Week 1 – PBL Solutions

1. (a) T, (b) F, (c) T, (d) F, (e) F, (f) T, (g) F, (h) F, (i) T, (j) F, (k) F), (l) T.

2. (a) $\{1, 3, 5, 7, 9, 11\}$, (b) $\{20, 22, 24, 26, 28, 30\}$.

3.

(a) The variable is discrete. The sample space is $\{1, 2, \ldots, 20\}$.

(b) The variable is continuous. The sample space is the set of real numbers x such that $0 \leq x < \infty$.

(c) The variable is continuous. The sample space is the set of real numbers x such that $-\infty < x < \infty$.

4.

Let G be the event 'article is good' , M_n be the event 'article has minor defect' and M_j be the event 'article has major defect'

(a) Here we require P(G). Obviously P$(G) = \dfrac{10}{16} = \dfrac{5}{8}$

(b) We require P$(M_j') = 1-$ P$(M_j) = 1 - \dfrac{2}{16} = \dfrac{7}{8}$

(c) The event we require is the complement of the event M_n.

Since P$(M_n) = \dfrac{4}{16} = \dfrac{1}{4}$ we have P$(M_n') =$ P$(G$ or $M_j) = 1 - \dfrac{1}{4} = \dfrac{3}{4}$.

Equivalently P(G) + P$(M_n) = \dfrac{10}{16} + \dfrac{2}{16} = \dfrac{12}{16} = \dfrac{3}{4}$

5.

(a) A total of 76 failures involved electrical faults. Of the 76 some 53 involved gas. Hence

$$\text{P}\{\text{Gas Failure} \mid \text{Electrical Failure}\} = \frac{53}{76} = 0.697$$

(b) A total of 64 failures involved electrical faults. Of the 64 some 53 involved gas. Hence

$$\text{P}\{\text{Electrical Failure} \mid \text{Gas Failure}\} = \frac{53}{64} = 0.828$$

6.

P$(A) = \dfrac{2}{16}$, P$(B) = \dfrac{4}{16}$, P$(C) = \dfrac{11}{16}$, P(A) + P(B) + P$(C) = \dfrac{17}{16}$

A, B and C are not mutually exclusive since events A and C have outcomes in common. This is the reason why P(A) + P(B) + P$(C) = \dfrac{17}{16}$; we are adding the probabilities corresponding to common outcomes more than once.

7.

(a) $\mathsf{P}(A \cup B) = \mathsf{P}(A) + \mathsf{P}(B)$ so $0.7 = 0.4 + p$ implying $p = 0.3$

(b) $\mathsf{P}(A \cup B) = \mathsf{P}(A) + \mathsf{P}(B) - \mathsf{P}(A) \times \mathsf{P}(B)$ so $0.7 = 0.4 + p - 0.4 \times p$ implying $p = 0.5$.

8.

The required number is

$$\binom{16}{2} = \frac{16 \times 15}{2 \times 1} = 120.$$

9.

x	0	1	2	3	4	5	6	7	8	9
$\mathsf{P}(X = x)$	$^1/_{10}$	$^1/_{10}$	$^1/_{10}$	$^1/_{10}$	$^1/_{10}$	$^1/_{10}$	$^1/_{10}$	$^1/_{10}$	$^1/_{10}$	$^1/_{10}$

$$\mathsf{E}(X) = \frac{1}{10}\{0 + 1 + 2 + 3 + \ldots + 9\} = 4.5$$

10.

Binomial distribution $\mathsf{P}(X = r) = {}^nC_r p^r (1-p)^{n-r}$ where p is the probability of single 'success' which is 'tyre burst'.

(a) $\mathsf{P}(X = 1) = {}^{17}C_1 (0.05)^1 (0.95)^{16} = 0.3741$

(b)

$$\begin{aligned} \mathsf{P}(X \le 3) &= \mathsf{P}(X=0) + \mathsf{P}(X=1) + \mathsf{P}(X=2) + \mathsf{P}(X=3) \\ &= (0.95)^{17} + 17(0.05)(0.95)^{16} + \frac{17 \times 16}{2 \times 1}(0.05)^2(0.95)^{15} \\ &\quad + \frac{17 \times 16 \times 15}{3 \times 2 \times 1}(0.05)^3(0.95)^{14} = 0.9912 \end{aligned}$$

(c) $\mathsf{P}(X \ge 2) = 1 - \mathsf{P}[(X = 0) \cup (X = 1)] = 1 - (0.95)^{17} - 17(0.05)(0.95)^{16} = 0.2077$

11.

(a) $1 = \int_0^k 3(1 - u/k)\,du = \left[3\left(u - \frac{u^2}{2k}\right)\right]_0^k = 3(k - k/2)$ so $k = 2/3$.

(b) $\mathsf{E}(T) = \int_0^{2/3} 3u(1 - 3u/2)\,du = 3\int_0^{2/3} u - 3u^2/2\,du$

$$3\left[\frac{u^2}{2} - \frac{u^3}{2}\right]_0^{2/3} = 3\left(\frac{2}{9} - \frac{4}{27}\right) = 3\left(\frac{6-4}{27}\right) = \frac{2}{9}.$$

(c) $\mathsf{E}(T^2) = \int_0^{2/3} 3u^2(1 - 3u/2)\,du = 3\int_0^{2/3} u^2 - 3u^3/2\,du$

$$= 3\left[\frac{u^3}{3} - \frac{3u^4}{8}\right]_0^{2/3} = 3\left(\frac{8}{81} - \frac{6}{81}\right) = 3\left(\frac{8-6}{81}\right) = \frac{2}{27}$$

(d) $\mathsf{V}(T) = \mathsf{E}(T^2) - \{\mathsf{E}(T)\}^2 = \frac{2}{27} - \frac{4}{81} = \frac{2}{81}$.