A Benchmark Problem for Robust Control Design

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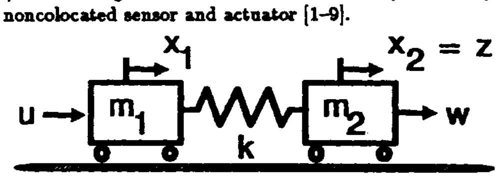
1. Introduction

The purpose of this paper is to formulate a simple, yet meaningful, control problem to highlight issues in robust controller design. The problem has been studied by several researchers under a variety of assumptions (see [1–9]).

In our formulation of the problem certain aspects, such as parameter uncertainty with given nominal parameter values and nominal desired performance, are given concretely, while other aspects, such as the measurement noise model, definition of settling time, measure of control effort and controller complexity, etc., are deliberately left vague. Each designer is thus free to inject into the problem any desired level of realism.

2. Benchmark Problem

Consider the two-mass/spring system shown in the following figure, which is a generic model of an uncertain dynamical system with noncolocated sensor and actuator [1–9].



A control force acts on body 1, and the position of body 2 is measured resulting in a noncolocated control problem. This system can be represented in state-space form as

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\end{bmatrix}=\begin{bmatrix}0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ -k/m_1 & k/m_1 & 0 & 0\\ k/m_2 & -k/m_2 & 0 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix}+\begin{bmatrix}0\\ 0\\ 1/m_1\\ 0\end{bmatrix}u+\begin{bmatrix}0\\ 0\\ 0\\ 1/m_2\end{bmatrix}w$$

$$\begin{aligned} y &= x_2 + v\\ z &= x_2 \end{aligned}$$

where

x_1 = position of body 1
x_2 = position of body 2
x_3 = velocity of body 1
x_4 = velocity of body 2
u = control input
w = plant disturbance
y = sensor measurement
v = sensor noise
z = performance variable (output to be controlled)

3. Design Problems

Design #1. Design a constant gain linear feedback compensator of the form

$$\begin{aligned} \dot{x}_c &= A_c x_c + B_c y\\ u &= C_c x_c + D_c y \end{aligned}$$

(any of these matrices may of course be zero) with the following properties:

i) The closed-loop system is stable for $m_1 = m_2 = 1$ and $0.5 < k < 2.0$.

ii) For $w(t)$ = unit impulse at $t = 0$, the performance variable z has a settling time of about 15 seconds for the nominal system $m_1 = m_2 = k = 1$.

iii) The measurement noise $v(t)$ is to be characterized by each designer to reflect realism and practical control design.

iv) Achieve reasonable performance/stability robustness.

v) Minimize controller effort.

vi) Minimize controller complexity.

Design #2 (optional). Same as Design #1 except in place of *ii*) insert:

ii) $w(t)$ is a sinusoidal disturbance of frequency 0.5 rad/sec but whose amplitude and phase, although constant, are not available to the designer. Achieve asymptotic rejection of $w(t)$ at the performance variable $z(t)$ (i.e., minimize $\limsup_{t\to\infty} z(t)$ with a 20 second settling time) for $m_1 = m_2 = 1$, $0.5 < k < 2.0$.

Design #3 (optional). Same as Design #1 except in place of *i*) insert:

i) Maximize a stability performance measure with respect to the three uncertain parameters m_1, m_2, k whose nominal values are $m_1 = m_2 = k = 1$.

For each design, please give

1) matrices A_c, B_c, C_c, D_c
2) poles and zeros of compensator
3) measurement noise model $v(t)$
4) evidence of robustness, settling time, disturbance rejection where applicable
5) assessment of controller complexity and control effort.

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