A simple t-test

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1 Statement

Every claims that the flags of the world are cut along a golden ratio, that is

$$\frac{width}{height} = \frac{1}{2} + \frac{\sqrt{5}}{2} \approx 1.618$$

... in average. Evaluate this statement with a signficance of $\alpha = 0.05$.

2 Data

We fetch the data from `https://en.wikipedia.org/wiki/Gallery_of_sovereign_state_flags` (TODO: cite) using a little web-page script as follows:

```
f = (img) => {
   let countryName = img.parentElement.parentElement.parentElement.parentElement.innerText;
   if (countryName.startsWith("Flag of ")) countryName = countryName.substring(8);
   let txt = '"' + countryName + '","' + img.width/img.height + '"\n';
   allText = allText + txt; console.log(txt)
}
var imgs = jQuery(".mw-parser-output img")
var allText = ""; imgs.each((n)=>{f(imgs[n]);})
```

This obtains the values in the neighbour CSV file with $n = 206$ flags and an average of about 1.67 and standard deviation of 0.257.

3 Interpretation

We make the hypothesis that the aspect ratio of the flags R is distributed along a normal distribution $R \sim \mathcal{N}(1.618, \sigma)$. Where σ is a standard-deviation that we do not know of.

We want to evaluate if the sample that we have follows a normal distribution with the same average with significance 0.01 and formulate the hypotheses:

- H_0: $\mu_R \approx 1.618$

- H_1: $\mu_R \not\approx 1.618$

We thus want to apply a two-tailed t-test TODO cite. e.g. [**?**, theorem 7.4.2] which will allow us to reject the H_0 in favour of $H1$ if $t \leqslant -t_{\alpha/2,n-1}$ or $t \geqslant t_{\alpha/2,n-1}$

4 Theory

The student-t-theorem tells us that, considering $R_1, ..., R_n$ independent normal random variables, the following variable follows a Student-t-distribution of $n-1$ degrees of freedom:

$$T_{n-1} = \frac{\overline{R} - \mu}{S_{R_1,\dots R_n}/\sqrt{n}}$$

where $S_{R_1,\dots R_n}$ is the sample standard deviation. TODO cite

Applying the t-test means testing if the probability of having this sample is at least $1 - \alpha$.

5 Computation

$$t \approx \frac{1.67 - 1.618}{0.257 \cdot \sqrt{206}} \approx 0.014$$

$$t_{\alpha/2,n-1} \approx 1.960$$

So t is by far not less than $-t_{\alpha/2,n-1}$ or bigger than $t_{\alpha/2,n-1}$.

6 Conclusion

We are not able to conclude that the flags follow a different distribution.

References