



Newington College

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Mathematics Extension 1

2021 Trial HSC Assessment point 4

General Instructions

- Working time: 1 hour
- Write using black or blue pen
- Attempt ALL questions
- Do NOT write in pencil
- Write your student number on the front of each exam writing booklet.

Section 1 - Multiple Choice
5 marks

Section 1 – Written (booklets)
30 marks

Section 2 - Multiple Choice
5 marks

Section 2 – Written (booklets)
30 marks

Section I (Multiple Choice)

5 marks

Attempt Questions 1-5

Allow approximately 8 minutes for this section

Mark your answers on the multiple-choice answer provided.

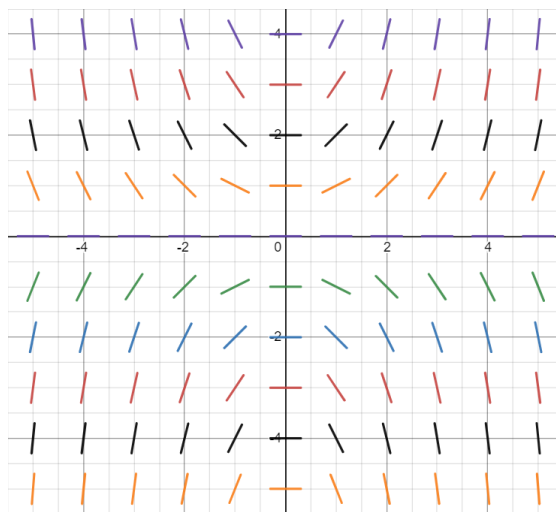
1. What is the solution set of the inequality $\frac{x^2 - 6}{x} \leq 1$?

- (A) $(\infty, -2] \cup (0, 3]$ (B) $[-2, 0) \cup [3, \infty)$
 (C) $[-2, 3]$ (D) $(\infty, -2] \cup [0, 3]$

2. What is the range of the function $y = \sin^{-1} x + \tan^{-1} x$?

- (A) $-\frac{\pi}{2} < y < \frac{\pi}{2}$ (B) $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$
 (C) $-\frac{3\pi}{4} \leq y \leq \frac{3\pi}{4}$ (D) $-\pi < y < \pi$

3. The slope field of a first order differential equation is shown here.



Which of the following could be the differential equation represented above?

- (A) $\frac{dy}{dx} = \frac{xy}{2}$ (B) $\frac{dy}{dx} = -\frac{xy}{2}$
 (C) $\frac{dy}{dx} = \frac{x}{2y}$ (D) $\frac{dy}{dx} = -\frac{x}{2y}$

4. In how many ways can the letters of the word OLYMPICS be arranged if the 2 vowels are **not** next to each other?

(A) 40320 (B) 30240
(C) 10080 (D) 2520

5. Given that $(1+x)^n(1+x)^n = (1+x)^{2n}$, which of the following is equivalent to

$$\binom{n}{0}^2 + \binom{n}{1}^2 + \binom{n}{2}^2 + \dots + \binom{n}{n}^2 ?$$

(A) $\binom{2n}{n}$ (B) $\binom{2n}{n}^2$
(C) $\binom{2n}{0} + \binom{2n}{1} + \binom{2n}{2} + \dots + \binom{2n}{n}$ (D) $\binom{2n}{0} + \binom{2n}{1} + \binom{2n}{2} + \dots + \binom{2n}{2n}$

Section I (Written response)

30 marks

Attempt Questions 6-7

Allow approximately 52 minutes for this section.

Write your answers in the writing booklets supplied. Use a separate writing booklet for each question.

Additional writing booklets are available.

Your responses should include relevant mathematical reasoning and/or calculations.

QUESTION 6 (15 MARKS) (Start a new Answer Booklet)

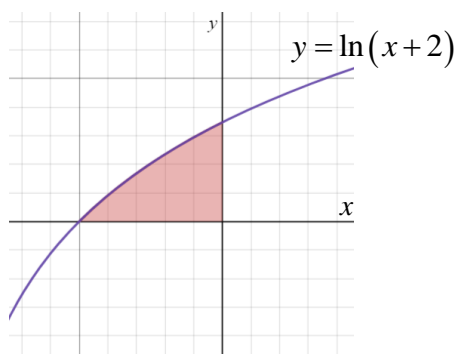
Marks

- (a) Using the substitution $u = x + 3$, find $\int \frac{x}{\sqrt{x+3}} dx$ **2**
- (b) Consider the polynomial $P(x) = x^3 - 4x^2 + kx + 12$, where k is a constant. It is given that α , β , and γ are the roots of the equation $P(x) = 0$.
- (i) Find the values of $\alpha + \beta + \gamma$ and $\alpha\beta\gamma$ **2**
- (ii) If two of the roots are equal in magnitude but opposite in sign, find the value of the roots and the value of k . **3**
- (c) How many numbers do you need to select from the integers 1 to 18 to ensure that there is a pair of numbers that add to give you 19? **2**
- (d) In how many ways can 10 people be split into two unequal groups? (Note, a group must contain at least one person). **2**
- (e) (i) Write the expression $\cos x + \cos 5x$ as the product of trigonometric ratios. **1**
- (ii) Hence solve the equation $\cos x + \cos 3x + \cos 5x = 0$ for $0 \leq x \leq \pi$ **3**

End of Question 6. Question 7 continues over the page.

QUESTION 7 (15 MARKS) (Start a new Answer Booklet)**Marks**

(a)



In the diagram, the region bounded by the curve $y = \ln(x + 2)$, the x -axis and the y -axis, is rotated one revolution about the **y -axis**. Find in simplest exact form, the volume of the solid formed.

4

- (b) In 2020 the world population was 7.8 billion. Projections are that the population will be 8.6 billion in 10 years' time and that the projected maximum population is 10.9 billion.

(i) show by differentiation that $P = 10.9 + Be^{-kt}$ is a solution of the differential equation

$$\frac{dP}{dt} = -k(P - 10.9) \text{ where } t \text{ is the number of years after 2020.}$$

1

(ii) In how many years will the population reach 10 billion? (answer to the nearest year)

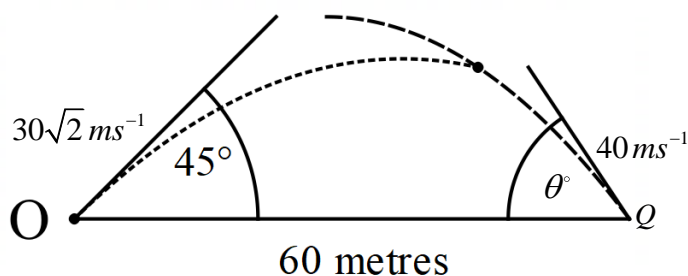
3

(iii) What will the rate of population growth be at that time?

1

Question 7 continues over the page

- (c) A stone P is projected from a point O with an initial velocity of $30\sqrt{2} \text{ ms}^{-1}$ at an angle of 45° from the horizontal. At the same instant a second stone Q is projected in the opposite direction with an initial speed of 40 ms^{-1} from the same horizontal plane 60 metres from O .



After T seconds, the stones collide. Assume $g = 10 \text{ ms}^{-2}$.

- (i) Show that the position vector of stone P is given by $\underline{r}(t) = 30t\underline{i} + (30t - 5t^2)\underline{j}$ 1
- (ii) Given that the position vector of stone Q is given by $\underline{r}(t) = 40t \cos \theta \underline{i} + (40t \sin \theta - 5t^2)\underline{j}$ (DO NOT SHOW THIS), find the angle of projection of stone Q to the nearest degree. 2
- (iii) Find the **exact** value of T , the time when the stones collide and the horizontal distance from O at the point of collision. 3

End of Question 7. End of Section I.



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5 marks

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30 marks

Section 2 - Multiple Choice
5 marks

Section 2 – Written (booklets)
30 marks

Section II (Multiple Choice)

5 marks

Attempt Questions 8-12

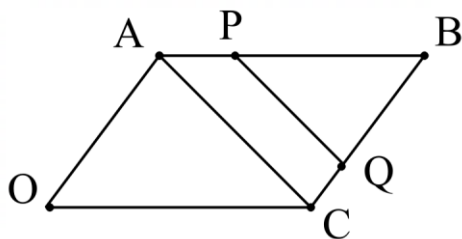
Allow approximately 8 minutes for this section

Mark your answers on the multiple-choice answer provided.

8. What is the remainder when the polynomial $P(x) = x^3 - 8x^2 - 4x + 5$ is divided by $2x + 1$?

- (A) $-\frac{1}{72}$ (B) $\frac{1}{2}$ (C) $\frac{39}{8}$ (D) $\frac{9}{8}$

9. OABC is a parallelogram. $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OC} = \mathbf{c}$. The points P and Q are chosen such that $AP:PB = 1:3$ and $CQ:QB = 1:3$.



Which of the following is \overrightarrow{PQ} ?

- (A) $\frac{3}{4}\mathbf{c} - \frac{3}{4}\mathbf{a}$ (B) $\frac{3}{4}\mathbf{c} + \frac{3}{4}\mathbf{a}$
 (C) $\frac{2}{3}\mathbf{c} - \frac{2}{3}\mathbf{a}$ (C) $\frac{2}{3}\mathbf{c} + \frac{2}{3}\mathbf{a}$

10. The graph of $y = \cos^{-1}(x + 2)$ is transformed by being dilated horizontally with a scale factor of $\frac{1}{2}$ then translated to the right by 1.

What is the equation of the transformed graph?

- (A) $y = \frac{1}{2}\cos^{-1}(x + 1)$ (B) $y = 2\cos^{-1}(x + 1)$
 (C) $y = \cos^{-1} 2x$ (D) $y = \cos^{-1}\left(\frac{x + 3}{2}\right)$

11. If $y = \sin^{-1} \frac{1}{x}$ for $x \geq 1$, what is $\frac{dy}{dx}$?

(A) $\cos^{-1} \frac{1}{x}$

(B) $\frac{x}{\sqrt{x^2 - 1}}$

(C) $-\operatorname{cosec}^2 y \sec y$

(D) $-\sin y \tan y$

12. A vertical tower of height h metres stands with its base on horizontal ground. A stone projected horizontally from the top of the tower O with speed $V \text{ ms}^{-1}$. The stone moves in a vertical plane under gravity, where the acceleration due to gravity is $g \text{ ms}^{-2}$. At time t seconds its position vector **relative to O** is given by $\vec{r}(t) = Vt\vec{i} - \frac{1}{2}gt^2\vec{j}$. The stone hits the ground at a horizontal distance $2V$ metres from the base of the tower. What is the height of the tower in terms of g ?

(A) $\frac{1}{2}g$ metres

(B) g metres

(C) $2g$ metres

(D) $4g$ metres

Section II (Written Section)

30 marks

Attempt Questions 13-14

Allow approximately 52 minutes for this section.

Write your answers in the writing booklets supplied. Use a separate writing booklet for each question.

Additional writing booklets are available.

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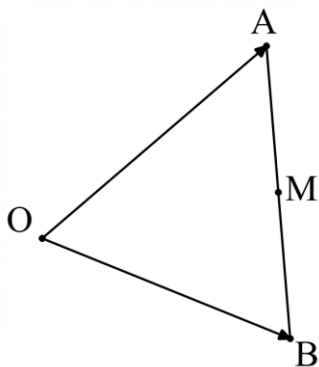
QUESTION 13 (15 MARKS) (Start a new Answer Booklet)

Marks

(a) Prove by mathematical induction that $13 \times 6^n + 2$ is divisible by 10 for all integers $n \geq 1$ 3

(b) In the diagram, OAB is a triangle with $\overrightarrow{OA} = \underline{a}$, $\overrightarrow{OB} = \underline{b}$ and $|\overrightarrow{OA}| = |\overrightarrow{OB}|$.

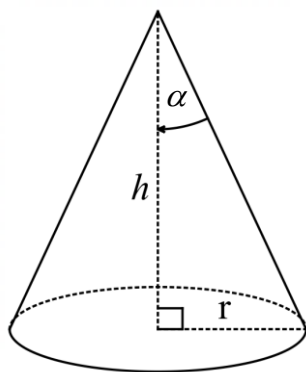
If M is the midpoint of AB , show that $\overrightarrow{OM} \perp \overrightarrow{AB}$. (Diagram is not to scale) 3



Question 13 continues over the page

- (c) Sand is poured from above onto a cone with a height of 10 cm, so that it remains conical in shape but the semi-vertical angle α is increasing at a rate of 0.0005 radians per second, while the height remains unchanged. Find the rate at which the volume of the cone is increasing when the semi-vertical angle is $\frac{\pi}{3}$. (Answer to 2 decimal places) 3

You are given that the volume of the cone is $V = \frac{1}{3}\pi h^3 \tan^2 \alpha$.

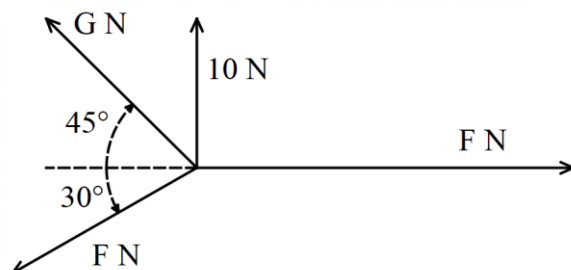


- (d) Solve the differential equation $y' = \frac{3x}{\cos y}$ given that $y(1) = 0$ 3
- (e) Find the exact value of $\sin\left[\cos^{-1}\frac{3}{5} + \tan^{-1}\left(\frac{-3}{4}\right)\right]$ 3

End of Question 13. Question 14 starts over the page.

QUESTION 14 (15 MARKS) (Start a new Answer Booklet)**Marks**

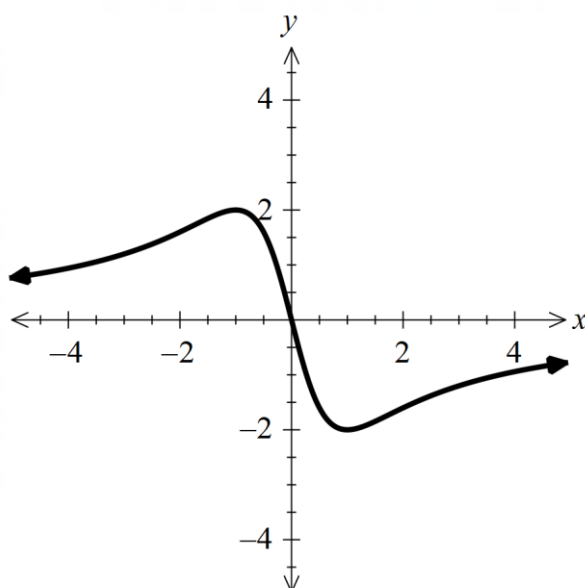
- (a) An object on level ground is subject to forces operating horizontally of magnitudes F Newtons, G Newtons and 10 Newtons as shown on the diagram. Find the exact value of F if the object is in equilibrium.

3

- (b) Find $\int_0^{\frac{\pi}{3}} \sin^3 x \, dx$

3

- (c) The graph of the function $f(x) = \frac{-4x}{x^2 + 1}$ is shown below.



- (i) What is the domain of the graph $y = \sqrt{f(x)}$? **1**
- (ii) Sketch the graph of $y = \frac{1}{\sqrt{f(x)}}$ showing clearly any turning points, intercepts and asymptotes **2**
- (iii) On a separate axis, Sketch the graph of $y^2 = \frac{1}{f(x)}$ **1**

Question 14 continues over the page

(d) (i) Show that $\tan(\alpha + \beta + \gamma) = \frac{\tan \alpha + \tan \beta + \tan \gamma - \tan \alpha \tan \beta \tan \gamma}{1 - \tan \alpha \tan \beta - \tan \alpha \tan \gamma - \tan \beta \tan \gamma}$ 2

(ii) If $\tan \alpha, \tan \beta, \tan \gamma$ are the roots of the equation $x^3 - (a+1)x^2 + (b-a)x - b = 0$, show that $\alpha + \beta + \gamma = n\pi + \frac{\pi}{4}$. 3

End of Section II. End of Examination.