

Characteristic values of distributions

Let X random variable with pdf $f(x)$ and cdf $F(x)$

1) Measures of central tendency: mean, median, mode

Mean: $E[X] = \int_{-\infty}^{\infty} x f(x) dx$

average of x weighted by the pdf

Median: $F^{-1}(0.5)$ (*)

50% quantile, middle value that separates lower half from upper half of distribution

Mode: $\arg\max_x f(x)$

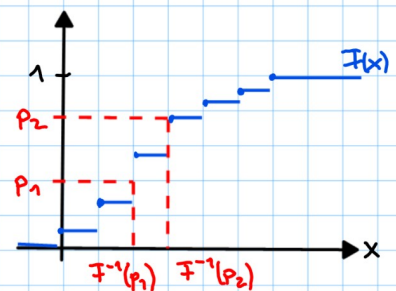
maximum value of pdf (need not be unique)

(*) On the quantile function F^{-1} :

• If F is invertible (strictly monotone increasing), then F^{-1} is the inverse function.

• Otherwise:

$$F^{-1}(p) = \inf \{x \in \mathbb{R} : p \leq F(x)\}$$



2) Measures of deviation from mean:

Variance $v = E[(X - \mu)^2]$

Standard deviation $\sigma = \sqrt{v}$ \leftarrow has same scale / units as X and μ

3) Measure of asymmetry:

Skewness $E\left[\left(\frac{X - \mu}{\sigma}\right)^3\right]$

4) Measure of heaviness of tails:

Kurtosis $E\left[\left(\frac{X - \mu}{\sigma}\right)^4\right]$

Standard normal distribution has kurtosis 3 (**)

\rightarrow Excess Kurtosis = Kurtosis - 3 \leftarrow

(**) i.e. $\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} x^4 \exp\left(-\frac{x^2}{2}\right) dx = 3$