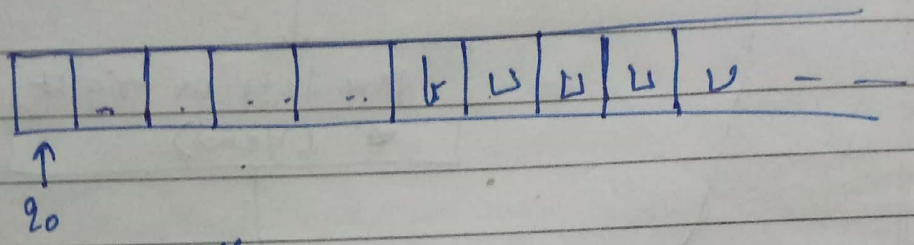
Q) Construct a T.M over $\Sigma = \{a, b\}$ for set of strings ending with 'b'.

A) $L = \{ b, ab, bb, aab, \dots \}$

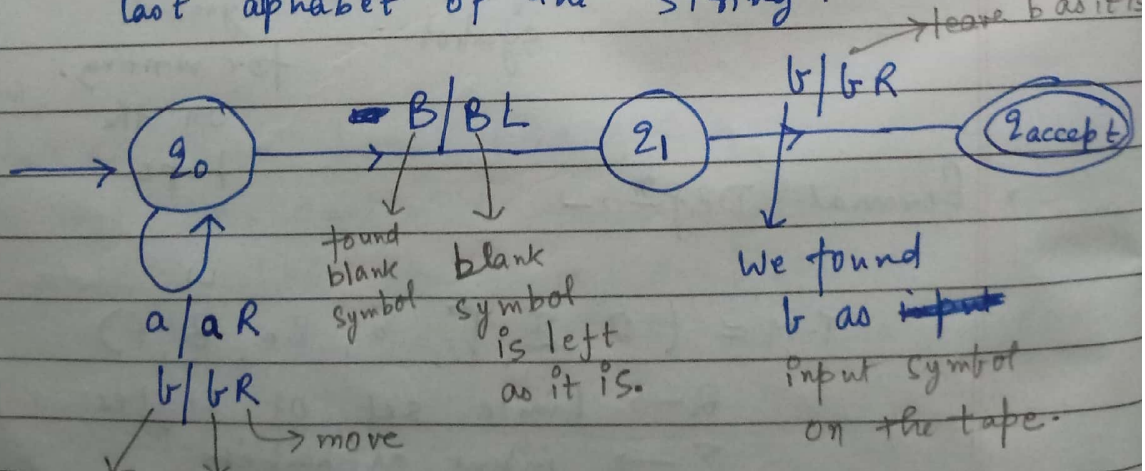
↙ Enumerating the language



q_0

Left-most cell is focused with the head with initial state as q_0).

Approach :- We need to move the head till we reach a blank symbol to check the last alphabet of the string.



* Checking the running of the T.M.

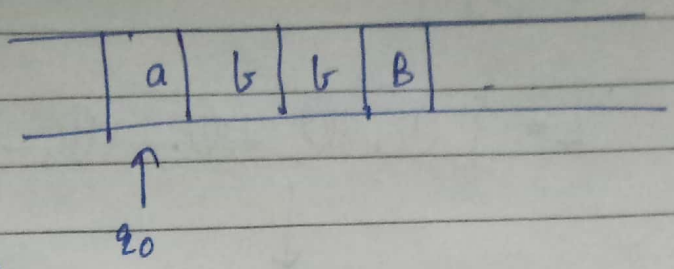
Eg :-

$w = abb$

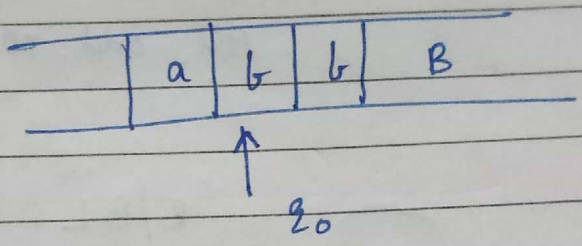
Inst. Desc

Tape Diagram :-

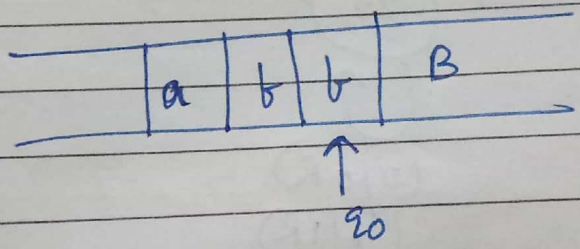
Each step denotes the instantaneous tape structure of T.M.



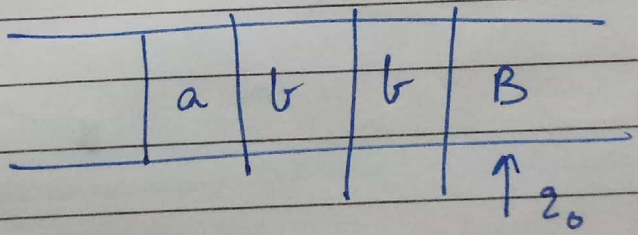
$q_0 abbB$



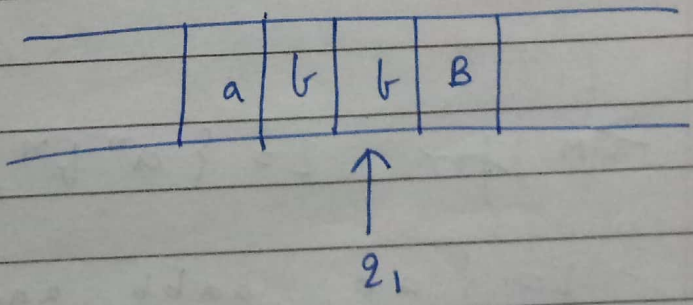
$a q_0 bbB$



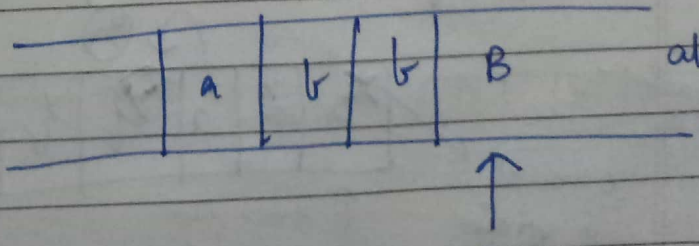
$ab q_0 bB$



$abb q_0 B$



$ab q_1 bB$



$abb q_{accept} B$

Formally, it can be represented as Instantaneous description.

Hence,

$q_0 abbB \vdash aq_0 bbB$

$\vdash^* abb q_0 B \vdash ab q_1 bB$

$\vdash abb q_{accept} B$

To club together multiple steps. (or skip steps).

Denoting transitions using instantaneous descrip^n.

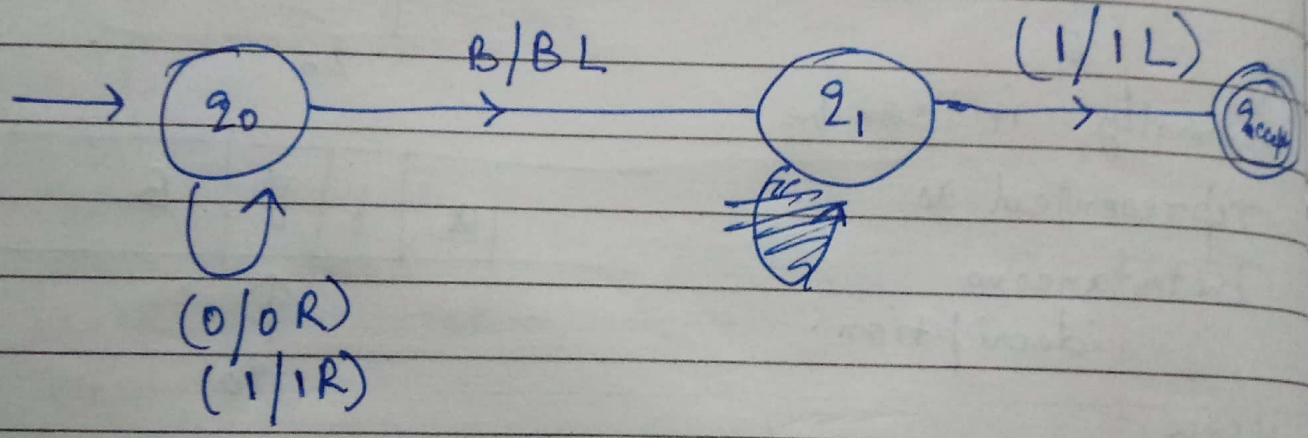
Q) T.M to accept every odd number if given in binary form.

A) $\Sigma = \{0, 1\}$

$L = \{ 1, 11, 101, 111, \ldots \}$

↓

All odd no.s end with 1.



$\delta(q_1, 0) = (q_{Reject}, 0, R)$

↓

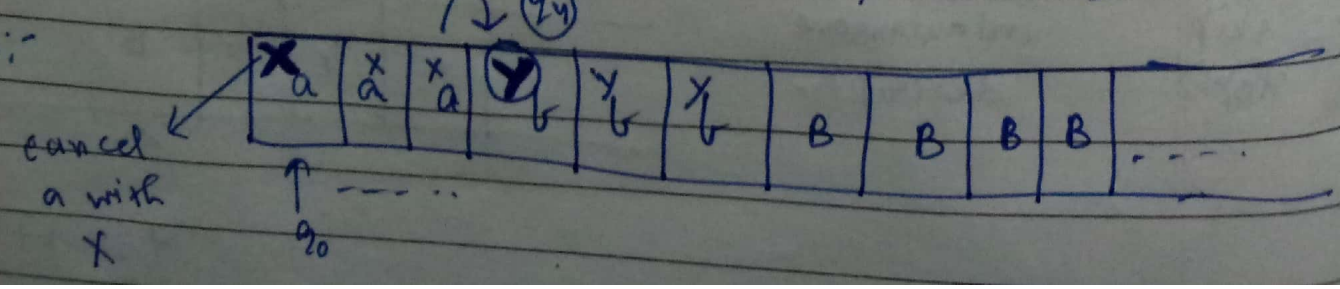
(for rejection)

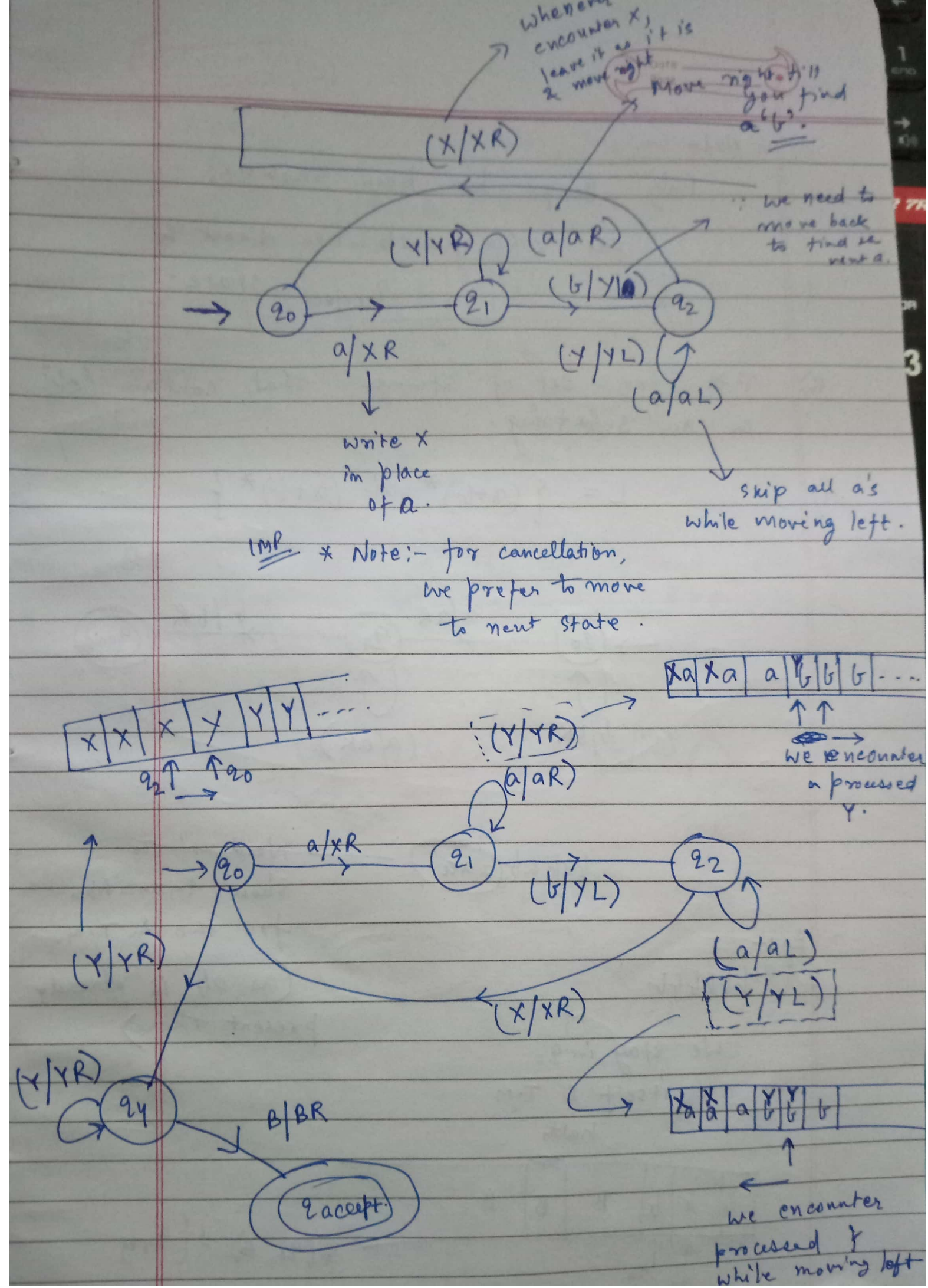
Q) TM for $L = \{ a^n b^n, n \geq 1 \}$

A)

$L = \{ ab, aabb, aaabbb, \ldots \}$

Approach:-

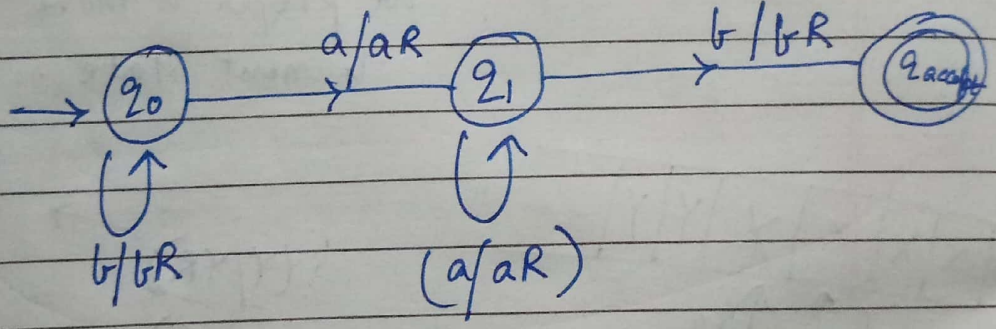




* Note :-
Only accept has been shown.
All other possibilities direct to q_{reject} state.

Q) T.M for set of strings that contain 'ab' as a substring.

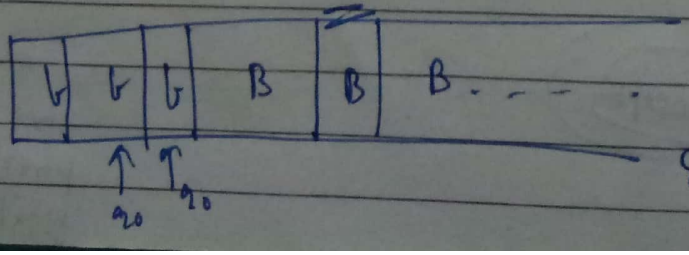
A) $L = \{(a+b)^* ab (a+b)^*\}$



bbb ab aaa → Need not to show transition for this in T.M. (as ab is already present there)

Eg :-
bbbbbb

We stay in q_0 itself & T.M halts

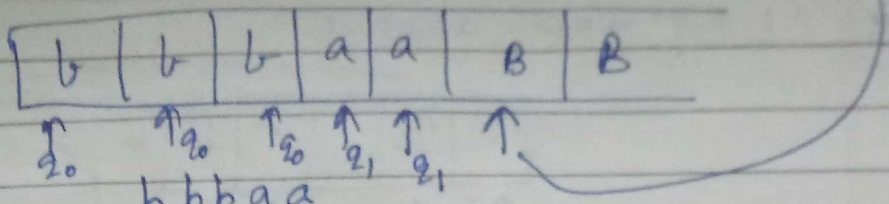


$\delta(q_0, B) = (q_{rej}$

Also,

Eg:-

$\delta(q_1, B) = q_{rej}$

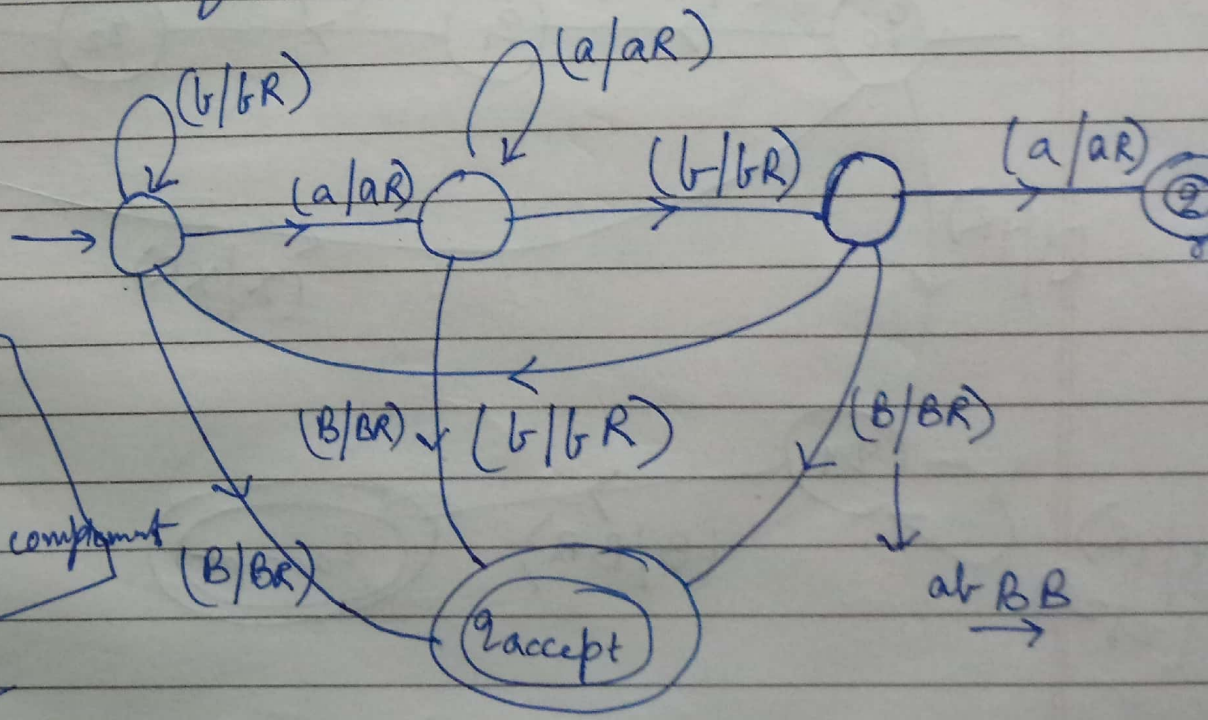


Important rejection states might be shown in STD.

(B/BR)

Q) T.M for set of all strings over $\Sigma = \{a, b\}$ that do not contain 'aba' as a substring.

A)



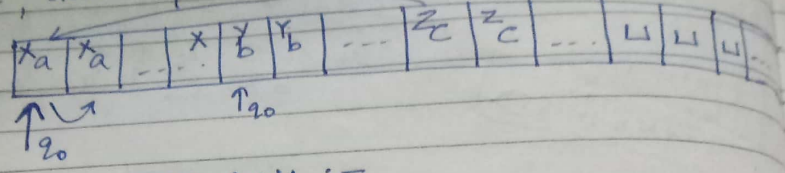
Taking that contain 'aba' then, we will ~~~~ complement it

* For taking complement in a T.M. just turn q_{acc} to q_{rej}

IMP

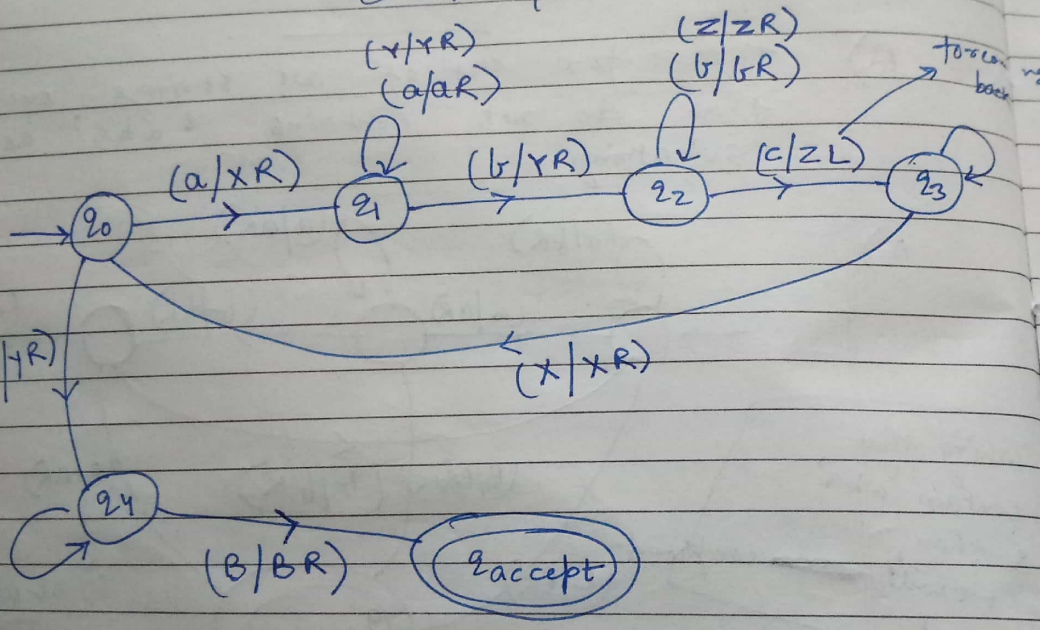
Q) TM for $L = \{ a^n b^n c^n \mid n \geq 1 \}$ over $\Sigma = \{a, b, c\}$.

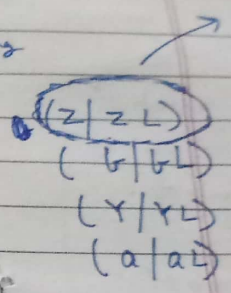
A) $L = \{ abc, aabbcc, \ldots \}$



*APPROACH :-

Cancel a, then b, then c.





→ Writing rejection states separately.

Extra b's/c's are present after all a's are cancelled ← $\delta(q_4, b) = (q_{rej}, b, R)$

← $\delta(q_4, c) = (q_{rej}, c, R)$

Q) Construct a TM that accepts a palindrome over $\Sigma = \{a, b\}$.

A) Eg :- abaaba → odd length palindrome
abba → even length palindrome

$\{waw^R / w \in (a+b)^*\}$

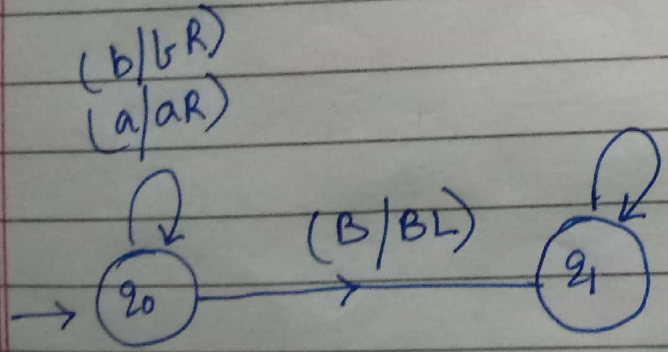
$\cup \{wbw^R / w \in (a+b)^*\}$

$\{ww^R / w \in (a+b)^*\}$

APPROACH → Match the first & Last symbol
&
2nd and 2nd last symbol.

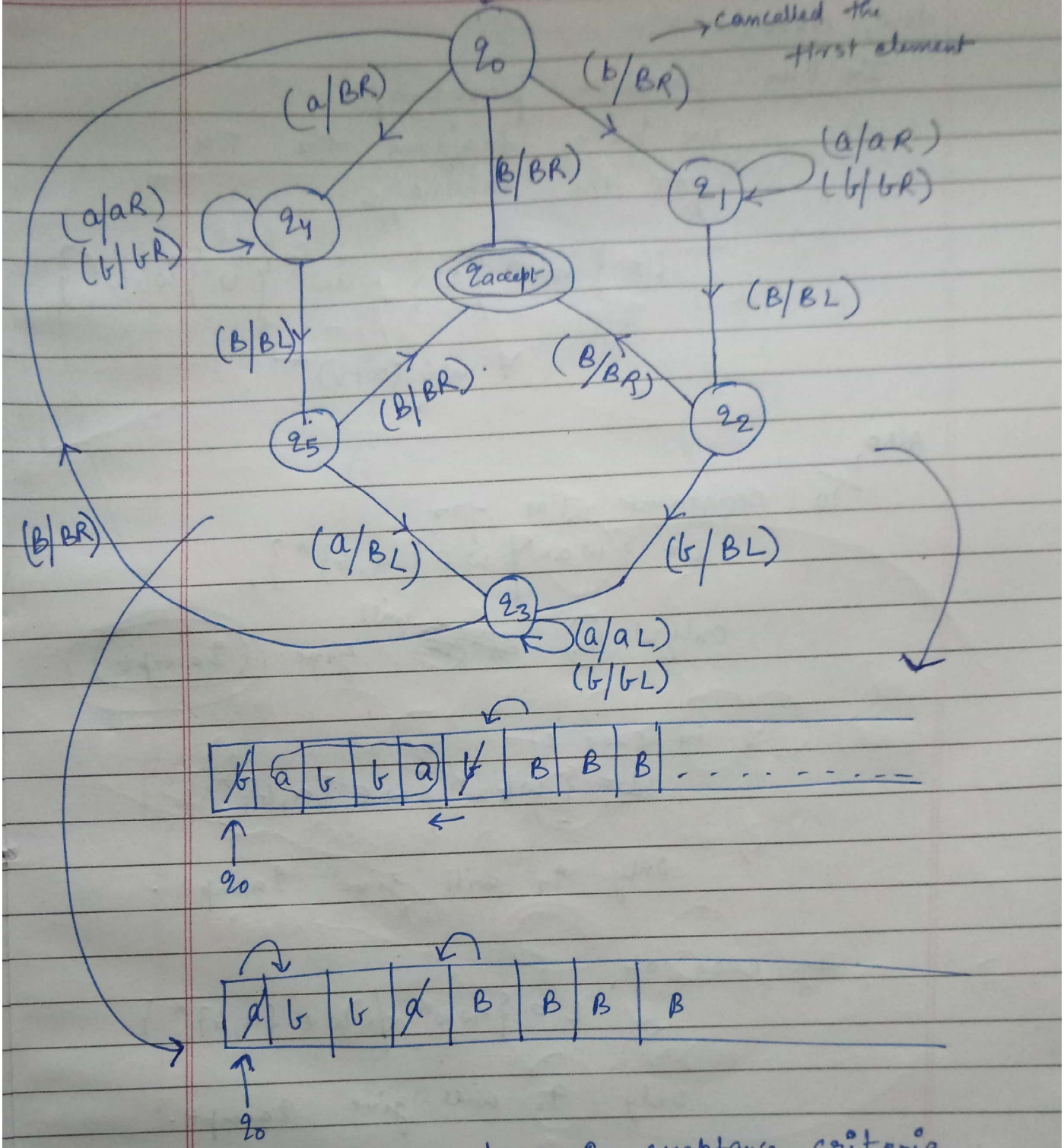
* In case of palindrome, check the middle symbol for odd length ones.

Now,



Hence, a ~~deterministic~~ TM cannot be drawn for such a problem in the above fashion, as the string may start with 'a' or 'b'. We need to consider both the possibilities.

* Note:- We have used the recursive approach for designing the turing machine.



In this problem, we have 2 acceptance criteria,

① in case of odd length palindrome.

② in case of even length palindrome.

→ no center (blank symbol found in q_0).

→ center is present (

B B B

q0 q1

q2

→ Some possible rejection criteria,

$$\begin{cases} \delta(q_5, b) = (q_{rej}, b, \blacktriangleright) \\ \delta(q_2, a) = (q_{rej}, a, \blacktriangleright) \end{cases}$$

Now, Note:-

In our previous problem, we have designed the TM for the language

$$L = \{ ww^R \} \cup \{ waw^R \} \cup \{ wbw^R \}$$

$$\forall\ w \in (a+b)^*$$

Also,

To construct TM for

$$L = \{ waw^R \mid w \in \{a, b\}^* \},$$

only q_5 ~~q_2~~ will give q_{accept}.

& in case of

$$L = \{ wbw^R \mid w \in \{a, b\}^* \}$$

Only q_2 will give q_{accept}.

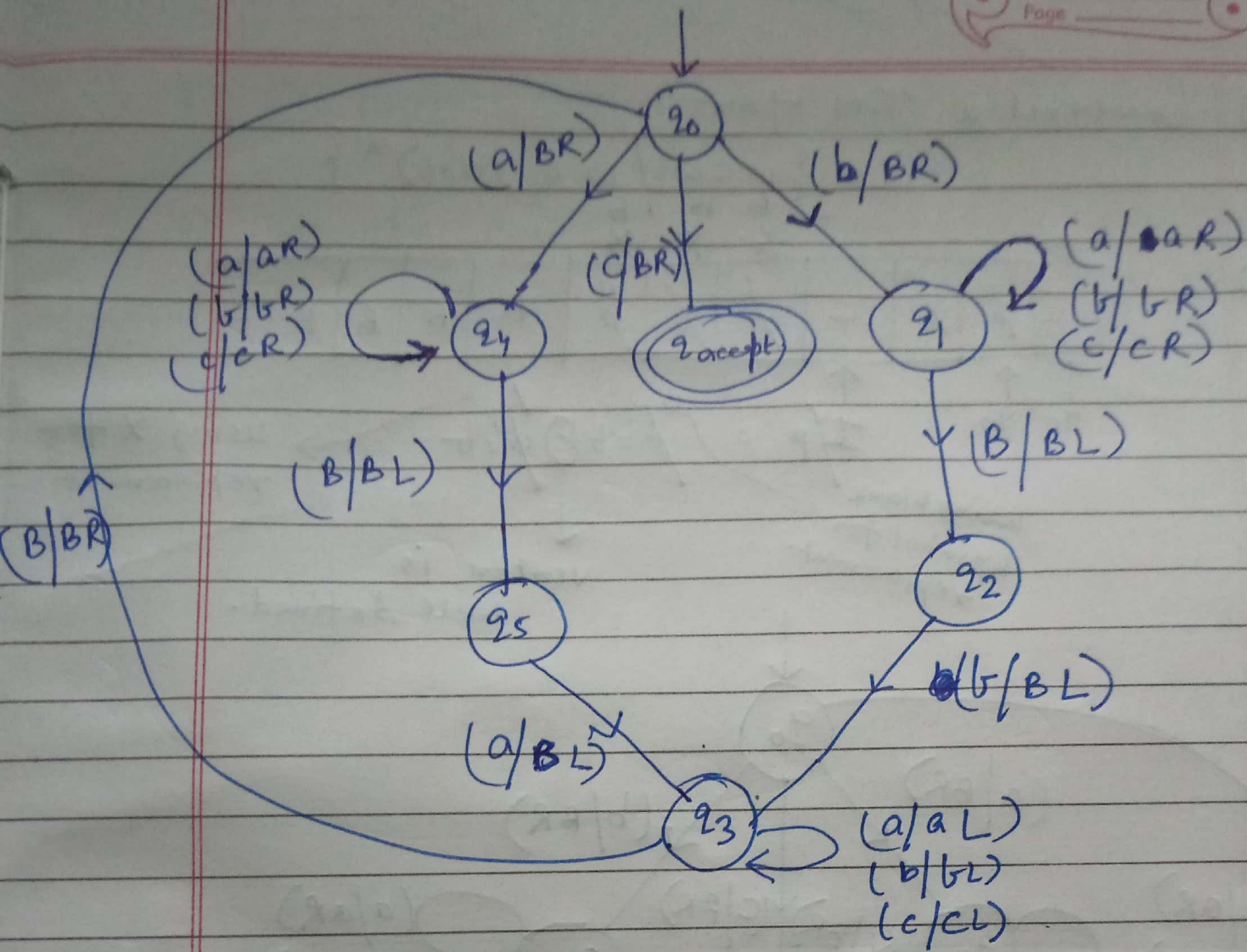
In case of,

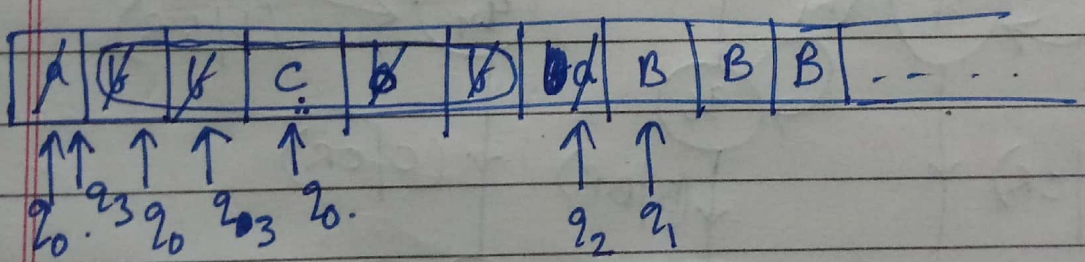
$$L = \{ ww^R \mid w \in \{a, b\}^* \}$$

only q_0 will give q_{accept}.

H.W

Q) Design a T.M for $L = \{ wcw^R \mid w \in (a+b)^* \}$.





Again, Recursive approach.

P.T.O

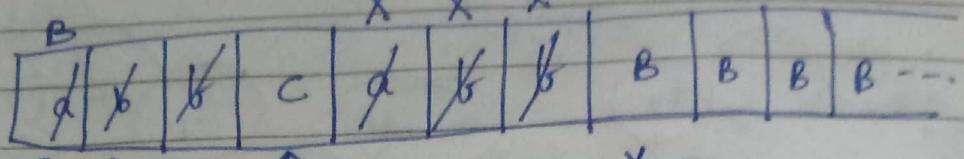
→ not a palindrome

Q) Construct a T.M for

$L = \{ wcw \mid w \in (a+b)^* \}$

B B B
X X X

A)



q_0 q_0

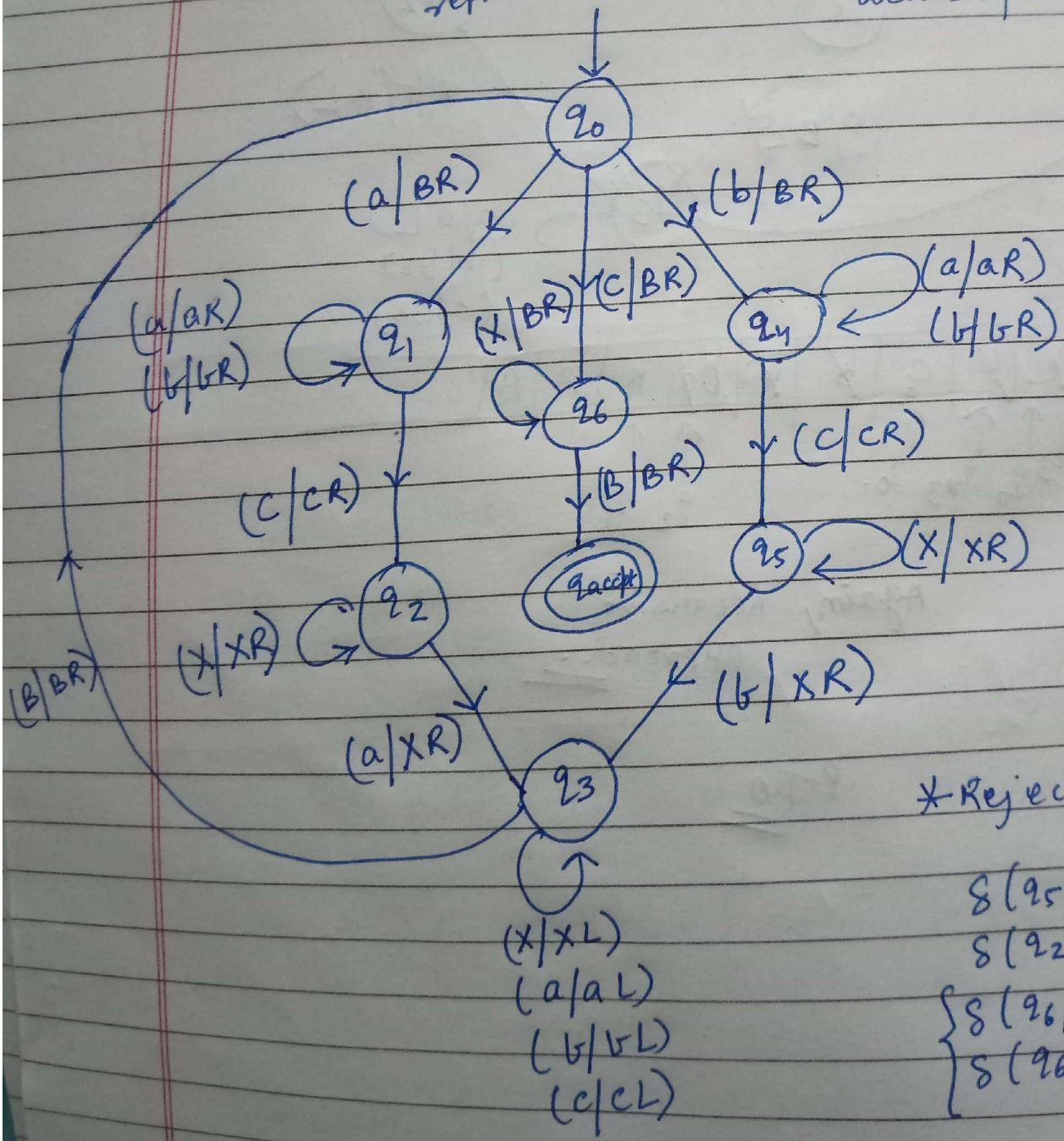
I/P = B a b b c a b b

X

→ using X for replacement

using blank symbol for replacement

vertex is well defined.



*Reject^n criteria

$\delta(q_5, a)$

$\delta(q_2, b)$

$\delta(q_6, a)$

$\delta(q_6, b)$

Q) Construct a T.M for

$$L = \{ ww \mid w \in (a+b)^* \}$$

A) In this case, we do not have the mid-point explicitly available.