

## Week 1 – Problems

**1.** Given a set  $A$ , its complement  $A'$  and a universal set  $S$ , state which of the following expressions are true and which are false.

- (a)  $A \cup A' = S$
- (b)  $A \cap S = \emptyset$
- (c)  $A \cap A' = \emptyset$
- (d)  $A \cap A' = S$
- (e)  $A \cup \emptyset = S$
- (f)  $A \cup \emptyset = A$
- (g)  $A \cup \emptyset = \emptyset$
- (h)  $A \cap \emptyset = A$
- (i)  $A \cap \emptyset = \emptyset$
- (j)  $A \cup S = A$
- (k)  $A \cup S = \emptyset$
- (l)  $A \cup S = S$

**2.** List the elements of the following sets:

- (a)  $A = \{x : x \text{ is odd and } x \text{ is greater than 0 and less than 12}\}$
- (b)  $B = \{x : x \text{ is even and } x \text{ is greater than 19 and less than 31}\}$

**3.** For each of the following experiments, state whether the variable is discrete or continuous. In each case state the sample space.

- (a) The number of defective items in a batch of twenty is noted.
- (b) The weight, in kg, of lubricating oil drained from a machine is determined using a spring balance.
- (c) The natural logarithm of the weight, in kg, according to a spring balance, of lubricating oil drained from a machine, is noted.

**4.** A lot consists of 10 good articles, 4 articles with minor defects and 2 with major defects. One article is chosen at random from the lot. Find the probability that:

- (a) it has no defects,
- (b) it has no major defects,
- (c) it is either good or has major defects.

- 5.** A central heating installation and maintenance engineer keeps a record of the causes of failure of systems he is called out to repair. The causes of failure are classified as ‘electrical’, ‘gas’ or in some cases ‘other’. A summary of the records kept of failures involving either gas or electrical faults is as follows:

		Electrical	
		Yes	No
Gas	Yes	53	11
	No	23	13

- (a) Find the probability that failure involves gas given that it involves electricity.  
 (b) Find the probability that failure involves electricity given that it involves gas.
- 6.** Obtain the sample space of an experiment that consists of a fair coin being tossed four times. Consider the following events:
- $A$  is the event ‘all four results are the same’.
  - $B$  is the event ‘exactly one Head occurs’.
  - $C$  is the event ‘at least two Heads occur’.
- Show that  $P(A) + P(B) + P(C) = \frac{17}{16}$  and explain why  $P(A) + P(B) + P(C) > 1$ .
- 7.** If  $A$  and  $B$  are two events associated with an experiment and  $P(A) = 0.4$ ,  $P(A \cup B) = 0.7$  and  $P(B) = p$ , find:
- the choice of  $p$  for which  $A$  and  $B$  are mutually exclusive
  - the choice of  $p$  for which  $A$  and  $B$  are independent.
- 8.** A machine is operated by two workers. There are sixteen workers available. How many possible teams of two workers are there?
- 9.** A random number generator produces sequences of independent digits, each of which is as likely to be any digit from 0 to 9 as any other. If  $X$  denotes any single digit, find  $E(X)$ .

**10.** The probability that a mountain-bike rider travelling along a certain track will have a tyre burst is 0.05. Find the probability that among 17 riders:

- (a) exactly one has a burst tyre
- (b) at most three have a burst tyre
- (c) two or more have burst tyres.

**11.** A continuous random variable  $T$  has the following probability density function.

$$f_T(u) = \begin{cases} 0 & (u < 0) \\ 3(1 - u/k) & (0 \leq u \leq k) \\ 0 & (u > k) \end{cases} .$$

Find

- (a)  $k$ .
- (b)  $E(T)$ .
- (c)  $E(T^2)$ .
- (d)  $V(T)$ .