



Multi-start heuristics for the profitable tour problem

Kasi Viswanath Dasari ^a✉, Venkatesh Pandiri ^{a b}✉, Alok Singh ^a✉

Show more ▾

Share Cite

<https://doi.org/10.1016/j.swevo.2021.100897> ↗

[Get rights and content](#) ↗

Abstract

This paper is concerned with an interesting variant of the traveling salesman problem (TSP) called a profitable tour problem (PTP). Unlike TSP, in PTP there is no need to visit all the cities, and each city is associated with a profit which the salesman gets in case he visits that city. Like TSP, a travel cost is incurred in visiting a city that depends on the city visited last before visiting the city in consideration. The goal of the problem is to maximize the total profit minus total travel cost. In this paper, we have proposed three methods, viz. a multi-start hyper-heuristic (MSHH), a multi-start iterated local search (MS-ILS) and a multi-start general variable neighborhood search (MS-GVNS) to solve the PTP. MSHH uses eight different low level heuristics, whereas MS-ILS and MS-GVNS utilize variable neighborhood descent search over five different neighborhoods for local search. To evaluate the performance of the proposed approaches, a set of benchmark instances is generated based on the publicly available TSPLIB instances. Computational results on these instances show the effectiveness of our proposed approaches.

Introduction

The profitable tour problem (PTP) is a variant of the traveling salesman problem (TSP). Given a set of n cities, the TSP is to find a minimum length tour that visits all the n cities. Unlike TSP, in PTP, these n

cities have associated profits, and there is no need to visit all the cities. The salesman gets a profit associated with a city by visiting that city. A travel cost is also incurred to visit a city that depends on the city visited immediately before visiting the city in consideration. The salesman always starts and ends its tour at a base city (depot), assumed to have zero profit. The tour always includes at least one city in addition to the depot. The objective of the PTP is to find a tour that maximizes the total profit minus total travel cost. Note that visiting all the n cities may not maximize the objective, and an optimal tour can contain any number of cities between 2 to n . The PTP comes under the category of traveling salesman problem with profits. *Feillet et al.* [18] classified the traveling salesman problem with profits into three generic problems based on the two values, viz. collected profit and travel cost incurred. These two values can be either part of the objective function or as a constraint. Three generic problems are as follows.

1. Both values are part of the objective function: the goal is to find a tour that maximizes the collected profit minus travel cost. Alternatively, it can be considered as minimizing the travel cost minus collected profit.
2. The value of travel cost as a constraint: the goal is to find a tour that maximizes collected profit in such a way that the travel cost does not exceed a predefined value.
3. The value of profit as a constraint: the goal is to find a tour that minimizes travel cost in such a way that the collected profit should not be less than a predefined value.

These three generic problems have appeared with many names in the literature. However, these three problems are usually referred to as profitable tour problem [14], orienteering problem [38] and prize collecting traveling salesman problem [1] respectively. In this paper, we address the profitable tour problem, which was first defined by Dell'Amico *et al.* [14]. The PTP is also addressed as an augmented traveling salesman problem in [34]. PTP, being a generalization of the TSP, is also \mathcal{NP} -hard [21]. The PTP finds applications in logistics and home delivery services.

Several variants of the PTP exist in the literature. A variant of the PTP, profitable arc tour problem was introduced in [17], where the profits are associated with arcs instead of cities or vertices. Sun *et al.* [36] introduced a time dependent capacitated PTP with time windows and precedence constraints. The objective of this problem is also to maximize the profit collected minus the total travel cost. The authors proposed two methods, viz. an exact solution method called tailored labeling algorithm and a restricted dynamic programming heuristic algorithm. The exact solution method's computational results show that most cases of up to 75 requests can be optimally resolved within the specified two weeks time period and some cases remain unresolved. The restricted dynamic programming heuristic algorithm can find the good quality solutions for all the cases in short execution times. Lera-Romero *et al.* [25] studied the time dependent PTP with resource constraints, and, proposed a mixed integer linear programming formulation and a tailored branch-and-cut algorithm. Bruni *et al.* [2] introduced a

fascinating variant of PTP with stochastic costs under a risk-averse perspective. To solve this problem, the authors proposed two metaheuristics approaches, viz. a genetic algorithm(GA) and a tabu search (TS) algorithm.

Zhang *et al.* [46] introduced a probabilistic version of PTP. This version can be used to resolve a scenario in which a collection of consumers, each with a probability of requesting service, is presented and only a subset must be chosen and served. The authors provided a non-linear mathematical programming formulation and developed a genetic algorithm to solve this problem. Chentli *et al.* [8] introduced PTP with simultaneous pickup and delivery services. The authors provided a mathematical formulation and developed an extension of the adaptive large neighborhood search heuristic called selective adaptive large neighborhood search (sALNS) heuristic for solving it. Another interesting variant of this problem called multi-vehicle profitable pickup and delivery problem is studied in [20]. Chentli *et al.* [6] addressed a well-known vehicle routing problem variant called capacitated profitable tour problem (CPTP). The authors presented a hybrid iterative local search (ILS) heuristic, which uses as local search a large neighborhood search (LNS) heuristic and a variable neighborhood descent with random neighborhood ordering (RVND). The authors presented the impact of iterated local search heuristic hybridization on CPTP in [7]. Cortés-Murcia *et al.* [9] showed a generalization of CPTP called the electric capacitated profitable tour problem with mandatory stops (ECPTPMS). To solve this problem, the authors proposed a mathematical programming formulation and developed a branch-and-price algorithm. Bulhões *et al.* [3] introduced the vehicle routing problem with service level constraints, which can be considered as an extension of CPTP. To solve this problem, a compact mathematical programming formulation, a branch-and-price algorithm and a hybrid genetic algorithm with population management have been proposed in [3].

Of late, there is an emerging trend to solve the combinatorial optimization problems utilizing machine learning based methods. These machine learning based methods can learn the appropriateness of different heuristics in different circumstances, can automatically produce a new heuristic depending on the problem instance at hand, and can be employed to solve the complete problem or a part of it. TSP and its variants are no exception, and several machine learning based approaches have been proposed for them, e.g., [24], [43], [44], [45].

Compared to other variants of the TSP, the PTP did not get that much attention from the researchers. Still, researchers from different fields of study including heuristics and metaheuristics need to explore the basic version of PTP, though a few heuristic and metaheuristic approaches exist in the literature for variants of PTP as mentioned in previous paragraphs. Developing heuristics and metaheuristics for PTP is harder than TSP as one has to deal with deciding which cities to visit in addition to deciding the ordering among cities to be visited, and both aspects are equally vital in order to find a good solution. In this paper, we have proposed a multi-start hyper-heuristic (MSHH) approach, a multi-start iterated local search (MS-ILS) and a multi-start general variable neighborhood search (MS-GVNS) for the PTP. Hyper-heuristics can be considered as high-level strategies that manage a set of low-level heuristics

and work either by selecting a heuristic from available heuristics or generating a new heuristic from components of available heuristics at each decision point in the search process and applying the heuristic selected/generated [4], [5]. Hence, to deal with a wide range of instances of PTP, a hyper-heuristic is used. This hyper-heuristic is designed keeping in mind the specific needs of PTP, and, utilizes several low-level heuristics, each catering to different characteristics of PTP. Our multi-start hyper-heuristic will be referred to as *MSHH* subsequently. Iterated local search is a simple and powerful metaheuristic for solving combinatorial optimization problems [26], [27]. We have developed a multi-start iterated local search *MS-ILS* for the PTP which utilizes variable neighborhood descent [22] for local search. Variable neighborhood search (VNS) is one among the most successful metaheuristic techniques that utilizes systemic changes to the neighbourhood structure within a local search to solve the global optimization problems. We have developed a multi-start general variable neighborhood search *MS-GVNS*. General variable neighborhood search (GVNS)[22] is a variant of VNS where the variable neighborhood descent (VND) method is used as the local search. Computational results on 77 benchmark instances under different scenarios show the effectiveness of our proposed approaches.

The remainder of this paper is organized as follows. Section 2 introduces the notational conventions, formally defines the PTP and analyzes its search space. Section 3 provides an overview of hyper-heuristics. Section 4 describes the proposed multi-start hyper-heuristic approach. Section 5 presents multi-start iterated local search (*MS-ILS*) approach. Section 6 presents multi-start general variable neighborhood search (*MS-GVNS*) approach. Section 7 presents the computational results and their analysis. Finally, Section 8 outlines some concluding remarks regarding contributions made and directions for future research.

Section snippets

Problem definition and notational conventions

Given a complete directed graph $G = (V, E)$, where $V = \{1, 2, \dots, n\}$ is the set of cities, $E = \{(i, j) | i, j \in V\}$ is the set of edges, a distance d_{ij} is associated with each edge $(i, j) \in E$, a profit p_i is associated with each city $i \in V$, a designated city $h \in V$ with zero profit ($p_h=0$) known as base city or depot where the salesman has to start and end his tour, and the travel cost per unit of distance is α . The cities that are part of the tour are termed visited cities, whereas the remaining cities are termed ...

Overview of Hyper-Heuristics

For a decade or so, hyper-heuristics are receiving growing attention from the researchers owing to their ability to swiftly adapt as per the problem instance at hand, thereby, ensuring good quality

solutions over a wide range of instances of a problem [4]. The term *hyper-heuristic* was first used in a technical report by Denzinger *et al.* [15] as a strategy to combine a range of artificial intelligence methods for automated theorem proving, and does not provide any definition of hyper-heuristics. ...

Multi-start hyper-heuristic approaches for PTP

Depending on the profit values associated with the cities, distance values associated with the edges and the value of α , number, composition and ordering of cities in the optimal tour differ widely from one instance to another. As a hyper-heuristic can quickly adapt according to the characteristics of the instance at hand, we have developed a hyper-heuristic approach for PTP where several low-level heuristics are used. Hereafter, this approach will be referred to as MSHH.

Following subsections ...

Multi-start iterated local search approach for PTP

We have also developed a multi-start iterated local search (MS-ILS) approach for PTP where variable neighborhood descent (VND) search is utilized as a local search. Iterated local search (ILS) is a metaheuristic that maintains a single solution which is improved in an iterative manner. ILS posses several desirable features [26], [27]: simplicity, ease of implementation, robustness, and effectiveness. ILS comprises four principal components, viz. initial solution generation, local search, ...

Multi-start general variable neighborhood search for PTP

Variable neighborhood search (VNS) is a metaheuristic for solving combinatorial optimization problems, proposed by Mladenovic and Hansen [28]. The key idea behind VNS is the systematic search of different neighborhood structures to achieve an optimal or a close-to-optimal solution. The VNS consists of executing alternately, one shake phase (diversification phase) to escape from local optima and one local search phase (intensification phase) to improve the solution together with neighborhood ...

Computational results

As our approaches are the first metaheuristic / hyper-heuristic approaches for the PTP, no benchmark instances were available in the literature. Hence, it is inevitable for us to generate new test instances. We generated 77 PTP benchmark instances based on the instances from standard TSPLIB¹. These instances have cities ranging from 14 to 1379. First city in these instances is always the base city or depot with zero profit. A profit for ...

Conclusions

In this paper, a variant of the traveling salesman problem (TSP) called the profitable tour problem (PTP) is studied. The objective of the PTP is to maximize the total profit collected from visited cities minus total travel cost. As our contribution, we have developed a mathematical model, generated new benchmark instances and proposed four multi-start approaches, viz. MSHH_RAND, MSHH_GREEDY, MS-ILS and MS-GVNS to solve the PTP. The first two approaches are based on hyper-heuristic and other ...

CRediT authorship contribution statement

Kasi Viswanath Dasari: Methodology, Software, Formal analysis, Investigation, Writing - original draft, Visualization. **Venkatesh Pandiri:** Methodology, Validation, Formal analysis, Visualization. **Alok Singh:** Supervision, Conceptualization, Validation, Formal analysis, Writing - review & editing, Resources, Project administration, Funding acquisition. ...

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper. ...

Acknowledgements

Author are grateful to four anonymous reviewers for their valuable comments and suggestions which helped in improving the quality of this manuscript. The first author acknowledges the financial assistance received from the University Grants Commission (UGC), Government of India in the form of a Junior Research Fellowship (JRF). The third author gratefully acknowledges the support of the research grant no. MTR/2017/000391 of Science & Engineering Research Board (SERB), Government of India. ...

Recommended articles

References (46)

T. Bulhões *et al.*

[The vehicle routing problem with service level constraints](#)

Eur J Oper Res (2018)

D.L. Cortés-Murcia *et al.*

A branch-and-price algorithm for the electric capacitated profitable tour problem with mandatory stops

IFAC-Papers OnLine (2019)

M. Dell'Amico *et al.*

On prize-collecting tours and the asymmetric travelling salesman problem

International Transactions in Operational Research (1995)

J. Derrac *et al.*

A practical tutorial on the use of nonparametric statistical tests as a methodology for comparing evolutionary and swarm intelligence algorithms

Swarm Evol Comput (2011)

P. Hansen *et al.*

Variable neighborhood search: basics and variants

EURO Journal on Computational Optimization (2017)

Y. Hu *et al.*

A reinforcement learning approach for optimizing multiple traveling salesman problems over graphs

Knowledge Based Systems (2020)

G. Lera-Romero *et al.*

A branch and cut algorithm for the time-dependent profitable tour problem with resource constraints

Eur J Oper Res (2021)

N. Mladenović *et al.*

Variable neighborhood search

Computers & operations research (1997)

A.W. Mohamed *et al.*

Novel mutation strategy for enhancing SHADE and LSHADE algorithms for global numerical optimization

Swarm Evol Comput (2019)

A. Sifaleras *et al.*

General variable neighborhood search for the multi-product dynamic lot sizing problem in closed-loop supply chain

Electronic Notes in Discrete Mathematics (2015)



[View more references](#)

Cited by (12)

[Two heuristic approaches for clustered traveling salesman problem with d-relaxed priority rule](#)

2023, Expert Systems with Applications

Citation Excerpt :

...Readers may refer to Burke et al. (2013) for a good survey on hyper-heuristics and their applications. Hyper-heuristics independently and in hybridization with other meta-heuristics already have been successfully applied to solve several variants of the TSP (Chen et al., 2007; Dasari et al., 2021; Kendall & Li, 2013; Pandiri & Singh, 2018, 2020). Inspired by the success of these hyper-heuristic based approaches in solving these TSP variants and also due to the fact that CTSP-d has a wide variety of instances with different characteristics, we have developed a multi-start hyper-heuristic approach for CTSP-d where three levels of heuristics are used....

[Show abstract](#) ▾

[Variable neighborhood search for a new practical dynamic pickup and delivery problem](#)

2022, Swarm and Evolutionary Computation

Citation Excerpt :

...Thus, this paper designs an effective and efficient variable neighborhood search (VNS) algorithm to solve this new DPDP variant well. As an important research area of evolutionary computation, VNS combines different local search heuristics with disturbance strategies, which has shown effectiveness on a wide variety of combinatorial optimization problems [3,19-22], such as Vehicle Routing Problems (VRPs) [23-31]. However, VNS is a generic framework, which should be appropriately instantiated for solving a specific problem....

[Show abstract](#) ▾

[Adaptive Ant Colony Optimization with node clustering applied to the Travelling Salesman Problem](#)

2022, Swarm and Evolutionary Computation

Citation Excerpt :

...A Novel Discrete Differential Evolution (NDDE) algorithm for improving the performance of the standard differential evolution algorithm when it is used to solve complex TSPs is introduced in [22]; it incorporates several efficient components such as an enhanced mapping method applied to the randomly generated initial population with continuous variables and an improved k-means clustering as a repairing method for enhancing the discrete

solutions in the initial phase. In [23], the authors propose three methods for a variant of TSP called the Profitable Tour Problem (PTP) where there is no need to visit all the cities as each city is associated with a profit which the salesman gets in case of visiting that city. The first method called a Multi-Start Hyper-Heuristic (MSHH) uses eight different low-level heuristics, whereas the other two methods (Multi-Start Iterated Local Search and Multi-Start General Variable Neighbourhood Search) utilize variable neighbourhood descent search over five different neighbourhoods for local search....

Show abstract ▾

Quantum Annealing Approach for Selective Traveling Salesman Problem ↗

2023, IEEE International Conference on Communications

A Novel Clustering Strategy-Based Sink Path Optimization for Wireless Sensor Network ↗

2022, IEEE Sensors Journal

An efficient hybrid genetic algorithm for solving truncated travelling salesman problem ↗

2022, Decision Science Letters



View all citing articles on Scopus ↗

[View full text](#)

© 2021 Elsevier B.V. All rights reserved.



All content on this site: Copyright © 2025 Elsevier B.V., its licensors, and contributors. All rights are reserved, including those for text and data mining, AI training, and similar technologies. For all open access content, the relevant licensing terms apply.

