Networked Fusion Estimation With Bounded Noises

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***Abstract*—Most of time-varying systems in fusion estimation are generally modeled without bounded noises. In this paper, we study the distributed fusion estimation problem for networked time-varying systems with bounded noises, where the resource constraints (i.e., bandwidth or energy) and quantization effect are described by a unified model. A new local estimator with time-varying gain is designed by solving a class of convex optimization problems such that the square error of the estimator is bounded. When each local estimate is transmitted to the fusion center over communication networks, the selection probability criterion is derived such that the mean square error of the each compensating state estimate is bounded. Then, a convex optimization problem on the design of an optimal weighting fusion criterion is established in terms of linear matrix inequalities, which can be solved by standard software packages. Target tracking system with time-varying sampling period is given to show the effectiveness of the proposed method.**

***Index Terms*—Bounded noises, communication constraints, convex optimization, networked fusion estimation.**

I. Introduction

Multisensor fusion estimation is one of the most important focuses in the area of state estimation and information fusion, and different fusion methods have been developed in [1]–[3]. More recently, networked multisensor fusion estimation has attracted considerable research interest, and has found applications in a broad range of areas such as networked filtering in wireless sensor networks [4] and cyber-physical systems [5]. Under the networked fusion framework, two issues must be taken into account: 1) communication delays and packet dropouts; 2) constraint of sensor energy and communication bandwidth. Notice that information loss is inevitable because of the above constraints, and such a fusion estimation with incomplete information degrades the estimation performance. Particularly, networked fusion estimation methods with delays and packet dropouts were presented in [6]–[9] to solve the first issue. In this paper, we will focus on the second issue.

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Since the lower communication traffic is able to save energy, and meanwhile meets the bandwidth constraint, the straightforward idea is to reduce the size of data packets. In this sense, to guarantee a satisfactory fusion estimation performance, various quantization methods and dimensionality reduction methods have been proposed in [10]–[15]. Meanwhile, when designing the estimator in energy-constrained sensor networks, it is not necessary for sensors to transmit messages at every sampling instants [16]. Although this method cannot deal with the bandwidth constrain problem, it can be combined with the idea of packet size reduction to design the fusion estimator in bandwidth and energy constrained sensor networks [12]. Notice that the statistical property of disturbance noises in [1]–[3], [6]–[8] and [10]–[14] was assumed to be known *a priori*; however, this assumption is not always satisfied in practical applications. On the other hand, H_∞ estimation method does not make any assumption on the statistical property of system noises, and the only assumption is that the external disturbance is the energy-bounded noise [17], [18]. Therefore, centralized and distributed H_∞ fusion estimation algorithms have been developed in [15] and [19]–[21] for various fusion systems with packet dropouts, time-delays, and quantization effect. It should be pointed out that when the time t goes to ∞, the energy-bounded noise $w(t)$ in the H_∞ fusion framework assumes that the noise $w(t)$ must be 0 (i.e., $\lim_{t\to\infty} w(t) = 0$). However, the noise $w(t)$ may always exist in the systems. Under this case, when the noise $w(t)$ is bounded at each time but $\lim_{t\to\infty} w(t) \neq 0$, H_∞ fusion methods cannot design the stable estimator for time-invariant/time-varying fusion systems.

Motivated by the aforementioned analysis, the aim in this paper is to design the stable fusion estimation algorithm for the networked time-varying fusion systems with bounded noises and resource constraints. Although the distributed mixed H_2/H_∞ fusion estimation algorithm was developed in [22] under the bandwidth constraints, it can only deal with the stable time-invariant fusion systems with the energy-bounded noises and Gaussian white noises. The main contributions are as follows: 1) To guarantee the boundedness of the estimation error, a new local estimator with time-varying gains is designed by using standard software packages to solve a class of convex optimization problems; 2) A unified signal transmission model, which is dependent on the selection probability of each component of the local estimate, is proposed to describe the resource constraints (i.e., bandwidth and energy) and quantization effect. Then, a probability-dependent stability condition is derived such that the mean square error of the local compensating state estimator is bounded. Under this condition, the distributed fusion estimator (DFE) with time-varying weighting matrices is designed by minimizing an upper bound of the square error of the DFE at each time.

Notations: The superscript $''\mathrm{T}''$ represents the transpose, while $''I''$ represents the identity matrix with appropriate dimension. $X > (<) 0$ denotes a positive-definite (negative-definite) matrix, $\mathrm{diag}\{\cdot\}$ stands for a block diagonal matrix. $||A||_2$ represents 2-norm of the matrix A, while $\mathrm{E}\{\cdot\}$ is the mathematical expectation. The symmetric terms in a symmetric matrix are denoted by "$*$", and $\mathrm{col}\{a_1, \ldots, a_n\}$ means a column vector whose elements are $a_1, \ldots, a_n$. Moreover, if $\tau_1 >$

τ_2, it will be specified that $\prod_{\tau=\tau_1}^{\tau_2} F(\tau) = I_m$, where $F(\tau) \in \mathbb{R}^{m\times m}$ represents a matrix function with respect to the variable τ. The operator "$\otimes$" denotes the Hadamard product.

II. Problem Statement

A. System Modeling

Consider a linear discrete-time system described by the following state–space model:

$$x(t+1) = A(t)x(t) + \Gamma(t)w(t) \tag{1}$$

$$y_i(t) = C_i(t)x(t) + B_i(t)v(t)(i = 1, \ldots, L) \tag{2}$$

where $x(t) \in \mathbb{R}^n$ $(n > 1)$ is the system state, $y_i(t) \in \mathbb{R}^{q_i}$ is the measured output from sensor i, and L is the number of sensors. $A(t)$, $\Gamma(t)$, $C_i(t)$, and $B_i(t)(i = 1, \ldots, L)$ are time-varying matrices with appropriate dimensions. $w(t)$ and $v(t)$ are the bounded noises, i.e.,

$$||w(t)||_2 \le \delta_w, ||v(t)||_2 \le \delta_v \tag{3}$$

where δ_w and δ_v are unknown. Based on the measurements $\{y_i(1), y_i(2), \ldots, y_i(t)\}$, the local estimator is given by the following recursive form:

$$\hat{x}_i(t+1) = A(t)\hat{x}_i(t) + \mathrm{K}_i(t)(y_i(t) - C_i(t)\hat{x}_i(t)) \tag{4}$$

where the time-varying estimator gain $\mathrm{K}_i(t)$ is to be designed such that the estimation performance is optimal.

Under the networked fusion framework, when each local estimate $\hat{x}_i(t)$ is transmitted to the fusion center (FC) through a constrained communication network, only r_i $(1 \le r_i < n)$ components of $\hat{x}_i(t)(\in \mathbb{R}^n)$ are transmitted to the FC at each time to satisfy finite bandwidth [12]. Then, the allowed sending components (ASC) of each $\hat{x}_i(t)$ has Δ_i possible cases, where $\Delta_i = \prod_{\ell_n=0}^{r_i-1}(n-\ell_n)/\prod_{\ell_r=1}^{r_i}\ell_r$. In this case, the selected allowed sending components (SASC), denoted by $\hat{x}_{s_i}(t)(\in \mathbb{R}^{r_i})$, only takes one element from the finite set $\Theta_i(t)$.

$$\Theta_i(t) \triangleq \{\hat{x}_{\hbar_i}^{s_i}(t) | \hbar_i = 1, 2, \ldots, \Delta_i\} \tag{5}$$

where $\hat{x}_{\hbar_i}^{s_i}(t)$ represents one group of ASCs. On the other hand, each sensor intermittently sends the SASC $\hat{x}_{s_i}(t)$ to save the sensor energy, which means that the FC may not receive the messages from sensors at arbitrary time. Under this condition, when the SASC $\hat{x}_{s_i}(t)$ is not sent by the sensor, the signal received by the FC will be regarded as "0." Notice that even if the SASC $\hat{x}_{s_i}(t)$ is transmitted to the FC over communication channels, the quantization error may be difficult to be avoided. Therefore, for the local estimate $\hat{x}_i(t)$, the signal received by the FC, denoted as $\hat{\mathrm{x}}_{u_i}(t)$, can be modeled by

$$\hat{\mathrm{x}}_{u_i}(t) = \eta_i(t)[\hat{\mathrm{x}}_{s_i}(t) + D_i\varsigma(t)] \tag{6}$$

where $\eta_i(t) \in \{0, 1\}$ denotes whether the ith sensor sends $\hat{x}_{s_i}(t)$ or not at time t, and "$D_i\varsigma(t)$" is used to model the quantization error with respective to $\hat{x}_{s_i}(t)$, where $\varsigma(t) \in \mathbb{R}$ is the bounded noise, i.e.,

$$||\varsigma(t)||_2 \le \delta_\varsigma \tag{7}$$

where δ_ς is unknown. To construct a simple mathematical model for the $\hat{\mathrm{x}}_{u_i}(t)$, let Δ_i elements of the set $\chi_i(t)$ be indexed from 1 to Δ_i. Then, we introduce the following indicator functions:

$$\sigma_{i,\hbar_i}(t) = \begin{cases} 1 & \text{if } \hat{x}_{s_i}(t) = \hat{x}_{\hbar_i}^{s_i}(t) \\ 0 & \text{if } \hat{x}_{s_i}(t) \ne \hat{x}_{\hbar_i}^{s_i}(t) \end{cases} \tag{8}$$

where $\sigma_{i,\hbar_i}(t)(\hbar_i = 1, 2, \ldots, \Delta_i)$ satisfy

$$\sigma_{i,\hbar_i}(t)\sigma_{i,\hbar_i^o}(t) = 0(\hbar_i \ne \hbar_i^o), \sum_{\hbar_i=0}^{\Delta_i} \sigma_{i,\hbar_i}(t) \in \{0, 1\}. \tag{9}$$

According to the definition of $\eta_i(t)$, one has by (8–9) that

$$\eta_i(t) = \sum_{\hbar_i=0}^{\Delta_i} \sigma_{i,\hbar_i}(t). \tag{10}$$

Then, it follows from (6), (9), and (10) that

$$\hat{\mathrm{x}}_{u_i}(t) = \sum_{\hbar_i=0}^{\Delta_i} \sigma_{i,\hbar_i}(t)\hat{x}_{\hbar_i}^{s_i}(t) + \left(\sum_{\hbar_i=0}^{\Delta_i} \sigma_{i,\hbar_i}(t)\right) D_i\varsigma(t). \tag{11}$$

At time t, if the fusion estimate of $x(t)$ is directly designed based on $\hat{\mathrm{x}}_{u_i}(t)$, fusion estimation performance may be poor because of the untransmitted components of $\hat{x}_i(t)$. In this sense, let each $H_{i,\hbar_i}$ denote a diagonal matrix that contains r_i diagonal elements "1" and $n - r_i$ diagonal elements "0," then the compensating state estimate (CSE) of $x(t)$, denoted by $\hat{\mathrm{x}}_i^{\mathrm{c}}(t)$, is given by

$$\begin{aligned}\hat{\mathrm{x}}_{\mathrm{i}}^{\mathrm{c}}(\mathrm{t}) = {} & [I - H_i(t)]A(t-1)\hat{\mathrm{x}}_i^{\mathrm{c}}(t-1) \\ & + H_i(t)\hat{x}_i(t) + Q_i(t)D_i\varsigma(t)\end{aligned} \tag{12}$$

where $H_i(t)$ is determined by

$$H_i(t) \triangleq \sum_{\hbar_i=1}^{\Delta_i} \sigma_{i,\hbar_i}(t)H_{i,\hbar_i} = \mathrm{diag}\{\gamma_{1i}(t), \ldots, \gamma_{ni}(t)\} \tag{13}$$

while $Q_i(t) \in \mathbb{R}^{n\times r_i}$ is a 0–1 matrix (i.e., each element of $Q_i(t)$ is "0" or "1") satisfying

$$H_i(t)Q_i(t) = Q_i(t) \tag{14}$$

and $Q_i(t)$ in (12) is used to describe the quantization effect for the transmitted SASC $\hat{\mathrm{x}}_{s_i}(t)$. Moreover, it follows from (8) and (9) that

$$\gamma_{\ell,i}(t) \in \{0, 1\}, \left(\sum_{\ell=1}^{n} \gamma_{\ell,i}(t)\right) \in \{r_i, 0\}(i = 1, \ldots, L) \tag{15}$$

where $\gamma_{\ell,i}(t) = 1$ means that the ℓth component of $\hat{x}_i(t)$ is selected and sent to the FC, while $\gamma_{\ell,i}(t) = 0$ means that the ℓth component of $\hat{x}_i(t)$ is discarded. Particularly, the CSE model (12) can describe the following two cases.

1) $\sum_{\ell=1}^{n} \gamma_{\ell,i}(t) = r_i$ means that $H_i(t) \ne 0$ and $Q_i(t) \ne 0$. Under this case, the untransmitted components of $\hat{x}_i(t)$ are compensated by "$[I - H_i(t)]A(t-1)\hat{\mathrm{x}}_i^{\mathrm{c}}(t-1)$."
2) $\sum_{\ell=1}^{n} \gamma_{\ell,i}(t) = 0$ means that $H_i(t) = 0$ and $Q_i(t) = 0$. Under this case, the ith sensor does not send the information to the FC at time t, and the one-step prediction "$A(t-1)\hat{\mathrm{x}}_i^{\mathrm{c}}(t-1)$" is used to estimate the state $x(t)$.

B. Problem of Interest

It is concluded from (11) that the received signal $\hat{\mathrm{x}}_{u_i}(t)$ is determined by the binary variables $\sigma_{i,\hbar_i}(t)(\hbar_i = 1, 2, \ldots, \Delta_i)$. Therefore, let each stochastic process $\{\sigma_{i,\hbar_i}(t)\}$ be independent and identically distributed (i.i.d.), i.e.,

$$\begin{aligned}&\mathrm{E}\{\sigma_{\mathrm{i},\hbar_{\mathrm{i}}}(\mathrm{t})\sigma_{\mathrm{j},\hbar_{\mathrm{j}}^{o}}(\mathrm{t}_1)\}\\ &=\begin{cases}0 & i=j, t=t_1, \hbar_i\neq\hbar_i^o\\ \mathrm{E}\{\sigma_{i,\hbar_i}(t)\} & i=j, t=t_1, \hbar_i=\hbar_i^o\\ \mathrm{E}\{\sigma_{i,\hbar_i}(t)\}\mathrm{E}\{\sigma_{i,\hbar_i^o}(t_1)\} & i=j, t\neq t_1, \forall\hbar_i,\hbar_i^o\\ \mathrm{E}\{\sigma_{i,\hbar_i}(t)\}\mathrm{E}\{\sigma_{j,\hbar_j^o}(t_1)\} & i\neq j, \forall t, t_1, \hbar_i, \hbar_j^o.\end{cases}\end{aligned} \tag{16}$$

Moreover, the selection probabilities for the cases $\sigma_{i,\hbar_i}(t)=1$ and $\sigma_{i,\hbar_i}(t)=0$ are taken as

$$\mathrm{Prob}\{\sigma_{i,\hbar_i}(t)=1\}=\pi_{i,\hbar_i}, \mathrm{Prob}\{\sigma_{i,\hbar_i}(t)=0\}=1-\pi_{i,\hbar_i} \tag{17}$$

where $0\leq\pi_{i,\hbar_i}\leq 1$. Then, it follows from (9), (10), (16), and (17) that the selection probabilities $\pi_{i,\hbar_i}(\hbar_i=1,\ldots,\Delta_i)$ must satisfy

$$\sum_{\hbar_i=1}^{\Delta_i}\pi_{i,\hbar_i}=\eta_i\ (\eta_i\leq 1) \tag{18}$$

where $\mathrm{E}\{\eta_i(t)\}=\eta_i$, and $1-\eta_i$ denotes the *energy saving rate* (ESR) of the ith sensor. Notice that the selection probabilities $\pi_{i,\hbar_i}(\hbar_i=1,2,\ldots,\Delta_i)$ are to be deigned such that the mean square error (MSE) of the CSE $\hat{\mathrm{x}}_i^c(t)$ is bounded.

According to the CSEs (12), the DFE for the addressed networked fusion systems is given by

$$\hat{\mathrm{x}}(t)=\sum_{i=1}^{L}\mathrm{W}_i(t)\hat{\mathrm{x}}_i^c(t) \tag{19}$$

where $\sum_{i=1}^{L}\mathrm{W}_i(t)=I$, then how to design the optimal weighting matrix will be discussed in Section III.

Consequently, the problems to be solved in this paper are described as follows.

1) For each local estimator (4), the aim is to design the time-varying estimator gain $\mathrm{K}_i(t)$ such that the square error (SE) of $\hat{x}_i(t)$ is bounded, and then to minimize an upper bound of the SE at each time step.
2) For each local CSE (12), the aim is to find the selection probabilities $\pi_{i,\hbar_i}(\hbar_i=1,2,\ldots,\Delta_i)$ in (18) such that the MSE of each $\hat{\mathrm{x}}_i^c(t)$ is bounded, and then to design the optimal $\mathrm{W}_i(t)(i=1,2,\ldots,L)$ in (19) by minimizing an upper bound of the SE of the DFE $\hat{\mathrm{x}}(t)$ at each time step.

Remark 1: When the covariance of a Gaussian white noise is bounded, the Gaussian disturbance in a practical system can be viewed as a bounded signal at each time step. Under this case, compared with the Kalman fusion estimation algorithms in [1]–[3], [6]–[8] and [10]–[12], the distributed fusion estimation algorithm in this paper does not need the statistical property of the disturbance noises that may be difficult to know in practical applications. At the same time, compared with the H_∞ fusion estimation algorithms in [15] and [19]–[22], the distributed fusion estimation algorithm in this paper is not require to satisfy $\lim_{t\to\infty}w(t)=0$ and $\lim_{t\to\infty}v(t)=0$, and can deal with the time-varying fusion systems. Notice that the assumptions $\lim_{t\to\infty}w(t)=0$ and $\lim_{t\to\infty}v(t)=0$ are somewhat rigorous for the systems (1–2) with disturbance noises, and cannot be satisfied in some practical applications (e.g., sensor's measurement noise generated from the external environment). Therefore, the noise assumption (3) is easier to be satisfied for a practical system, and the proposed fusion estimation algorithm is applicable to a more general case.

III. Main Results

In this section, the optimal estimator gain $\mathrm{K}_i(t)$ in (4) will be given in Theorem 1. The probability selection criterion, which can determine whether each component of $\hat{x}_i(t)$ is sent to the FC, will be derived such that the MSE of $\hat{\mathrm{x}}_i^c(t)$ is bounded, and an optimal weighting fusion criterion will be presented in Theorem 2. Before deriving the result of Theorem 1, let us define

$$\begin{cases}e_i(t)\triangleq x(t)-\hat{x}_i(t), \bar{w}(t)\triangleq\mathrm{col}\{w(t),v(t)\}\\ G(t)\triangleq[\Gamma(t)\ \ 0], G_i(t)\triangleq[0\ \ B_i(t)].\end{cases} \tag{20}$$

Theorem 1: For a given $\mu_i(0<\mu_i<1)$, the optimal estimator gain $\mathrm{K}_i(t)$ can be obtained by solving the following convex optimization problem:

$$\begin{aligned}&\min_{P_i(t)>0,\mathrm{K}_i(t),\chi_{i1}(t),\chi_{i2}(t)}\ \mu_i\chi_{i1}(t)+(1-\mu_i)\chi_{i2}(t)\\ &\text{s.t.}:\begin{cases}\begin{bmatrix}-I & A(t)-\mathrm{K}_i(t)C_i(t) & G(t)-\mathrm{K}_i(t)G_i(t)\\ * & -P_i(t) & 0\\ * & * & -\chi_{i2}(t)I\end{bmatrix}<0\\ P_i(t)-\chi_{i1}(t)I<0\\ 0<\chi_{i1}(t)<1\end{cases}\end{aligned} \tag{21}$$

where "s.t." is the abbreviation of "subject to." Under this case, an upper bound of the SE of $\hat{x}_i(t)$ (i.e., an upper bound of $e_i^{\mathrm{T}}(t)e_i(t)$) can be taken as

$$\begin{aligned}f_u(e_i(t))=\ &\chi_{i1}(t-1)e_i^{\mathrm{T}}(t-1)e_i(t-1)\\ &+\chi_{i2}(t-1)\bar{w}^{\mathrm{T}}(t-1)\bar{w}(t-1).\end{aligned} \tag{22}$$

Moreover, the SE of $\hat{x}_i(t)$ is bounded, i.e., there exists a positive scalar $p_i>0$ such that

$$\lim_{t\to\infty}e_i^{\mathrm{T}}(t)e_i(t)<p_i. \tag{23}$$

Proof: It follows from (1) and (4) that

$$\begin{aligned}e_i(t+1)=\ &[A(t)-\mathrm{K}_i(t)C_i(t)]e_i(t)\\ &+[G(t)-\mathrm{K}_i(t)G_i(t)]\bar{w}(t)\end{aligned} \tag{24}$$

where $e_i(t)$, $G(t)$, $G_i(t)$, and $\bar{w}(t)$ are defined by (20). Then, one has by (24) that

$$\begin{aligned}J_i(t+1)\triangleq\ &e_i^{\mathrm{T}}(t+1)e_i(t+1)-e_i^{\mathrm{T}}(t)P_i(t)e_i(t)\\ &-\chi_{i2}(t)\bar{w}^{\mathrm{T}}(t)\bar{w}(t)=\begin{bmatrix}e_i(t)\\ \bar{w}(t)\end{bmatrix}^{\mathrm{T}}\Sigma_i(t)\begin{bmatrix}e_i(t)\\ \bar{w}(t)\end{bmatrix}\end{aligned} \tag{25}$$

where

$$\Sigma_i(t)=\begin{bmatrix}A_{f_i}^{\mathrm{T}}(t)A_{f_i}(t)-P_i(t) & A_{f_i}^{\mathrm{T}}(t)G_{f_i}(t)\\ * & -\chi_{i2}(t)I+G_{f_i}^{\mathrm{T}}(t)G_{f_i}(t)\end{bmatrix} \tag{26}$$

with $P_i(t)>0$, $\chi_{i2}(t)>0$ and

$$\begin{cases}A_{f_i}(t)\triangleq A(t)-\mathrm{K}_i(t)C_i(t)\\ G_{f_i}(t)\triangleq G(t)-\mathrm{K}_i(t)G_i(t).\end{cases} \tag{27}$$

According to the Schur Complement lemma [23], the first inequality in (21) is equivalent to $\Sigma_i(t)<0$. In this case, it is concluded from (25) that when the first inequality in (21) holds, one has $J_i(t+1)<0$, i.e., $e_i^{\mathrm{T}}(t+1)e_i(t+1)<e_i^{\mathrm{T}}(t)P_i(t)e_i(t)+\chi_{i2}(t)\bar{w}^{\mathrm{T}}(t)\bar{w}(t)$. Since $P_i(t)>0$, it can be obtained that

$$e_i^{\mathrm{T}}(t)P_i(t)e_i(t)\leq\lambda_{\max}(P_i(t))e_i^{\mathrm{T}}(t)e_i(t). \tag{28}$$

Notice that the second inequality "$P_i(t)-\chi_{i1}(t)I<0$" in (21) is equivalent to

$$D_{P_i}(t)<\chi_{i1}(t)U_{P_i}(t)U_{P_i}^{\mathrm{T}}(t)=\chi_{i1}(t)I \tag{29}$$

where $D_{P_i}(t)$ is a diagonal matrix formed by the eigenvalues of $P_i(t)$, and $U_{P_i}(t)$ is an orthogonal matrix whose columns are normalized eigenvectors. Then, it follows from (28) and (29) that

$$e_i^{\mathrm{T}}(t+1)e_i(t+1) < \chi_{i1}(t)e_i^{\mathrm{T}}(t)e_i(t) + \chi_{i2}(t)\bar{w}^{\mathrm{T}}(t)\bar{w}(t) \quad (30)$$

which means that (22) can be chosen as an upper bound of $e_i^{\mathrm{T}}(t)e_i(t)$.

At time $t+1$, it is known from (4) and (30) that the estimation error $e_i(t+1)$ is determined by the time-varying gain $\mathrm{K}_i(t)$, and $e_i(t)$ has been determined by the previous time. Thus, to minimize the upper bound of $e_i^{\mathrm{T}}(t+1)e_i(t+1)$, only two parameters $\chi_{i1}(t)$ and $\chi_{i2}(t)$ in (30) can be optimized to determine the gain $\mathrm{K}_i(t)$. Meanwhile, it follows from (26) and (29) that

$$A_{f_i}^{\mathrm{T}}(t)A_{f_i}(t) < \chi_{i1}(t)I, G_{f_i}^{\mathrm{T}}(t)G_{f_i}(t) < \chi_{i2}(t)I. \quad (31)$$

Then, (31) implies that it may be difficult to simultaneously minimize the parameters $\chi_{i1}(t)$ and $\chi_{i2}(t)$. In this case, the weight $\mu_i (0 < \mu_i < 1)$ is introduced to model the optimization object.

On the other hand, it is derived from (30) that

$$\begin{aligned} e_i^{\mathrm{T}}(t)e_i(t) < {} & \left(\prod\nolimits_{\kappa=1}^{t-1} \chi_{i1}(t-\kappa)\right) e_i^{\mathrm{T}}(1)e_i(1) \\ & + \sum_{\kappa=1}^{t-1} \Big\{ \left(\prod\nolimits_{\tau=1}^{\kappa-1} \chi_{i1}(t-\tau)\right) \\ & \times \chi_{i2}(t-\kappa)\bar{w}^{\mathrm{T}}(t-\kappa)\bar{w}(t-\kappa)\Big\}. \end{aligned} \quad (32)$$

Notice that $t \to \infty$ is equivalent to $\kappa \to \infty$ in (32). When the third condition "$0 < \chi_{i1}(t) < 1$" in (21) holds, one has

$$\begin{cases} \lim\limits_{t\to\infty} \prod_{\kappa=1}^{t-1} \chi_{i1}(t-\kappa) = 0 \\ \lim\limits_{\kappa\to\infty} \prod_{\tau=1}^{\kappa-1} \chi_{i1}(t-\tau) = 0. \end{cases} \quad (33)$$

Since $\bar{w}(t)$ is the bounded noise, it can be concluded from (32) and (33) that $\lim\limits_{t\to\infty} e_i^{\mathrm{T}}(t)e_i(t)$ is bounded, i.e., the result (23) holds. ■

Remark 2: For the standard H_∞ estimation method (see [17] and [18]), the estimator gain is obtained by minimizing the performance index ϕ_i, and ϕ_i satisfies

$$\sum_{i=1}^{\infty} e_i^{\mathrm{T}}(t)e_i(t) < \phi_i \sum_{i=1}^{\infty} \bar{w}^{\mathrm{T}}(t)\bar{w}(t) \quad (34)$$

where it is assumed that $\bar{w}(t)$ is the energy-bounded noise (i.e., $\sum_{i=1}^{\infty} \bar{w}^{\mathrm{T}}(t)\bar{w}(t)$ is bounded). Notice that H_∞ estimation method may be difficult to design the time-varying estimator gains for time-varying systems such that the SE of the H_∞ estimator is bounded for $t \to \infty$. Moreover, when $\bar{w}(t)$ is the bounded noise, one has $\left(\sum_{i=1}^{\infty} \bar{w}^{\mathrm{T}}(t)\bar{w}(t)\right) \to \infty$. This implies that the condition (34) is invalid, and the stable estimator cannot be designed by using the H_∞ estimation method. Therefore, this paper proposes a new estimation strategy to design the stable estimator with time-varying gains, and the core idea is to find an upper bound of the SE, and then minimize this upper bound to determine the estimator gain $\mathrm{K}_i(t)$ at each time.

Before deriving the result of Theorem 2, let us define

$$\begin{cases} \mathrm{W(t)} \triangleq [\mathrm{W_1(t)}, \ldots, \mathrm{W_{L-1}(t)}, \mathrm{I} - \sum_{\mathrm{i=1}}^{\mathrm{L-1}} \mathrm{W_i(t)}] \\ H(t) \triangleq \mathrm{diag}\{H_1(t), \ldots, H_L(t)\}, \Xi_i \triangleq \mathrm{E}\{H_i(t)\} \\ \bar{A}_f(t-1) \triangleq \mathrm{diag}\{A_{f_1}(t-1), \ldots, A_{f_L}(t-1)\} \\ \bar{A}_L(t-1) \triangleq \mathrm{diag}\{\underbrace{A(t-1), \ldots, A(t-1)}_{L \text{ elements}}\} \\ \bar{G}_m(t-1) \triangleq H(t)\bar{G}_f(t-1) + (I - H(t))\bar{B}_L(t-1) \\ \bar{Q}(t) \triangleq \mathrm{col}\{Q_1(t)D_1, \ldots, Q_L(t)D_L\} \\ \bar{G}_f(t-1) = \mathrm{col}\{G_{f_1}(t-1), \ldots, G_{f_L}(t-1)\} \\ \bar{B}_L(t-1) = \mathrm{col}\{\underbrace{G(t-1), \ldots, G(t-1)}_{L \text{ elements}}\} \\ \bar{H}_i(t) \triangleq \mathrm{col}\{1-\gamma_{i1}(t), \ldots, 1-\gamma_{in}(t)\}, \Upsilon_i \triangleq \mathrm{E}\{\bar{H}_i(t)\bar{H}_i^{\mathrm{T}}(t)\} \end{cases} \quad (35)$$

where $\gamma_{ij}(t)$ is defined in (13), while $A_{f_i}(t-1)$ and $G_{f_i}(t-1)$ are calculated by (27).

Theorem 2: For a given ESR $1-\eta_i$ $(0 < \eta_i \le 1)$, if there exist integers $N_i \ge 1$ and $M_i \ge 0$ such that the selection probabilities $\pi_{i,\hbar_i}$ $(\hbar_i = 1, 2, \ldots, \Delta_i)$ in (18) satisfy

$$\left\| \prod\nolimits_{\kappa=1}^{N_i} (I - \Xi_i)A(t-\kappa) \right\|_2 < 1 \ (t > N_i) \quad (36)$$

$$\|f_i(t-M_i, f_i(t-M_i-1, f_i(\ldots, f_i(t,I))))\|_2 < 1 (t > M_i) \quad (37)$$

where $f_i(t,Z) \triangleq A^{\mathrm{T}}(t)[\Upsilon_i \otimes Z]A(t)$ $(\forall Z)$. Then, the MSE of the CSE $\hat{\mathrm{x}}_i^{\mathrm{c}}(t)$ will be bounded. Moreover, when each selection probability matrix Ξ_i is determined by (36) and (37), the MSE of the DFE $\hat{\mathrm{x}}(t)$ is bounded. In this case, for a given parameter $\alpha > 0$, the optimal weighting matrices $\mathrm{W}_1(t), \ldots, \mathrm{W}_L(t)$ in (19) can be obtained by the following convex optimization problem:

$$\min_{\mathrm{W}(t), \chi(t), \Omega(t-1) > 0} \chi(t)$$

$$\text{s.t. :} \begin{cases} \begin{bmatrix} -I & \Lambda_{12}(t) & \Lambda_{13}(t) \\ 0 & -\Omega(t-1) & 0 \\ 0 & 0 & -\chi(t)I \end{bmatrix} < 0 \\ \Omega(t-1) - \alpha I \le 0 \end{cases} \quad (38)$$

where $\Lambda_{12}(t) \triangleq [\mathrm{W}(t)H(t)\bar{A}_f(t-1) \quad \mathrm{W}(t)(I - H(t))\bar{A}_L(t-1)]$ and $\Lambda_{13}(t) \triangleq [\mathrm{W}(t)\bar{G}_m(t-1) \quad -\mathrm{W}(t)\bar{Q}(t)]$.

Proof: Let $\mathrm{e}_i^{\mathrm{c}}(t) \triangleq x(t) - \hat{\mathrm{x}}_i^{\mathrm{c}}(t)$. Then, it follows from (12) that

$$\mathrm{e}_i^{\mathrm{c}}(t) = [I - H_i(t)]A(t-1)\mathrm{e}_i^{\mathrm{c}}(t-1) + \bar{v}_i^{\mathrm{c}}(t) \quad (39)$$

where $\bar{v}_i^{\mathrm{c}}(t) \triangleq H_i(t)e_i(t) + [I - H_i(t)]\Gamma(t-1)w(t-1) - Q_i(t)D_i\varsigma(t)$. Then, it is derived from (39) that

$$\begin{aligned} \mathrm{E}\{\mathrm{e}_i^{\mathrm{c}}(\mathrm{t})\} = {} & \left(\prod\nolimits_{\tau=1}^{N_i} (I - \Xi_i)A(t-\tau)\right) \\ & \times \mathrm{E}\{\mathrm{e}_i^{\mathrm{c}}(t-N_i)\} + \hat{v}_i^{\mathrm{c}}(t) \end{aligned} \quad (40)$$

where $\hat{v}_i^{\mathrm{c}}(t) \triangleq \sum_{\tau=1}^{N_i} \{(\prod_{\varepsilon=1}^{\tau-1} (I - \Xi_i)A(t-\varepsilon))\mathrm{E}\{\bar{v}_i^{\mathrm{c}}(t+1-\tau)\}\}$. Meanwhile, it is deduced from (17) and (18) that

$$0 \le \Xi_i \le I, 0 \le \mathrm{E}\{q_{\kappa j}(t)\} \le 1 \quad (41)$$

where $Q_i(t) = (q_{\kappa j}(t))_{n \times r_i}$. In this case, one has by (3), (7), and (40) that $\mathrm{E}\{\bar{v}_i^{\mathrm{c}}(t)\}$ and $\hat{v}_i^{\mathrm{c}}(t)$ in (40) are bounded. Moreover, it can be derived from (40) that

$$||\mathrm{E}\{\mathrm{e}_i^{\mathrm{c}}(t)\}||_2 \leq g_{i1}(t,N_i)||\mathrm{E}\{\mathrm{e}_i^{\mathrm{c}}(t-N_i)\}||_2 + ||\hat{v}_i^{\mathrm{c}}(t)||_2 \tag{42}$$

where $g_{i1}(t,N_i) \triangleq \left\|\prod_{\kappa=1}^{N_i}(I-\Xi_i)A(t-\kappa)\right\|_2$. Then, when the condition (36) holds, it is concluded from (42) and the similar derivation of (23) that there must exist a scalar $\delta_{\mathrm{e}_i^{\mathrm{c}}} > 0$ to satisfy

$$\lim_{t\to\infty} ||\mathrm{E}\{\mathrm{e}_i^{\mathrm{c}}(t)\}||_2 < \delta_{\mathrm{e}_i^{\mathrm{c}}}. \tag{43}$$

At the same time, it follows from (39) that

$$\mathrm{e}_i^{\mathrm{c}}(t+1) = \Phi_i(t,M_i)\mathrm{e}_i^{\mathrm{c}}(t-M_i) + \xi_i(t) \tag{44}$$

where

$$\begin{cases} \Phi_i(t,M_i) \triangleq \prod_{\tau=0}^{M_i}(I-H_i(t+1-\tau)A(t-\tau) \\ \xi_i(t) \triangleq \sum_{\tau=0}^{M_i}\{\Phi_i(t,\tau-1)\{H_i(t+1-\tau)e_i(t+1-\tau) \\ \quad +(I-H_i(t+1-\tau))G(t-\tau)\bar{w}(t-\tau) \\ \quad -Q_i(t+1-\tau)D_i\varsigma(t+1-\tau)\}\}. \end{cases} \tag{45}$$

Since $\bar{w}_i(t)$ and $\varsigma(t)$ are the bounded noises, then it is derived from (16) and (45) that

$$\begin{cases} \mathrm{E}\{\Phi_{\mathrm{i}}(\mathrm{t},\mathrm{M_i})\mathrm{e}_{\mathrm{i}}^{\mathrm{c}}(\mathrm{t}-\mathrm{M_i})\} = \mathrm{E}\{\Phi_{\mathrm{i}}(\mathrm{t},\mathrm{M_i})\}\mathrm{E}\{\mathrm{e}_{\mathrm{i}}^{\mathrm{c}}(\mathrm{t}-\mathrm{M_i})\} \\ \mathrm{E}\{[\mathrm{e}_i^{\mathrm{c}}(t-M_i)]^{\mathrm{T}}e_i(t)\} = \mathrm{E}\{[\mathrm{e}_i^{\mathrm{c}}(t-M_i)]^{\mathrm{T}}\}e_i(t) \\ \mathrm{E}\{[\mathrm{e}_i^{\mathrm{c}}(t-M_i)]^{\mathrm{T}}\bar{w}_i(t)\} = \mathrm{E}\{[\mathrm{e}_i^{\mathrm{c}}(t-M_i)]^{\mathrm{T}}\}\bar{w}_i(t) \\ \mathrm{E}\{[\mathrm{e}_i^{\mathrm{c}}(t-M_i)]^{\mathrm{T}}\varsigma(t)\} = \mathrm{E}\{[\mathrm{e}_i^{\mathrm{c}}(t-M_i)]^{\mathrm{T}}\}\varsigma(t). \end{cases} \tag{46}$$

Define $\bar{f}_i(t,Z) \triangleq A^{\mathrm{T}}(t)H_i(t+1)ZH_i(t+1)A(t)$ and $G_{e_i}(t) \triangleq \Phi_i^{\mathrm{T}}(t,M_i)\Phi_i(t,M_i)$. Then, one has

$$\begin{aligned} G_{e_i}(t) = &\left(\prod_{\tau=t-M_i}^{t} A^{\mathrm{T}}(\tau)[I-H_i(\tau+1)]\right) \\ &\times\left(\prod_{\tau=0}^{t-M_i}[I-H_i(t+1-\tau)]A(t-\tau)\right) \\ = &\ \bar{f}_i(t-M_i,\bar{f}_i(t-M_i-1,\bar{f}_i(\ldots,\bar{f}_i(t,I)))). \end{aligned} \tag{47}$$

It is concluded from (16) and (47) that

$$\begin{cases} \mathrm{E}\{\mathrm{G_{e_i}}(\mathrm{t})\} = \mathrm{E}\{\bar{\mathrm{f}}_{\mathrm{i}}(\mathrm{t}-\mathrm{M_i},\mathrm{E}\{\bar{\mathrm{f}}_{\mathrm{i}}(\mathrm{t}-\mathrm{M_i}-1, \\ \qquad \mathrm{E}\{\bar{f}_i(\ldots,\mathrm{E}\{\bar{f}_i(t,I)\})\})\})\} \\ \mathrm{E}\{\bar{f}_i(t,Z)\} = f_i(t,Z) \end{cases} \tag{48}$$

where $f_i(t,Z)$ is defined in (37). In this case, $\mathrm{E}\{G_e(t)\}$ can be determined by

$$\mathrm{E}\{G_{e_i}(t)\} = f_i(t-M_i,f_i(t-M_i-1,f_i(\ldots,f_i(t,I)))). \tag{49}$$

Moreover, it is derived from (44) and (46) that

$$\begin{cases} \mathrm{E}\{[\mathrm{e}_{\mathrm{i}}^{\mathrm{c}}(\mathrm{t}+1)]^{\mathrm{T}}\mathrm{e}_{\mathrm{i}}^{\mathrm{c}}(\mathrm{t}+1)\} = \Delta\mathrm{e}_{\mathrm{i}}^{\mathrm{c}}(\mathrm{t}) \\ \qquad + \mathrm{E}\{[\mathrm{e}_i^{\mathrm{c}}(t-M_i)]^{\mathrm{T}}\mathrm{E}\{G_{e_i}(t)\}\mathrm{e}_i^{\mathrm{c}}(t-M_i)\} \\ \Delta\mathrm{e}_i^{\mathrm{c}}(t) = \mathrm{E}\{\xi_i^{\mathrm{T}}(t)\xi_i(t)\} \\ \qquad + 2\mathrm{E}\{[\mathrm{e}_i^{\mathrm{c}}(t-M_i)]^{\mathrm{T}}\}\mathrm{E}\{\Phi_i^{\mathrm{T}}(t,M_i)\xi_i(t)\} \end{cases} \tag{50}$$

where $\xi_i(t)$ and $\Phi_i(t,M_i)$ are defined in (45), while $G_{e_i}(t)$ is calculated by (47). Then, it follows from (50) that

$$\begin{aligned} &\mathrm{E}\{[\mathrm{e}_{\mathrm{i}}^{\mathrm{c}}(\mathrm{t}+1)]^{\mathrm{T}}\mathrm{e}_{\mathrm{i}}^{\mathrm{c}}(\mathrm{t}+1)\} \leq \lambda_{\max}(\mathrm{E}\{\mathrm{G_{e_i}}(\mathrm{t})\}) \\ &\quad \times \mathrm{E}\{[\mathrm{e}_i^{\mathrm{c}}(t-M_i)]^{\mathrm{T}}\mathrm{e}_i^{\mathrm{c}}(t-M_i)\} + \Delta\mathrm{e}_i^{\mathrm{c}}(t). \end{aligned} \tag{51}$$

Particularly, it is concluded from (3), (7), (36), (41), and (43) that $\Delta\mathrm{e}_i^{\mathrm{c}}(t)$ is bounded. When the condition (37) holds (i.e., $\lambda_{\max}(\mathrm{E}\{G_{e_i}(t)\}) < 1$), it follows from (49), (51), and the similar derivation of (23) that $\lim_{t\to\infty}\mathrm{E}\{[\mathrm{e}_i^{\mathrm{c}}(t)]^{\mathrm{T}}\mathrm{e}_i^{\mathrm{c}}(t)\}$ is bounded, i.e., the MSE of the local estimate $\hat{\mathrm{x}}_i^{\mathrm{c}}(t)$ is bounded.

On the other hand, it follows from (24) and (39) that

$$\begin{aligned} \mathrm{e}_{\mathrm{i}}^{\mathrm{c}}(\mathrm{t}) = &\ [I-H_i(t)]A(t-1)\mathrm{e}_i^{\mathrm{c}}(t-1) \\ &-Q_i(t)D_i\varsigma(t) + H_i(t)A_{f_i}(t-1)e_i(t-1) \\ &+[H_i(t)G_{f_i}(t-1)+[I-H_i(t)]G(t-1)]\bar{w}(t-1) \end{aligned} \tag{52}$$

where $A_{f_i}(t-1)$ and $G_{f_i}(t-1)$ are given by (27). Define $\vartheta(t-1) \triangleq \mathrm{col}\{\bar{w}(t-1),\varsigma(t)\}$, $\mathrm{e_a}(t) \triangleq \mathrm{col}\{\mathrm{e_o}(t),\mathrm{e_c}(t)\}$, $\mathrm{e_o}(t) \triangleq \mathrm{col}\{\mathrm{e}_1(t),\ldots,\mathrm{e}_L(t)\}$, and $\mathrm{e_c}(t) \triangleq \mathrm{col}\{\mathrm{e}_1^{\mathrm{c}}(t),\ldots,\mathrm{e}_L^{\mathrm{c}}(t)\}$, then one has by (24) and (52) that

$$\mathrm{e_a}(t) = \bar{A}(t-1)\mathrm{e_a}(t-1) + \bar{B}(t-1)\vartheta(t-1) \tag{53}$$

where $\bar{A}(t-1) \triangleq \left[\begin{smallmatrix} \bar{A}_f(t-1) & 0 \\ H(t)\bar{A}_f(t-1) & [I-H(t)]\bar{A}_L(t-1) \end{smallmatrix}\right]$ and $\bar{B}(t-1) \triangleq \left[\begin{smallmatrix} \bar{G}_f(t-1) & 0 \\ \bar{G}_m(t-1) & -\bar{Q}(t) \end{smallmatrix}\right]$. Here, $\bar{A}_f(t-1)$, $\bar{A}_L(t-1)$, $\bar{G}_f(t-1)$, $\bar{G}_m(t-1)$, $\bar{Q}(t)$ and $H(t)$ are defined in (35). Let $\mathrm{e}(t) \triangleq x(t)-\hat{\mathrm{x}}(t)$, then one has by (19) that the fusion estimation error $\mathrm{e}(t)$ is equivalent to

$$\mathrm{e}(t) = \sum_{i=1}^{L}\mathrm{W}_i(t)\mathrm{e}_i^{\mathrm{c}}(t) = \bar{\mathrm{W}}(t)\mathrm{e}_a(t) \tag{54}$$

where $\bar{\mathrm{W}}(t) \triangleq [0\ \mathrm{W}(t)]$, and $\mathrm{W}(t)$ is defined in (35). Then, it is derived from (54) that $\mathrm{E}\{\mathrm{e}^{\mathrm{T}}(t)\mathrm{e}(t)\} \leq \lambda_{\max}(\mathrm{W}^{\mathrm{T}}(t)\mathrm{W}(t))$ $\mathrm{E}\{\mathrm{e}_{\mathrm{c}}^{\mathrm{T}}(t)\mathrm{e}_c(t)\}$. Thus, when the selection probabilities for the L CSEs are determined by (36) and (37), the MSE of the DFE $\hat{\mathrm{x}}(t)$ is bounded, i.e., $\lim_{t\to\infty}\mathrm{E}\{\mathrm{e}^{\mathrm{T}}(t)\mathrm{e}(t)\}$ is bounded.

Define $\mathrm{e}_u(t) \triangleq \mathrm{col}\{\mathrm{e}_a(t),\vartheta(t)\}$. Then, for a matrix $\Omega(t-1) > 0$, it follows from (53) and (54) that

$$\begin{aligned} J_e(t) \triangleq &\ \mathrm{e}^{\mathrm{T}}(t)\mathrm{e}(t) - \mathrm{e}_a^{\mathrm{T}}(t-1)\Omega(t-1)\mathrm{e}_a(t-1) \\ &-\chi(t)\vartheta^{\mathrm{T}}(t-1)\vartheta(t-1) = \mathrm{e}_u^{\mathrm{T}}(t-1)\Psi(t)\mathrm{e}_u(t-1) \end{aligned} \tag{55}$$

where

$$\Psi(t) \triangleq \begin{bmatrix} \Psi_{11}(t) & \Psi_{12}(t) \\ * & \Psi_{22}(t) \end{bmatrix}$$

with $\Psi_{11}(t) = \bar{A}^{\mathrm{T}}(t-1)\bar{\mathrm{W}}^{\mathrm{T}}(t)\bar{\mathrm{W}}(t)\bar{A}(t-1) - \Omega(t-1)$, $\Psi_{12}(t) = \bar{A}^{\mathrm{T}}(t-1)\bar{\mathrm{W}}^{\mathrm{T}}(t)\bar{\mathrm{W}}(t)\bar{B}(t-1)$, and $\Psi_{22}(t) = -\chi(t)I + \bar{B}^{\mathrm{T}}(t-1)\bar{\mathrm{W}}^{\mathrm{T}}(t)\bar{\mathrm{W}}(t)\bar{B}(t-1)$. By using the Schur Complement lemma [23], the first inequality in (38) is equivalent to "$\Psi(t) < 0$," which means that $J_e(t) < 0$. Under this case, when the second inequality in (38) holds, it is derived from (55) that

$$\mathrm{e}^{\mathrm{T}}(t)\mathrm{e}(t) < \alpha\mathrm{e}_a^{\mathrm{T}}(t-1)\mathrm{e}_a(t-1) + \chi(t)\vartheta^{\mathrm{T}}(t-1)\vartheta(t-1). \tag{56}$$

Therefore, at each time step, when the right term in (56) is chosen as an upper bound of the SE of the DFE $\hat{\mathrm{x}}(t)$, the weighting matrices $\mathrm{W}_1(t),\mathrm{W}_2(t),\ldots,\mathrm{W}_L(t)$ can be obtained by solving the convex optimization problem (38). ■

Based on Theorems 1–2, the computation procedures for the DFE $\hat{\mathrm{x}}(t)$ are summarized as follows.

Remark 3: The convex optimization problems (21) and (38) are established in terms of linear matrix inequalities (LMIs), and thus they

Algorithm 1: For the given ESRs $1-\eta_i (i = 1, \ldots, L)$, the selection probabilities $\pi_{i,\hbar_i} (\hbar_i = 1, \ldots, \Delta_i, i = 1, \ldots, L)$ satisfying (18) and the parameters $\mu_i (i = 1, \ldots, L)$ and α.

1: Determine each local estimator gain $\mathrm{K}_i(t-1)$ by solving the optimization problem (21);
2: Determine the optimal weighting matrices $\mathrm{W}_i(t)(i = 1, \ldots, L)$ by solving the optimization problem (38);
3: Calculate local CSEs $\hat{\mathrm{x}}_i^c(t)(i = 1, \ldots, L)$ by substituting (4) into (12);
4: Calculate the DFE $\hat{\mathrm{x}}(t)$ by substituting $\mathrm{W}_i(t)(i = 1, \ldots, L)$ and (12) into (19);
5: Return to Step 1 and implement Steps 1-4 for obtaining $\hat{\mathrm{x}}(t+1)$.

can be directly solved by the function "*mincx*" of MATLAB LMI Toolbox [23]. Since the design of time-varying gain $\mathrm{K}_i(t)$ in (21) is independent of the measurements, it can be separately completed in the FC. This means that only the SASC $\hat{\mathrm{x}}_{s_i}(t)$ satisfying finite bandwidth is required to be sent to the FC by each sensor. On the other hand, if the statistical properties of $H_i(t)(i = 1, \ldots, L)$ are taken into account, the corresponding weighing fusion matrices will be designed in the mean-square sense. This method may not reflect the actual estimation error at each time, and thus lead to certain conservatism. Therefore, when designing the weighing matrices in Theorem 2, an upper bound of the SE is chosen as the optimization performance index at each time.

Remark 4: It is concluded from (16) and (17) that only if the selection probabilities $\pi_{i,\hbar_i} (\hbar_i = 1, \ldots, L)$ are given for the sensors, the binary variable $\sigma_{i,\hbar_i}(t)(\hbar_i = 1, \ldots, L)$ obeying i.i.d. can be randomly generated at the sensor side. Notice that the signal transmission model (11) and the local CSE model (12) are all determined by $\sigma_{i,\hbar_i}(t)(\hbar_i = 1, \ldots, L)$, thus the stability conditions (36) and (37) in Theorem 2 can provide an effective probability selection criterion such that the boundedness of the MSE and the limited resources (i.e., bandwidth and energy) are simultaneously satisfied for the distributed fusion estimation algorithm.

IV. Simulation Examples

Consider a maneuvering target that is monitored by two sensors, where a point moving in *X–Y* plane can be described by its two-dimensional position and velocity vectors. Define the state vector $x(t)$ in *X–Y* plane by $x(t) \triangleq \mathrm{col}\{\mathrm{X}_s(t), \dot{\mathrm{X}}_s(t), \mathrm{Y}_s(t), \dot{\mathrm{Y}}_s(t)\}$, where $(\mathrm{X}_s(t), \mathrm{Y}_s(t))$ are, respectively, the position coordinates along *X*- and *Y*- axes, while $(\dot{\mathrm{X}}_s(t), \dot{\mathrm{Y}}_s(t))$ are the corresponding velocities. Then, the target's position and velocity in *XY* plane can be modeled by [24]

$$x(t+1) = \begin{bmatrix} 1 & f(t) & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & f(t) \\ 0 & 0 & 0 & 1 \end{bmatrix} x(t) + \begin{bmatrix} \frac{f^2(t)}{2} \\ f(t) \\ \frac{f^2(t)}{2} \\ f(t) \end{bmatrix} w(t) \tag{57}$$

where $f(t)$ is the time-varying sampling period, and $f(t) (\in [0.8, 1])$ is taken as $f(t) = 0.9 + 0.1\sin(t)$ in the simulation. Meanwhile, the measurement parameters in (2) are taken as $C_1 = \begin{bmatrix} 0.5 & 1 & 0 & 0 \\ 0 & 0 & 0.9 & 0.6 \end{bmatrix}$, $C_2 = \begin{bmatrix} 0.9 & 0.8 & 0 & 0 \\ 0 & 0 & 0.5 & 1 \end{bmatrix}$, $B_1 = \begin{bmatrix} 0.5 \\ 0.7 \end{bmatrix}$ and $B_2 = \begin{bmatrix} 0.7 \\ 0.5 \end{bmatrix}$, while $w(t)$ and $v(t)$ are the bounded noises given by

$$w(t) = 2\varphi_1(t) - 1, v(t) = \varphi_2(t) - 0.5 \tag{58}$$

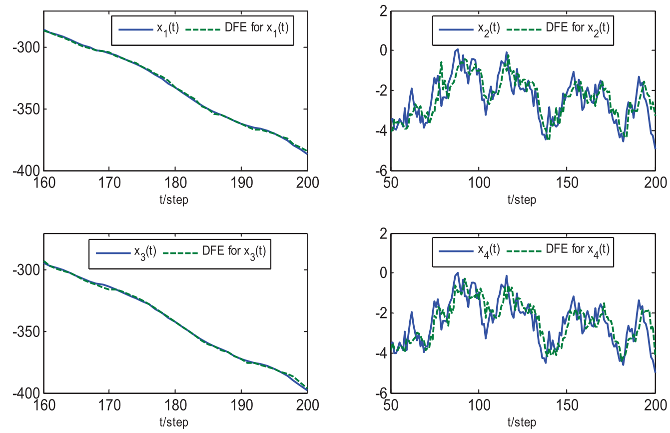

Fig. 1. Trajectories of the state $x(t)$ and the DFE $\hat{x}(t)$.

where $\varphi_i(t) (\in [0, 1])$ is a random variable that can be generated by the function "rand" of MATLAB. When each local estimate $\hat{x}_i(t)$ is transmitted to the FC, at most two component of $\hat{x}_i(t)$ is sent to the FC due to the finite bandwidth and limited sensor energy. Thus, one has $r_1 = r_2 = 2$, $\Delta_1 = \Delta_2 = 6$. Then, $H_i(t)$ in (13) is given by

$$\begin{cases} H_i(t) = \mathrm{diag}\{\sigma_{i,1}(t) + \sigma_{i,2}(t) + \sigma_{i,3}(t), \sigma_{i,1}(t) + \sigma_{i,4}(t) \\ \qquad + \sigma_{i,5}(t), \sigma_{i,2}(t) + \sigma_{i,4}(t) + \sigma_{i,6}(t), \sigma_{i,3}(t) + \sigma_{i,5}(t) \\ \qquad + \sigma_{i,6}(t)\} \\ \left(\sum_{\hbar_i=1}^{6} \sigma_{i,\hbar_i}(t)\right) \in \{0, 1\} \end{cases} \tag{59}$$

where $\sigma_{i,\hbar_i}(t) \in \{0, 1\}(\hbar_i = 1, 2, 3, 4, 5, 6)$ are determined by (8). Let each stochastic process $\{\sigma_{i,\hbar_i}(t)\}$ obey the i.i.d., i.e., the statistical property (16) holds. Meanwhile, the quantization effect for the transmitted components is modeled by $D_i\varsigma(t)$ in (6), where $D_1 = 0.1$, $D_2 = 0.2$. The bounded noise $\varsigma(t)$ is given by $\varsigma(t) = 0.1\sin(0.2\varsigma_0(t))$, where $\varsigma_0(t)$ is the unit Gauss white noise. Moreover, $Q_i(t)$ in (12) satisfying (14) is given by

$$\begin{aligned} Q_i(t) = \ & \mathrm{col}\{\sigma_{i,1}(t) + \sigma_{i,2}(t) + \sigma_{i,3}(t), \sigma_{i,1}(t) + \sigma_{i,4}(t) \\ & + \sigma_{i,5}(t), \sigma_{i,2}(t) + \sigma_{i,4}(t) + \sigma_{i,6}(t), \sigma_{i,3}(t) + \sigma_{i,5}(t) \\ & + \sigma_{i,6}(t)\}(i = 1, 2). \end{aligned} \tag{60}$$

To determine the received signal $\hat{\mathrm{x}}_{u_i}(t)$ (11), when the ESR for each sensor is given by $1-\eta_1 = 1-\eta_2 = 0.1$, the selection probabilities in (18) are chosen by $\pi_{1,1} = 0.1, \pi_{1,2} = 0.2, \pi_{1,3} = 0.1, \pi_{1,4} = 0.3, \pi_{1,5} = 0.1, \pi_{1,6} = 0.1$ and $\pi_{2,1} = 0.2, \pi_{2,2} = 0, \pi_{2,3} = 0.3, \pi_{2,4} = 0.2, \pi_{2,5} = 0, \pi_{2,6} = 0.2$. In this case, one can find the integers $N_1 = N_2 = 1$ and $M_1 = M_2 = 2$ such that the conditions (36) and (37) hold, i.e.,

$$\begin{cases} \left(g_{i1}(t, N_i) \triangleq \|(I - \Xi_i)A(t)\|_2\right) < 1 \\ \left(g_{i2}(t, M_i) \triangleq \|f_i(t-2, f_i(t-1, f_i(t, I)))\|_2\right) < 1 \end{cases} \tag{61}$$

Then, it can be concluded from Theorem 2 that the MSEs of the CSEs $\hat{\mathrm{x}}_i^c(t)$ and the DFE $\hat{\mathrm{x}}(t)$ are bounded.

When the parameters μ_1, μ_2 in (21) and α in (38) are taken as $\mu_1 = \mu_2 = 0.9$ and $\alpha = 2.6$, by using Algorithm 1, the trajectories of $x(t)$ and $\hat{x}(t)$ are depicted in Fig. 1, which shows that the designed DFE can track the maneuvering target well in the presence of disturbances and information loss caused by energy and bandwidth constraints.

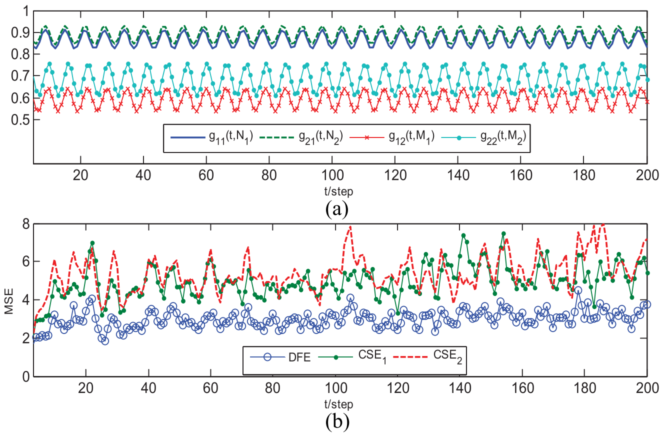

Fig. 2. a) The judgement conditions (61); b) Comparison of the estimation precision for the local CSEs and DFE.

Meanwhile, Fig. 2(a) shows that $g_{i1}(t, N_i)$ and $g_{i2}(t, M_i)$ are all less than one, which accords with the judgement condition (61). Since the estimation errors of $\hat{x}_i^c(t)$ and $\hat{x}(t)$ are random due to the random matrix $H_i(t)$ in (59), the Monte Carlo method is adopted to approach the theory value. Then, the MSEs of the local CSEs and the DFE, which are calculated by an average of 300 runs of Monte Carlo method, are plotted in Fig. 2(b). It can be seen from this figure that the fusion estimation precision of the DFE is higher than that of each CSE, which is as expected for the fusion estimation systems.

V. Conclusion

In this paper, the distributed fusion estimation problem has been investigated for the networked time-varying fusion systems with bounded noises. A new local estimator with time-varying gain was designed such that the SE of the estimator was bounded at each time. When the quantization effect and resource constraints were proposed to be characterized by a unified mathematical model with compensation strategy, the probability selection criterion was derived such that the MSE of each CSE was bounded. Then, the optimal weighting fusion criterion was designed by solving the optimization problems including the LMI constraints. Finally, an illustrative example was given to show the effectiveness of the proposed methods.

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