens 2015-WIP)

Alternative approach for cross-validation criterion

- Use nearest-neighbor matching to estimate treatment effect for test observations
- Categorize as misclassified at the individual unit level
- Loss function is mis-classification error for misclassified units
- When comparing two policies (classifiers), for unit where policies have different recommendations, the difference in loss function is the estimated treatment effect for that unit
- If sample is large s.t. close matches can be found, this criteria may be lower variance than transformed outcome in small samples, and thus a better fit is obtained
  - Two alternative policies may be similar except in small regions, so small sample properties are key

Alternative: Causal Policy Tree (Athey-Imbens 2015-WIP)

Inference

- Not considered in contextual bandit literature
- As before, split the sample for estimating classification tree and for conducting inference.
- Within each leaf:
  - Optimal policy is determined by sign of estimated treatment effect.
  - Simply use conventional test of one-sided hypothesis that estimate of sign of treatment effect is wrong.

Estimating Policy Functions: Summary

- Existing ML Literature
  - With minimal coding, apply pre-packaged classification tree using transformed outcome as weights
  - Resulting tree is an estimate of optimal policy.
- For multiple treatment options
  - Follow contextual bandit literature and construct tree of binary classifiers. Race pairs of alternatives, then race winners against each other on successively smaller subsets of the covariate space.
  - Also propose further transformations (offsets)
- Our observation:
  - Can do inference on hold-out sample taking tree as fixed.
- Can improve on pre-packaged tree with causal policy tree.
- Also relates to a small literature in economics:
  - e.g. Hirano & Porter (2009), Batthacharya & Dupas (2012)

Other Related Topics

- Online learning
  - See e.g. Langford
- Explore/exploit
  - Auer et al '95, more recent work by Langford et al
- Doubly robust estimation
  - Elad Hazan, Satyen Kale, Better Algorithms for Benign Bandits, SODA 2009.
  - David Chan, Rong Ge, Ori Gershony, Tim Hesterberg, Diane Lambert, Evaluating Online Ad Campaigns in a Pipeline: Causal Models at Scale, KDD 2010
  - Dudick, Langford, and Li (2011)

Inference

Inference for Causal Effects v. Attributes: Abadie, Athey, Imbens & Wooldridge (2014)

Approach

- Formally define a population of interest and how sampling occurs
- Define an estimand that answers the economic question using these objects (effects versus attributes)
- Specify: "What data are missing, and how is the difference between your estimator and the estimand uncertain?"
  - Given data on 50 states from 2003, we know with certainty the difference in average income between coast and interior
  - Although we could contemplate using data from 2003 to estimate the 2004, difference this depends on serial correlation within states, no direct info in cross-section

Application to Effects v. Attributes in Regression Models

- Sampling: Sample/population does not go to zero, finite sample
- Causal effects have missing data: don't observe both treatments for any unit
- Huber-White robust standard errors are conservative but best feasible estimate for causal effects
- Standard errors on fixed attributes may be much smaller if sample is large relative to population
  - Conventional approaches take into account sampling variance that should not be there

Robustness

Robustness of Causal Estimates
Athey and Imbens (AER P&P, 2015)

- General nonlinear models/estimation methods
- Causal effect is defined as a function of model parameters
  - Simple case with binary treatment, effect is $\tau_i = Y_i(1) - Y_i(0)$
- Consider other variables/features as "attributes"
- Proposed metric for robustness:
  - Use a series of "tree" models to partition the sample by attributes
    - Simple case: take each attribute one by one
  - Re-estimate model within each partition
  - For each tree, calculate overall sample average effect as a weighted average of effects within each partition
  - This yields a set of sample average effects
  - Propose the standard deviation of effects as robustness measure

Robustness of Causal Estimates
Athey and Imbens (AER P&P, 2015)

- Four Applications:
  - Randomly assigned training program
  - Treated individuals with artificial control group from census data (Lalonde)
  - Lottery data (Imbens, Rubin & Sacerdote (2001))
  - Regression of earnings on education from NLSY
- Findings
  - Robustness measure better for randomized experiments, worse in observational studies

Lalonde data

Variable	Exper $\hat{\theta}_B$	Exper s.e.	Non-exper $\hat{\theta}_B$	Non-exper s.e.
Base Model	1.67	(0.67)	1.07	(0.63)
$\hat{\sigma}_\theta$		[0.13]		[2.13]
Split on	est.	$\chi^2(10)$	est.	$\chi^2(10)$
treatment	1.58	22.8	-4.26	45.0
age	1.55	10.1	1.97	144.5
black	1.71	11.4	1.38	26.2
hispanic	1.61	7.2	1.54	59.7
married	1.87	11.0	1.06	10.4
education	1.77	14.7	1.25	74.5
nodegree	1.33	18.6	1.77	46.1
re74	1.64	11.2	0.58	71.0
re75	1.63	8.9	-0.88	94.1
u74	1.64	11.2	-2.44	88.6
u75	1.71	7.0	-0.83	81.9

Comparing Robustness

Variation of $\hat{\theta}$ over Model Specifications

Lalonde

	Exper	Non-exp	IRS	NLS
Est	1.67	1.07	-0.44	0.059
(s.e.)	(0.67)	(0.63)	(0.012)	(0.010)
σ_θ	[0.13]	[2.13]	[0.10]	[0.004]
ratio	0.20	3.38	0.83	0.40

Robustness Metrics: Desiderata

- Invariant to:
  - Scaling of explanatory variables
  - Transformations of vector of explanatory variables
  - Adding irrelevant variables
- Each member model must be somehow distinct to create variance, yet we want to allow lots of interactions
  - Need to add lots of rich but different models
- Well-grounded way to weight models
  - This paper had equal weighting

Robustness Metrics: Work In Progress

Std Deviation versus Worst-Case

- Desire for set of alternative models that grows richer
  - New additions are similar to previous ones, lower std dev
- Standard dev metric:
  - Need to weight models to put more weight on *distinct* alternative models
- "Worst-case" or "bounds":
  - Find the lowest and highest parameter estimates from a set of models
  - Ok to add more models that are similar to existing ones.
  - But worst-case is very sensitive to outliers—how do you rule out "bad" models?

Theoretical underpinnings

- Subjective versus objective uncertainty
  - Subjective uncertainty: correct model
  - Objective uncertainty: distribution of model estimates given correct model
- What are the preferences of the "decision-maker" who values robustness?
  - "Variational preferences"
    - "Worst-case" in set of possible beliefs, allow for a "cost" of beliefs that captures beliefs that are "less likely." (see Strzalecki, 2011)
  - Our approach for exog. covariate case:
    - Convex cost to models that perform poorly out of sample from a predictive perspective
- Good model
  - Low obj. uncertainty: tightly estimated
  - Other models that predict outcomes well also yield similar parameter estimates

Conclusions on Robustness

- ML inspires us to be both systematic and pragmatic
- Big data gives us more choices in model selection and ability to evaluate alternatives
- Maybe we can finally make progress on robustness