Q1.

a)

$F = \{AB \rightarrow C, BC \rightarrow D\}$

X	X^+	Key?
A	A	
B	B	
C	C	
D	D	
AB	A, B, C, D	YES
AC	A, C	
AD	A, D	
BC	B, C, D	
BD	B, C, D	
CD	C, D	
ABC	A, B, C, D	SK
ABD	A, B, C, D	SK
ACD	A, C, D	
BCD	B, C, D	

Key is AB, ABC and ABD are superkeys.

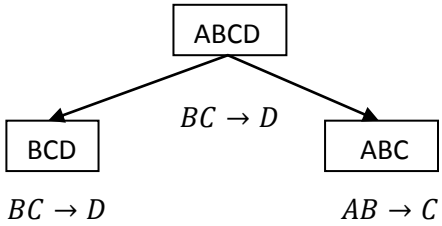
b)

$F^+ = \{AB \rightarrow C, AB \rightarrow D, BC \rightarrow D\}$

R is not in BCNF because at least one of the FDs, eg. $BC \rightarrow D$ violates BCNF.

R is not in 3NF because D is not part of a key.

Decompose R based on $BC \rightarrow D$:



Decomposition is BCD, ABC. Remaining FDs are BC->D and AB->C. It can be shown easily by computing closure of attribute set AB with respect to remaining FD set that we can derive AB->D, therefore decomposition is dependency preserving.

Q2.

a)

First, we split the right hand side of the final FD into C->A and C->D

$F = \{BC \rightarrow A, AB \rightarrow C, C \rightarrow D, C \rightarrow A\}$

X	X^+	Key?
A	A	
B	B	
C	A, C, D	
D	D	
AB	A, B, C, D	yes
AC	A, C, D	
AD	A, D	
BC	A,B, C, D	yes
BD	B, D	
CD	A, C, D	
ABC	A, B, C, D	SK
ABD	A, B, C, D	SK
ACD	A, C, D	
BCD	A, B, C, D	SK

Keys are AB and BC.

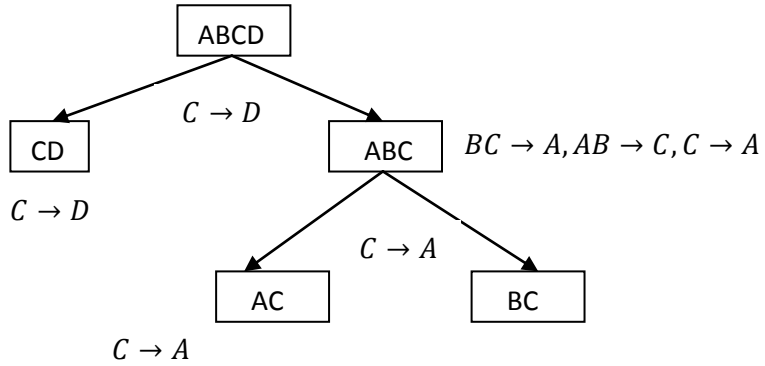
b)

$F^+ = \{BC \to A, C \to A, C \to D, AB \to C, AB \to D, BC \to D\}$

R is not in BCNF because at least one of the FDs, eg. $C \to A$ violates BCNF.

R is not in 3NF because D is not part of a key.

Decompose R based on $C \to D$:



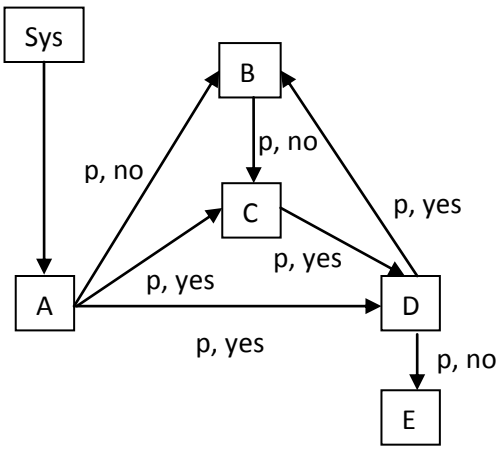
CD is BCNF, but ABC is not, because the keys are AB and BC, and C does not contain any of the keys. Hence C->A is a BCNF violation, and we split again around it, obtaining AC and BC.

Final decomposition is CD, AC, BC with remaining FDs C->A and C->D. It can be easily shown that this is not dependency preserving (e.g., closure of attribute set AB wrt remaining FDs does not contain D, hence AB->D is not preserved).

Q3.

At the end, both C and E can still exercise *p*.

After Step 7:



After Step 8: