

Week 1 – Problems

1. Given a set A , its complement A' and a universal set S , state which of the following expressions are true and which are false.

(a) $A \cup A' = S$ (b) $A \cap S = \emptyset$ (c) $A \cap A' = \emptyset$
(d) $A \cap A' = S$ (e) $A \cup \emptyset = S$ (f) $A \cup \emptyset = A$
(g) $A \cup \emptyset = \emptyset$ (h) $A \cap \emptyset = A$ (i) $A \cap \emptyset = \emptyset$
(j) $A \cup S = A$ (k) $A \cup S = \emptyset$ (l) $A \cup S = S$

2. List the elements of the following sets:

(a) $A = \{x : x \text{ is odd and } x \text{ is greater than 0 and less than 12}\}$
(b) $B = \{x : x \text{ is even and } x \text{ is greater than 19 and less than 31}\}$

3. For each of the following experiments, state whether the variable is discrete or continuous. In each case state the sample space.

- (a) The number of defective items in a batch of twenty is noted.
(b) The weight, in kg, of lubricating oil drained from a machine is determined using a spring balance.
(c) The natural logarithm of the weight, in kg, according to a spring balance, of lubricating oil drained from a machine, is noted.

4. A lot consists of 10 good articles, 4 articles with minor defects and 2 with major defects. One article is chosen at random from the lot. Find the probability that:

- (a) it has no defects,
(b) it has no major defects,
(c) it is either good or has major defects.

5. A central heating installation and maintenance engineer keeps a record of the causes of failure of systems he is called out to repair. The causes of failure are classified as 'electrical', 'gas' or in some cases 'other'. A summary of the records kept of failures involving either gas or electrical faults is as follows:

		Electrical	
		Yes	No
Gas	Yes	53	11
	No	23	13

- (a) Find the probability that failure involves gas given that it involves electricity.
 (b) Find the probability that failure involves electricity given that it involves gas.
6. Obtain the sample space of an experiment that consists of a fair coin being tossed four times. Consider the following events:
- A is the event 'all four results are the same'.
 - B is the event 'exactly one Head occurs'.
 - C is the event 'at least two Heads occur'.
- Show that $P(A) + P(B) + P(C) = \frac{17}{16}$ and explain why $P(A) + P(B) + P(C) > 1$.
7. If A and B are two events associated with an experiment and $P(A) = 0.4$, $P(A \cup B) = 0.7$ and $P(B) = p$, find:
- (a) the choice of p for which A and B are mutually exclusive
 (b) the choice of p for which A and B are independent.
8. A machine is operated by two workers. There are sixteen workers available. How many possible teams of two workers are there?
9. A random number generator produces sequences of independent digits, each of which is as likely to be any digit from 0 to 9 as any other. If X denotes any single digit, find $E(X)$.

- 10.** The probability that a mountain-bike rider travelling along a certain track will have a tyre burst is 0.05. Find the probability that among 17 riders:
- (a) exactly one has a burst tyre
 - (b) at most three have a burst tyre
 - (c) two or more have burst tyres.
- 11.** A continuous random variable T has the following probability density function.

$$f_T(u) = \begin{cases} 0 & (u < 0) \\ 3(1 - u/k) & (0 \leq u \leq k) \\ 0 & (u > k) \end{cases} .$$

Find

- (a) k .
- (b) $E(T)$.
- (c) $E(T^2)$.
- (d) $V(T)$.