Week 2 – PBL Solutions

1.

Mean $=$ 5.50, standard deviation $=$ 2.87, mid-spread $=$ 6. Very little to choose between the summary statistics, mean $=$ median and inter-quartile range is approximately twice the standard deviation.

For the second set of data mean $=$ 14.50, standard deviation $=$ 29.99 and the

inter-quartile range $=$ 6. Here the median and inter-quartile range are preferable to the mean and standard deviation - they represent the bulk of the data much more realistically.

2.

The Head of Department is right. The lecturer is only correct if both classes have the same number of students. Example: if class A has 20 students and class B has 60 students, the average mark will be: $(20 \times 36 + 60 \times 40)/(20 + 60) = 39\%$.

3.

(a) Stem and leaf diagram (2 digit leaves – tens and units).

7	66
8	03,12,29
9	47,69,74,74,78,78
10	32,45,55,65
11	03,10,24,31,36,66,67,77
12	14,29,35,77,83,89
13	00,30,37,39,78,85
14	24,36,37,38,70,88,93,94
15	21,94
16	04,52,72,82
17	27,33

(b) The sum of the lifetimes is $\sum x = 62802$. So the mean is

$$\frac{62802}{50} = 1256.04.$$

(c) Yes. The mean lifetime gives a reasonable indication of what can be expected since the distribution is fairly symmetrical. However it does not, of course, give any indication of the spread.

4.

(a) The required probability is $\mathsf{P}(W < 1000) = \dfrac{1000 - 98}{1030 - 980} = \dfrac{20}{50} = 0.4$

(b) The required probability is $\mathsf{P}(W < w) = \dfrac{w - 980}{1030 - 980} = \dfrac{w - 980}{50}$

(c) The probability that all five weigh less than w g is $\left(\dfrac{w - 980}{50}\right)^5$ so the pdf of the heaviest is

$$\frac{d}{dw}\left(\frac{w - 980}{50}\right)^5 = \frac{5}{50}\left(\frac{w - 980}{50}\right)^4 = 0.1\left(\frac{w - 980}{50}\right)^4 \quad \text{for } 980 < w < 1030.$$

5.

We have $\mu = 8$ so $\lambda = 0.125$.

(a) The probability is

$$\mathsf{P}(T < 5) = \int_0^5 0.125e^{-0.125t}\, dt = 1 - e^{-0.125\times 5} = 0.4647.$$

(b) We require

$$\int_t^{\infty} 0.125e^{-0.125x}\, dx = e^{-0.125t} = 0.95.$$

So $-0.125t = \log 0.95$ and

$$t = -\frac{\log 0.95}{0.125} = 0.4103.$$

That is, 24.6 s.

6.

(a) 0.9772 (b) 0.0049 (c) 0.3413 (d) 0.1510

7.

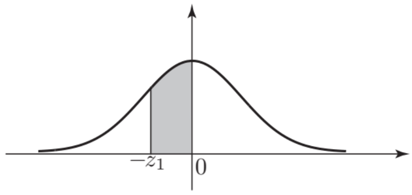
First, note that 99% corresponds to a probability of 0.99. Find z_1 such that

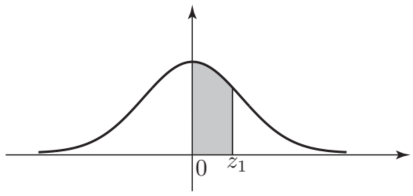
$$\mathsf{P}(0 < Z < z_1) = \frac{1}{2} \times 0.99 = 0.495 :$$

We look for a table value of 4950. The nearest we get is 4949 and 4951 corresponding to $Z = 2.57$ and $Z = 2.58$ respectively. We choose $Z = 2.58$.

8.

First sketch the standard normal curve marking the values z_1, z_2 between which 99% of the area under the curve is located:





By symmetry this shaded area is equal in value to the one above.

Now deduce the corresponding values x_1, x_2 for the general normal distribution:

$x_1 = \mu - 2.58\sigma, \quad x_2 = \mu + 2.58\sigma$

$x_1 = 2000 - 2.58 \times 40 = 1896.8$ hours

$x_2 = 2000 + 2.58 \times 40 = 2103.2$ hours

Answer

(1896.8 hours, 2103.2 hours).

9.

$X \sim N(0.8,\ 0.0004)$

(a) $\mathsf{P}(X > 0.81) = \mathsf{P}\left(Z > \dfrac{0.81 - 0.8}{0.02}\right)$

$= \mathsf{P}(Z > 0.5) = 0.5 - \mathsf{P}(0 < Z < 0.5) = 0.5 - 0.1915 = 0.3085$

(b) $\mathsf{P}[(X > 0.825) \cup (X < 0.785)] = 2\mathsf{P}(X > 0.825)$

$= 2\mathsf{P}\left(Z > \dfrac{0.025}{0.02}\right) = 2\mathsf{P}(Z > 1.25)$

$= 2[-\mathsf{P}(0 < Z < 1.25) + 0.5] = 2[-0.3944 + 0.5] = 0.2112$

10.

Let X be the time taken to assemble the component; then $X \sim N(110, 10)$

(a) $\mathsf{P}(X < 95) = \mathsf{P}(Z < \dfrac{95.0 - 110.0}{10.0}) = \mathsf{P}(Z < -1.5) = 1 - 0.9332 = 0.0668$ from Z-tables

Hence, from a group of 20 teams, the number completing the assembly within

95 minutes $= 20 \times 0.0668 = 1.336$ so the number of teams is 1.

(b) $\mathsf{P}(X > 120) = \mathsf{P}(Z > \dfrac{120.0 - 110.0}{10.0}) = \mathsf{P}(Z > 1.0) = 0.1587$ from Z-tables

Hence, from a group of 20 teams, the number completing the assembly in more than

2 hours $= 20 \times 0.1587 = 3.174$ so the number of teams is 3.

If 95% of teams are to complete the assembly 'on time', then 5% take longer than the set time, k, and $\mathsf{P}(X > k) = 0.05$ hence, $Z = 1.64$

Therefore, $\dfrac{k - 110.0}{10.0} = 1.64$ or, $k = 10(1.64) + 110.0 = 126.4$ minutes.

11.

Let the diameter of a nut be N. Let the diameter of a bolt be B. A bolt is too large for its nut if $N - B < 0$.

$$
\begin{aligned}
\mathsf{E}(N - B) &= 10 - 9.5 = 0.5 \\
\mathsf{V}(N - B) &= 0.02 + 0.02 = 0.04 \\
N - B &\sim N(0.5, 0.04) \\
\mathsf{P}(N - B < 0) &= \mathsf{P}\left(\frac{N - B - 0.5}{0.2} < \frac{0 - 0.5}{0.2}\right) = \mathsf{P}(Z < -2.5) \\
&= \Phi(-2.5) = 1 - \Phi(2.5) = 1 - 0.99379 \\
&= 0.00621.
\end{aligned}
$$

The probability that a bolt is too large for its nut is 0.00621.