GIRRAWEEN HIGH SCHOOL

2021 PRACTICE TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

MATHEMATICS EXTENSION 1

General Instructions

- Reading time – 10 minutes
- Working time – 2 hours
- Write using black pen
- NESA approved calculators may be used.
- In section II, Show relevant mathematical reasoning and/or calculations

Total marks: 70

Section I – 10 marks

- Attempt Questions 1-10
- Allow about 15 minutes for this section

Section II – 60 marks

- Attempt all questions
- Allow about 1 hour and 45 minutes for this section

Upload your answers as a single PDF in your google classroom in the assignment folder.

SECTION 1

10 marks

Attempt questions 1 – 10

Allow about 15 minutes for this section

1. Given the vectors $x = 5i + 3j$ and $y = -2i - 5j$. The magnitude and direction of $x + y$ is

(A) 3.6 ; 326°

(B) 3.6 ; 34°

(C) 3.6 ; 146°

(D) 3.6 ; 214°

2. In the expansion of $(2x+k)^6$, the coefficients of x and x^2 are equal. What is the value of k?

(A) 5 **(B)** 6 **(C)** 11 **(D)** 12

3. The coefficient of x^{-5} in the expansion of $\left(2x^2 - \frac{1}{x}\right)^{20}$ is

(A) -77520 **(B)** -155040 **(C)** -248064 **(D)** -496128

4. The domain and inverse of $f(x) = 4\log_e(x+3) - 2$ are

(A) $x > 3;\quad y = e^{\frac{x+2}{4}} - 3$

(B) $x > -3;\ y = e^{\frac{x+2}{4}} - 2$

(C) $x > -3;\ y = e^{\frac{x+2}{4}} - 3$

(D) $x > 3;\quad y = e^{\frac{x+2}{4}} - 2$

5. Consider the parametric equation $x = 5\cos\theta - 2$ and $y = 5\sin\theta + 3$. Which of these is the corresponding cartesian equation?

(A) $x^2 - 4x + y^2 - 6y = 12$

(B) $x^2 + 4x + y^2 + 6y = 12$

(C) $x^2 - 4x + y^2 + 6y = 12$

(D) $x^2 + 4x + y^2 - 6y = 12$

6. What is the derivative of $y = \cos^{-1}\left(\dfrac{x}{4}\right)$

(A) $-\dfrac{1}{\sqrt{16-x^2}}$ **(B)** $-\dfrac{2}{\sqrt{16-x^2}}$

(C) $-\dfrac{4}{\sqrt{16-x^2}}$ **(D)** $-\dfrac{6}{\sqrt{16-x^2}}$

7. What is the domain and range of $f(x) = 2\sin^{-1}\left(\dfrac{x}{2}\right)$?

(A) $D: -2 \le x \le 2,\ \mathrm{R}: -\dfrac{\pi}{2} \le y \le \dfrac{\pi}{2}$

(B) $D: -2 \le x \le 2,\ R: -\pi \le y \le \pi$

(C) $D: -\dfrac{1}{2} \le x \le \dfrac{1}{2},\ R: -\dfrac{\pi}{2} \le \mathrm{y} \le \dfrac{\pi}{2}$

(D) $D: -\dfrac{1}{2} \le x \le \dfrac{1}{2}, R: -\pi \le y \le \pi$

8. $\int \sin^2 3x\, dx$ is equal to which of the following?

(A) $\dfrac{x}{2} - \dfrac{\sin 6x}{3} + C$

(B) $\dfrac{x}{2} - \dfrac{\sin 6x}{6} + C$

(C) $\dfrac{x}{2} - \dfrac{\sin 6x}{9} + C$

(D) $\dfrac{x}{2} - \dfrac{\sin 6x}{12} + C$

9. What is the value of k such that $\displaystyle\int_0^k \frac{dx}{1+(x-1)^2} = \frac{\pi}{2}$

(A) $2\sqrt{3}$ **(B)** $\sqrt{3}$ **(C)** 2 **(D)** 1

10. Which of the following is a factor of $2x^4 - 4x^3 - 10x^2 + 12x$?

(A) $x+1$ **(B)** $x-2$ **(C)** $x-3$ **(D)** $x+4$

Section II

60 marks

Attempt all questions

Allow about 1 hour and 45 minutes for this section

Start each question on a new page in the answer booklet provided.

Your responses should include relevant mathematical reasoning and /or calculations.

Question 11 (12 marks) **Marks**

(a) Solve $\dfrac{6}{5x-2} \le 2$ 3

(b) Prove that $\cot 2x + \cot x = \dfrac{\sin 3x}{\sin 2x \sin x}$ 2

(c) Use the substitution $u = \ln 3x$, to find $\displaystyle\int \frac{dx}{x(\ln 3x)^2}$ 3

(d) Let $f(x) = \dfrac{2x}{\sqrt{1-x^2}}$

(i) For what values of x is $f(x)$ undefined? 1

(ii) Find $\displaystyle\int_0^{\frac{1}{2}} \frac{2x\,dx}{\sqrt{1-x^2}}$ using the substitution $x = \sin u$. 3

End of Question 11

Question 12 (12 marks)

(a) (i) Express $5\sin x + 12\cos x$ in the form $A\sin(x+\alpha)$ where $0 \le \alpha \le \frac{\pi}{2}$ (Give the value of α in radians, correct to 2 decimal places) 3

(ii) Hence solve $5\sin x + 12\cos x = 8$ for $0 \le x \le \pi$ (Give the value or values of x in radians correct to 2 decimal places) 2

(b) Six people attend a dinner party.

(i) In how many different ways can they be arranged around a round table? 1

(ii) In how many different ways can they be arranged if a particular couple must sit together? 1

(iii) What is the probability that, if the people are seated at random, the couple are sitting apart from each other? 1

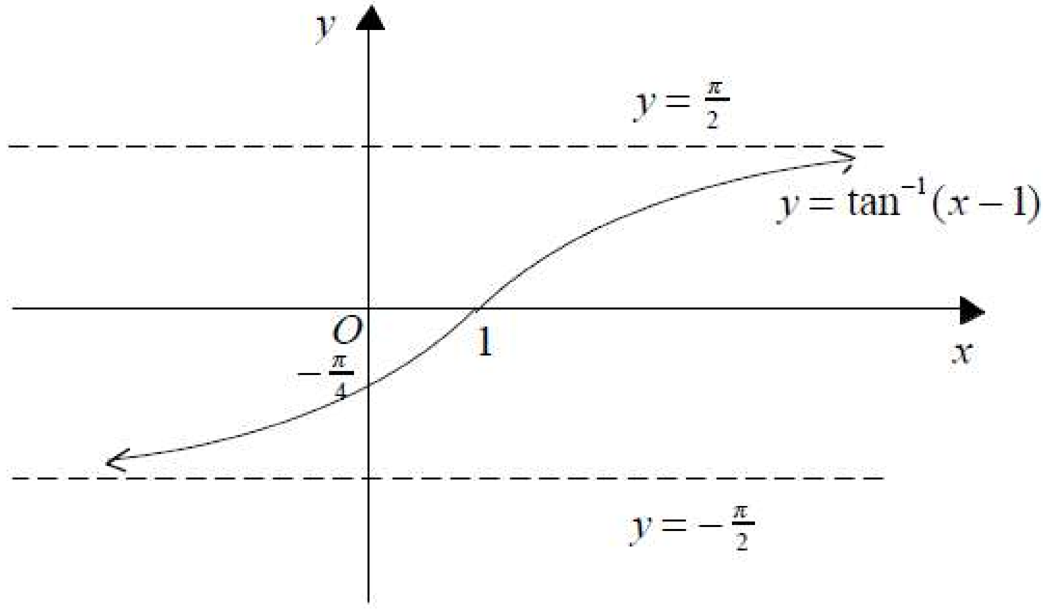
(c) Use mathematical induction to prove that

$(1^2+1)\,1! + (2^2+1)\,2! + (3^2+1)\,3! + ... + (n^2+1)\,n! = n\,(n+1)!$ for all positive integers $n \ge 1$. 4

End of Question12

Question 13 (12 marks)

(a)



The region in the first quadrant bounded by the curve $y = \tan^{-1}(x-1)$ and the $y-$axis between the lines $y = 0$ and $y = \dfrac{\pi}{4}$ is rotated through one complete revolution about the $y-$axis.

(i) Show that the volume V of the solid of revolution is given by

$$V = \pi \int_0^{\frac{\pi}{4}} (1 + \tan y)^2 \, dy.$$ 1

(ii) Hence find the value of V in simplest exact form. 3

(b) A particle is projected from a point O with velocity V m/s at an angle θ to the horizontal. At any time t seconds the horizontal and vertical components of displacement are given by

$x = Vt\cos\theta$ and $y = Vt\sin\theta - \frac{1}{2}gt^2$ where g is the acceleration due to gravity.

Show that the cartesian equation of the path is given by $y = x\tan\theta - \frac{gx^2}{2V^2}\left(1+\tan^2\theta\right)$ 2

(c) A particle is projected from O with velocity $60\ m/s$ at an angle α to the horizontal.

T seconds later, another particle is projected from O with velocity $60\ m/s$ at an angle β

To the horizontal where $\beta < \alpha$. The two particles collide 240 metres horizontally from O

and at a height of 80 metres above O. Taking $g = 10m/s^2$ and using results from (b)

(i) Show that $\tan\alpha = 2$ and $\tan\beta = 1$. 3

(ii) Find the value of T in simplest exact form. 3

End of Question 13

Question 14 (12 marks)

(a) (i) Differentiate $y = x\cos^{-1}x - \sqrt{1-x^2}$. 2

(ii) Hence calculate the exact value of $\int_0^{\frac{1}{2}} \cos^{-1} x dx$ 2

(b) Solve $x^4 - 5x^3 - 9x^2 + 81x - 108 = 0$, given that $P(x) = x^4 - 5x^3 - 9x^2 + 81x - 108$

has a triple zero. 3

(c) A bottle of medicine which is initially at a temperature of $10°C$ is placed into a room which has a constant temperature of $25°C$. The medicine warms at a rate proportional to the difference between the temperature of the room and the temperature (T) of the medicine. That is, T satisfies the equation $\frac{dT}{dt} = -k(T-25)$

(i) Show that $T = 25 + Ae^{-kt}$ is a solution of this equation. 2

(ii) If the temperature of the medicine after 10 minutes is $16°C$, find its temperature after 40 minutes. 3

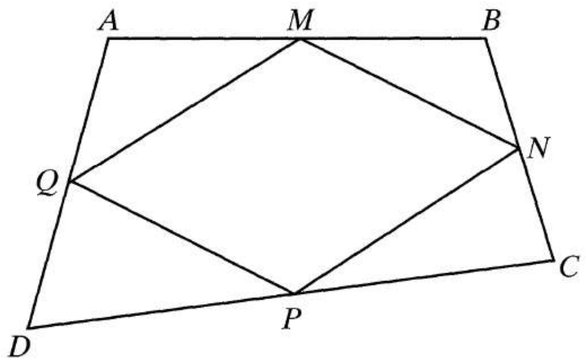
End of Question 14

Question 15 (12 marks)

(a) For what value(s) of m are the vectors $\begin{pmatrix} 10m-17 \\ 3 \end{pmatrix}$ and $\begin{pmatrix} m \\ 2 \end{pmatrix}$ perpendicular? 3

(b) Consider the vectors given by $u = bi + 2j$ and $w = 2i + bj$ where b is a real number. If the acute angle between the two vectors is $60°$, find the two possible values for b. 4

(c) Consider the quadrilateral $ABCD$. The midpoints of AB, BC, CD and DA are M, N, P and Q respectively.



Let $\overrightarrow{AB} = \underset{\sim}{a}$, $\overrightarrow{BC} = \underset{\sim}{b}$, $\overrightarrow{CD} = \underset{\sim}{c}$ and $\overrightarrow{DA} = \underset{\sim}{d}$

(i) Prove that $\underset{\sim}{a} + \underset{\sim}{b} + \underset{\sim}{c} + \underset{\sim}{d} = 0$ 2

(ii) Hence prove that $MNPQ$ is a parallelogram. 3

END OF TEST