Week 3 - Problems

1. The voltages of sixty nominally 10-volt cells are measured. Assuming these to be independent observations from a normal distribution with mean μ and variance σ^2, estimate μ and σ^2. Regarding this as a "large" sample, find a 99% confidence interval for μ. The data are:

10.3	10.5	9.6	9.7	10.6	9.9	10.1	10.1	9.9	10.5
10.1	10.1	9.9	9.8	10.6	10.0	9.9	10.0	10.3	10.1
10.1	10.3	10.5	9.7	10.1	9.7	9.8	10.3	10.2	10.2
10.1	10.5	10.0	10.0	10.6	10.9	10.1	10.1	9.8	10.7
10.3	10.4	10.4	10.3	10.4	9.9	9.9	10.5	10.0	10.7
10.1	10.6	10.0	10.7	9.8	10.4	10.3	10.0	10.5	10.1

2. Measurements are made on the lengths, in mm, of a sample of twenty wooden components for self-assembly furniture. Assume that these may be regarded as twenty independent observations from a normal distribution with unknown mean μ and unknown variance σ^2. The data are as follows.

581	580	581	577	580	581	577	579	579	578
581	583	577	578	582	581	582	580	582	579

3. A machine fills packets with powder. At intervals a sample of ten packets is taken and the packets are weighed. The ten weights may be regarded as a sample of ten independent observations from a normal distribution with unknown mean.

Find limits L, U such that the probability that $L < S^2 < U$ is 0.9 when the population variance is $\sigma^2 = 3.0$ and S^2 is the sample variance.

4. Dishwasher powder is poured into the cartons in which it is sold by an automatic dispensing machine which is set to dispense 3 kg of powder into each carton. In order to check that the dispensing machine is working to an acceptable standard (i.e. does not need adjustment), a production engineer takes a random samples of 40 cartons and weighs them. It is found that the mean weight of the sample is 3.005 kg. It is known that the dispensing machine operates with a standard deviation of 0.015 kg and that the manufacturer of the powder is willing to rely on a 5% level of significance. Does the sample provide the engineer with sufficient evidence that the true mean is not 3.00 kg and so the machine requires adjustment?

5. The average useful life of a random sample of 33 similar calculator batteries made on a production line is found to be 99.5 hours continuous use. The sample variance is 18.49 hours^2. Test the null hypothesis that the population mean lifetime is 100 hours against the alternative that it is less. Use the 5% level of significance.

6. In order to test the hypothesis that two standard turbochargers *A* and *B* have the same throttle reaction times, a random sample of 7 cars were fitted with the turbochargers and the throttle reaction times measured. The results were as follows:

Car	**1**	**2**	**3**	**4**	**5**	**6**	**7**
Throttle Reaction time for $A; R1$	0.223	0.212	0.201	0.205	0.216	0.211	0.209
Throttle Reaction time for $B; R2$	0.208	0.207	0.203	0.204	0.205	0.202	0.206
$D = R1 - R2$	0.015	0.005	-0.002	0.001	0.011	0.009	0.003

7. A motor manufacturer wishes to replace steel suspension components by aluminium components to save weight and thereby improve performance and fuel consumption. Tensile strength tests are carried out on randomly chosen samples of two possible components before a final choice is made. The results are:

Component Number	Sample Size	Mean Tensile Strength (kg mm^{-2})	Standard Deviation (kg mm^{-2})
1	15	90	2.3
2	10	88	2.2

Is there any difference between the measured tensile strengths at the 5% level of significance?

8. A manufacturer of electronic equipment has developed a circuit to feed current to a particular component in a computer display screen. While the new design is cheaper to manufacture, it can only be adopted for mass production if it passes the same average current to the component. In tests involving the two circuits, the following results are obtained.

Test Number	Circuit 1 - Current (mA)	Circuit 2 - Current (mA)
1	80.1	80.7
2	82.3	81.3
3	84.1	84.6
4	82.6	81.7
5	85.3	86.3
6	81.3	84.3
7	83.2	83.7
8	81.7	84.7
9	82.2	82.8
10	81.4	84.4
11		85.2
12		84.9

(a) On the assumption that the populations from which the samples are drawn **have equal variances**, should the manufacturer replace the old circuit design by the new one? Use the 5% level of significance.

(b) On the assumption that the populations from which the samples are drawn **do not have equal variances**, should the manufacturer replace the old circuit design by the new one? Use the 5% level of significance.

9. An automotive development engineer is investigating the properties of two fuel injection systems in order to determine whether they exhibit any significant difference in the level of fuel economy measured on different cars. The systems are fitted to 12 cars and a test is run ensuring that each injection system is used on each car under conditions which are as uniform as possible. The fuel consumption figures (in miles per gallon) obtained are given in the table below. Use the Wilcoxon signed-rank test applied to the differences in the paired data to decide whether the median fuel consumption figures are significantly different at the 5% level of significance.

	Fuel Injection System	
Car	1	2
1	27.6	26.3
2	29.4	31.0
3	29.5	28.2
4	27.2	26.1
5	25.8	27.6
6	26.9	25.8
7	26.7	28.2
8	28.9	27.6
9	27.3	26.9
10	29.2	30.3
11	27.8	26.9
12	29.2	28.3