



The Scots College

2021

HSC Year 12 Trial Examination

Mathematics Extension 1

General Instructions

- Reading time – 10 minutes
- Working time – 2 hours
- Write using black pen
- Calculators approved by NESA may be used.
- A reference sheet is provided.
- For questions in Section II, show all relevant mathematical reasonings and/or calculations.
- Answer each question at the start of a NEW page, using either writing booklets or A4 paper.

Total marks: 70

- **Section I – 10 marks** (pages 3–6)
 - Attempt Questions 1–10
 - Allow about 15 minutes for this section
 - Use the Multiple Choice Answer Sheet provided
- **Section II – 60 marks** (pages 7 – 13)
 - Attempt Questions 11–14
 - Allow about 1 hour and 45 minutes for this section

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Section 1

10 marks

Attempt Questions 1–10.

Allow about 15 minutes for this section.

Use the multiple-choice answer sheet for Questions 1 – 10.

1. Which of the following is equivalent to $\cos(2x - y) \cos x + \sin(2x - y) \sin x$?

(A) $\cos(x - y)$
(B) $\cos(3x - y)$
(C) $\sin(x - y)$
(D) $\sin(3x - y)$

2. Five Year 11 students and five Year 12 students are seated around a circular table.
How many ways can the students be seated if the Year 11 students and Year 12 students must alternate?

(A) 576
(B) 2 880
(C) 5 040
(D) 14 400

3. Which of the following represents the vector projection of $\begin{pmatrix} -1 \\ 1 \end{pmatrix}$ onto $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$?

(A) $\frac{1}{5}$
(B) $\frac{1}{25} \begin{pmatrix} -1 \\ 1 \end{pmatrix}$
(C) $\frac{1}{25} \begin{pmatrix} 3 \\ 4 \end{pmatrix}$
(D) $\frac{7}{25} \begin{pmatrix} 3 \\ 4 \end{pmatrix}$

4. What are the solutions to $|3x - 4| \leq 16$?

(A) $-6\frac{2}{3} \leq x \leq 4$

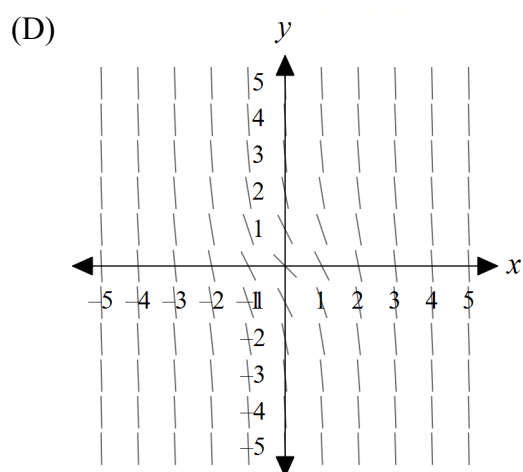
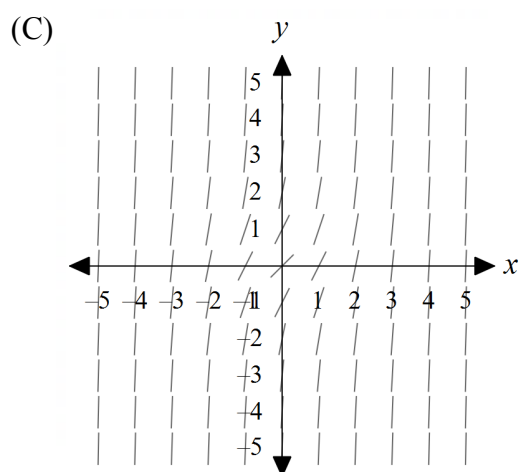
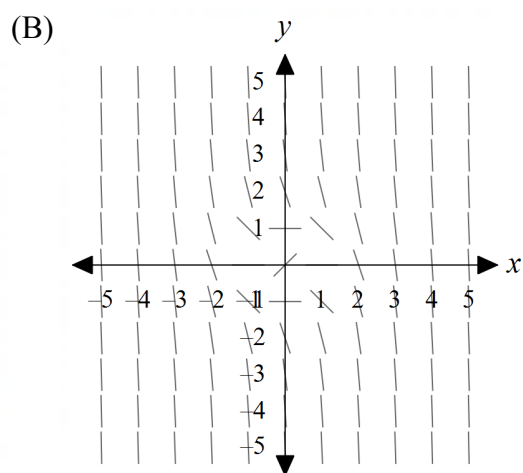
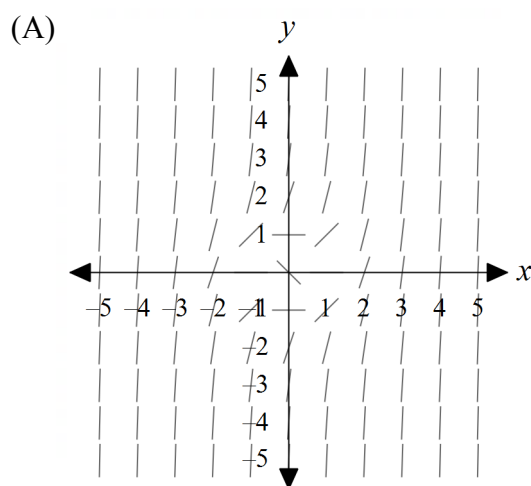
(B) $-4 \leq x \leq 6\frac{2}{3}$

(C) $x \leq -6\frac{2}{3}$ and $x \geq 4$

(D) $x \leq -4$ and $x \geq 6\frac{2}{3}$

5. A differential equation is given by $y' = x^2 + y^2 + 1$

Which of the following slope fields best represents the differential equation?



6. Which of the following is the primitive of $\frac{1}{\sqrt{4-9x^2}}$?

(A) $\frac{1}{3} \sin^{-1} \left(\frac{2x}{3} \right)$

(B) $\frac{1}{6} \sin^{-1} \left(\frac{2x}{3} \right)$

(C) $\frac{1}{3} \sin^{-1} \left(\frac{3x}{2} \right)$

(D) $\frac{1}{6} \sin^{-1} \left(\frac{3x}{2} \right)$

7. Suppose the rate of change of a quantity N is proportional to the excess of N over a fixed quantity P , that is

$$\frac{dN}{dt} = -k(N - P)$$

where k is a constant and t is time.

If $A = P - N$ when $t = 0$, then which of the following represents the equation of N ?

(A) $N = P - Ae^{-kt}$

(B) $N = P - Ae^{kt}$

(C) $N = P + Ae^{-kt}$

(D) $N = P + Ae^{kt}$

8. A curve is defined by the parametric equations below

$$x = \sin \theta$$

$$y = \cos^2 \theta - 3.$$

Which of the following represents the Cartesian equation of the curve?

(A) $y = x^2 - 3$

(B) $y = -x^2 - 2$

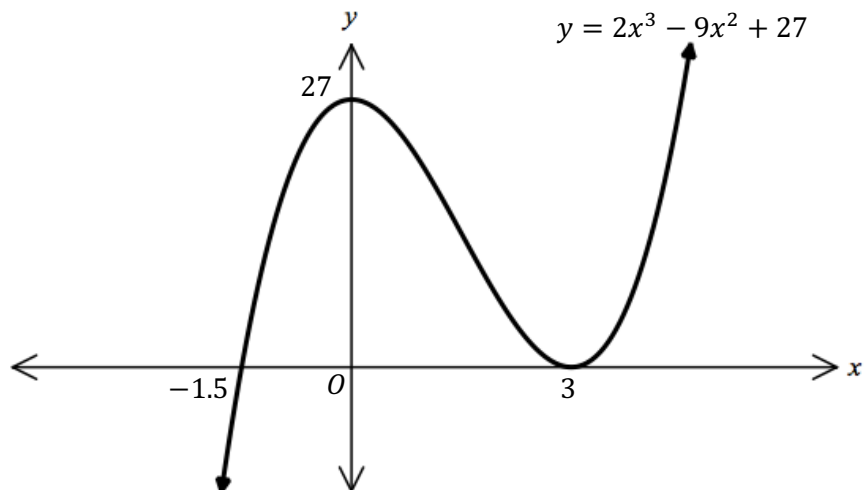
(C) $y = -x - 2$

(D) $y = -2x^2 - 2$

9. A particle's displacement equation is given by $x = 2 \sin \pi t + 1$.

Which of the following statements is true?

- (A) The maximum velocity of the particle occurs at $t = 2$ when $x = 1$.
- (B) The maximum velocity of the particle occurs at $t = 0.5$ when $x = 3$.
- (C) The maximum velocity of the particle occurs at $t = 1.5$ when $x = -1$.
- (D) The maximum velocity of the particle occurs at $t = 2$ when $x = 2\pi$.
10. The diagram below shows the graph of $y = 2x^3 - 9x^2 + 27$ with x intercepts at $x = 3$ and $x = -1.5$ and turning points at $x = 3$ and $y = 27$.



For what values of k does the function $y = 2x^3 - 9x^2 + k$ have one x intercept?

- (A) $0 < k < 27$
- (B) $k < -1.5$ or $k > 27$
- (C) $k < -1.5$ or $k > 3$
- (D) $k < 0$ or $k > 27$

Section II

60 marks

Attempt Questions 11–14.

Allow about 1 hour and 45 minutes for this section.

Answer each question on a NEW page.

In Questions 11–14, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (16 marks) Start a NEW page.

- (a) Solve $\frac{3}{x-2} \leq 1$. **3**
- (b) Consider the polynomial $P(x) = x^3 - 2px + q$, where p and q are constants and $p \neq 0$.
It is given that α , β and $\alpha + \beta$ are the roots of the equation $P(x) = 0$.
- (i) Show that $\alpha = -\beta$ and $q = 0$. **2**
- (ii) Hence, or otherwise, find all the roots of $P(x) = 0$ in terms of p . **2**
- (c) (i) Express $\sqrt{3} \sin x + \cos x$ in the form $R \sin(x + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. **2**
- (ii) Hence, or otherwise solve $\sqrt{3} \sin x = 1 - \cos x$ for $0 \leq x \leq 2\pi$. **3**
- (d) Using the substitution $u = 3 + e^x$, evaluate the integral below: **4**

$$\int_0^{\log_e 6} \frac{e^x}{\sqrt{3 + e^x}} dx.$$

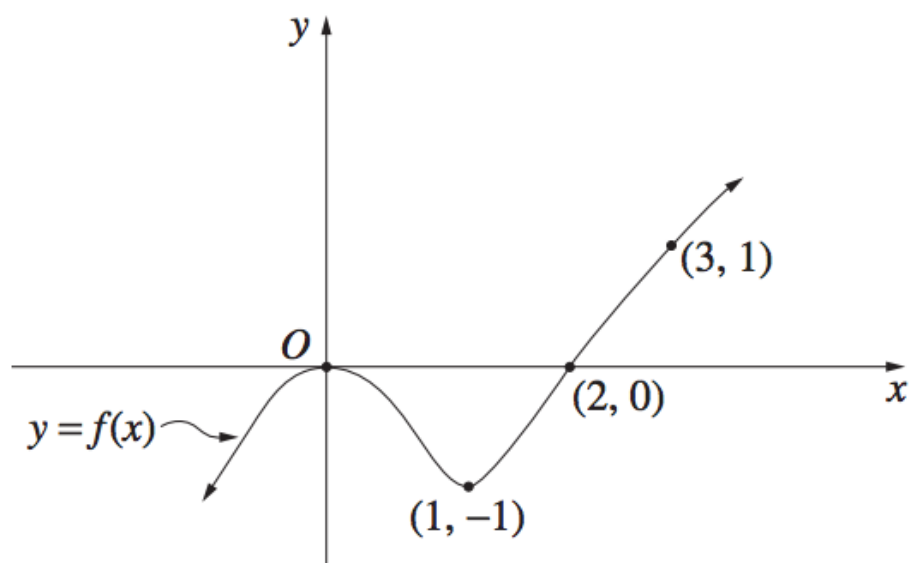
Question 12 (15 marks) Start a NEW page

- (a) Evaluate the following indefinite integral:

2

$$\int \sin 5x \cos 3x \, dx.$$

- (b) Consider the graph of $y = f(x)$ below.



Sketch each of the following curves, labelling the important points given.

Ensure your diagram is at least ONE-THIRD of the page.

(i) $y = \frac{1}{f(x)}$

3

(ii) $y^2 = f(x)$

2

Question 12 continues over the page

Question 12 (continued)

(c) Solve the differential equation $\frac{dy}{dx} = \frac{\sin^2 x}{y}$ given $y\left(\frac{\pi}{2}\right) = \sqrt{\frac{\pi}{2}}$, making y the subject. **4**

(d) An online furniture claims that 90% of all orders are shipped within 48 hours of a customer placing an order through the store's website. Adam ordered a total of 200 various pieces of furniture from the store for his company.

(i) Find the expected value and the standard deviation of the sampling proportion to 3 significant figures where necessary. **2**

(ii) The table below shows part of a table of $P(Z \leq z)$ values, where Z is a standard normal variable. **2**

Z	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
1	0.84134	0.84375	0.84614	0.84849	0.85083	0.85314	0.85543	0.85769	0.85993	0.86214
1.1	0.86433	0.86650	0.86864	0.87076	0.87286	0.87493	0.87698	0.87900	0.88100	0.88298
1.2	0.88493	0.88686	0.88877	0.89065	0.89251	0.89435	0.89617	0.89796	0.89973	0.90147

Using this table and ignoring continuity correction, find the approximate probability that less than 15 furniture pieces will NOT be delivered within 48 hours.

Question 13 (15 marks) Start a NEW page

- (a) Prove by mathematical induction for all positive integers n , that:

4

$$1 \times 1 + 3 \times 2 + 5 \times 3 + \cdots + (2n - 1) \times 2^{n-1} = 3 + 2^n(2n - 3).$$

- (b) (i) Given the identity $(1 + x)^{m+n} = (1 + x)^m(1 + x)^n$, prove that

2

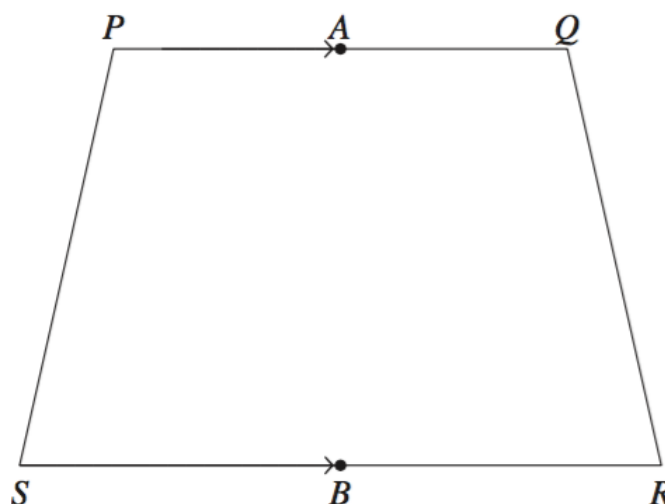
$$\binom{m+n}{4} = \binom{n}{4} + \binom{n}{3}\binom{m}{1} + \binom{n}{2}\binom{m}{2} + \binom{n}{1}\binom{m}{3} + \binom{m}{4}.$$

- (ii) Hence, by showing working, find the value of

1

$$\binom{16}{3}\binom{4}{1} + \binom{16}{2}\binom{4}{2} + \binom{16}{1}\binom{4}{3}$$

- (c) $PQRS$ is a trapezium with A and B being the midpoints of PQ and RS respectively.



Let $\overrightarrow{PA} = \mathbf{a}$ and $\overrightarrow{SB} = \mathbf{b}$

- (i) Prove, with reasons, that $\overrightarrow{QR} = \mathbf{b} - \mathbf{a} + \overrightarrow{AB}$.

2

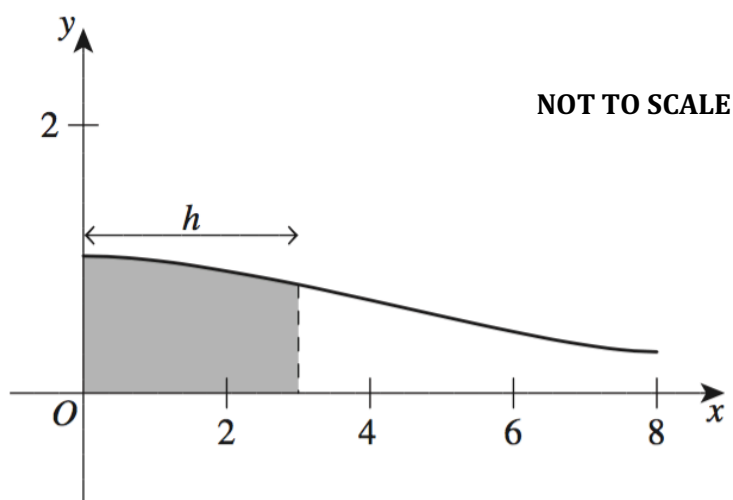
- (ii) Hence, or otherwise, show that $\overrightarrow{AB} = \frac{1}{2}(\overrightarrow{PS} + \overrightarrow{QR})$.

2

Question 13 continues over the page

Question 13 (continued)

- (d) The curve $y = \frac{3}{\sqrt{9+x^2}}$ for $x > 0$ is shown below.



- (i) A vase is formed by rotating the area underneath the curve from $x = 0$ to $x = 8$ around the horizontal axis. **2**

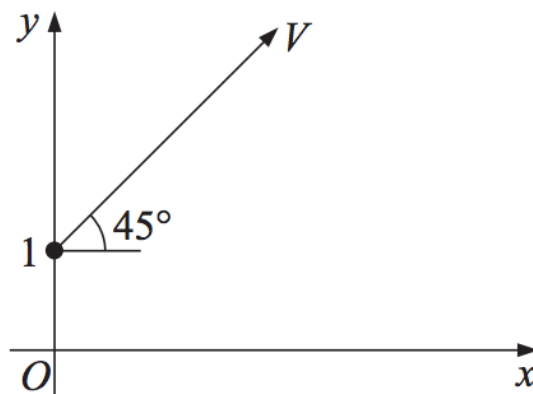
Show that the volume of the vase at any height h is given by $V = 3\pi \tan^{-1}\left(\frac{h}{3}\right)$ cubic units.

- (ii) The vase is now standing upright and water is poured into it. The height (h) of the water is increasing at a rate of 3 units per second. **2**

Find the exact rate at which the volume (V) of water is increasing when the height is 3 units.

Question 14 (14 marks) Start a NEW page

- (a) The diagram below shows a volleyball player serving a ball with initial speed V metres per second, 1 metre above ground with an angle of projection of 45° .



The horizontal component of the ball's displacement is given by $x = \frac{vt}{\sqrt{2}}$

The vertical component of the ball's velocity is given by $\dot{y} = -gt + \frac{v}{\sqrt{2}}$.

(DO NOT PROVE THESE)

- (i) Show that the Cartesian equation of the ball's path is given by **2**

$$y = 1 + x - \frac{gx^2}{V^2}.$$

- (ii) At a horizontal distance 9.3 metres away from the projection, the ball just clears the net of 2.3 metres high. **1**

Using $g = 9.8 \text{ m.s}^{-2}$, show that the initial speed of the ball is approximately 10.3 metres per second.

- (iii) Using the values from part (ii), find the distance from the net to the point where the ball lands? **3**
(Give you answer to 1 decimal place)

Question 14 continues over the page

Question 14 (continued)

- (b) A tank contains 10 tagged fish and 50 untagged fish. On each day, 4 fish are selected at random from the tank and placed together in a separate tank for observation. Later the same day the 4 fish are returned to the original tank.

(i) What is the probability of selecting no tagged fish on a given day? **1**

(ii) Given 7 days straight, what is the probability of selecting no tagged fish on exactly 3 of these 7 days? **2**

(Give your answer to 3 decimal places)

- (c) A triangle XYZ has an area of 40 square units and with angle XYZ being acute. **5**

The vector $\overrightarrow{YX} = \begin{pmatrix} 6 \\ 2 \end{pmatrix}$, $\overrightarrow{YZ} = \begin{pmatrix} p \\ q \end{pmatrix}$ and $|\overrightarrow{YZ}| = 8\sqrt{5}$.

Find the possible values of p and q .

End of Examination