

Algebraic Topology Notes

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Here are some of my notes from my reading of “Algebraic Topology” by Hatcher. The primary purpose will be to write down definitions, clarify things in need of clarification, and complete the exercises.

Chapter 0

Chapter zero discusses the geometric notions of homotopies (a definition to be clarified later). It can be summarized by stating that two spaces are equivalent if the space can be “shrunk” down to a core lines which are then homeomorphic (e.g. B and 8 would both shrink down to homeomorphic curves). Hatcher goes on to parameterize such shrinkage and give our first definition. A **deformation retraction** of a space X onto a subspace A is a family of maps $f_t : X \rightarrow X$, $t \in I$, such that $f_0 = \text{id}_X$, $f_1 = A$, and $f_t|_A = \text{id}_A$ for all t . The family f_t should be continuous in the sense that the associated map $X \times I \rightarrow X$, $(x, t) \mapsto f_t(x)$, is continuous. To make intuitive such continuity, we have that for some open set $B \in X$, $f_t(B)$ is an open subset of X for all $t \in I$ (e.g. an open disk in X encapsulating some open subset of A which gets mapped to that subset of A).