# NEURAL NETWORKS

Ref: Artificial Intelligence A Guide to Intelligent Systems, Michael Negnevitsky

## Lecture overview

Machine learning

Artificial neural networks

Neurons

Perception and learning algorithms

Multi-layered ANN and back propagation

Learning with adaptive learning rate

## Machine learning

Involves adaptive mechanisms that enable computers to

- learn from experience
- learn by example
- learn by analogy

Learning capabilities can improve the performance of an intelligent system over time.

## Artificial neural networks (ANNs)

A model of reasoning based on the human brain.

The brain consists of a densely interconnected set of nerve cells, or basic information-processing units, called neurons.

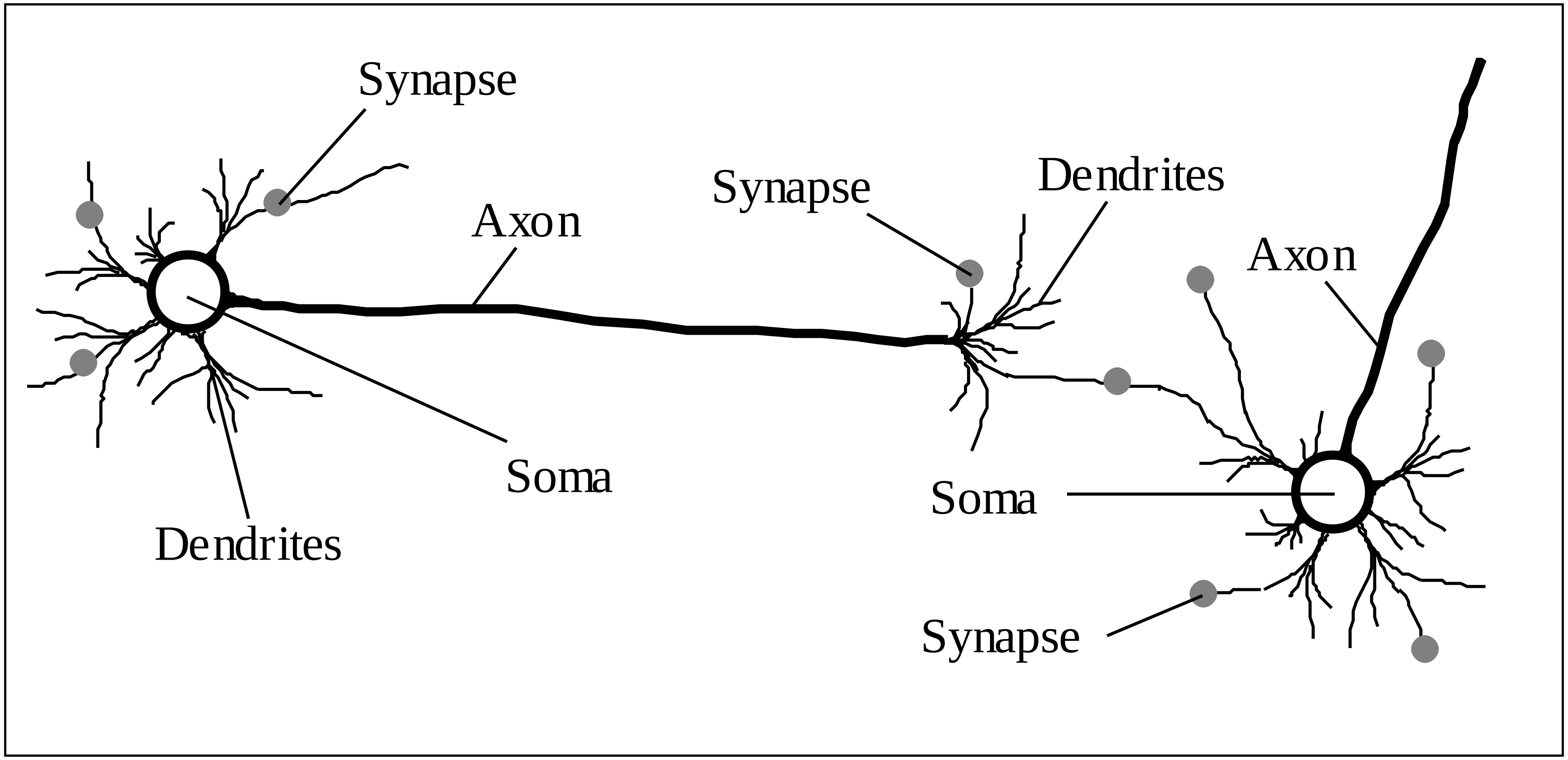
Brain

- nearly 10 billion neurons
- 60 trillion connections (synapses) between them.

A neuron consists of

- a cell body, soma
- a number of fibres called dendrites
- and a single long fibre called the axon.

# Biological neural network



## What is an ANN

Consists of a number of very simple processors, called neurons

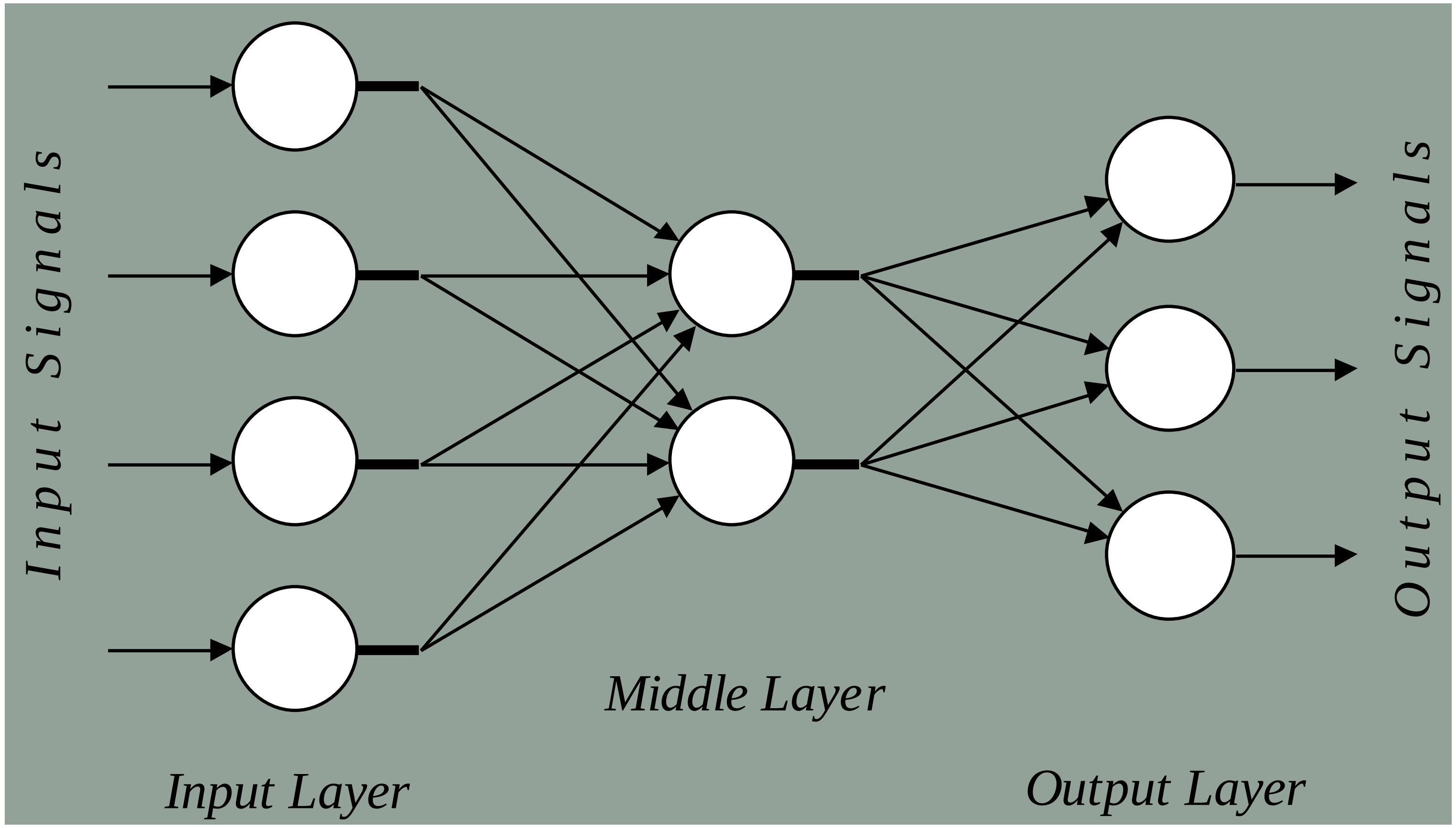
The neurons are connected by weighted links passing signals from one neuron to another.

Each neuron receives a number of input signals through its connections and transmits the output signal through the neuron's outgoing connection.

The outgoing connection splits into a number of branches that transmit the same signal (the signal is not split).

The outgoing branches terminate at the incoming connections of other neurons in the network.

# Architecture of a typical ANN



# What is an ANN

Each neuron receives a number of **input signals $x_i$**, through its connections

A set of real **weighted values $w_i$**, are used to describe connection strengths.

The neurons **activation level $\Sigma x_i w_i$**, determined by the cumulative strength of its input signals.

A **threshold value θ** to compute the neuron's final output state.

The weights are the long-term memory of an ANN

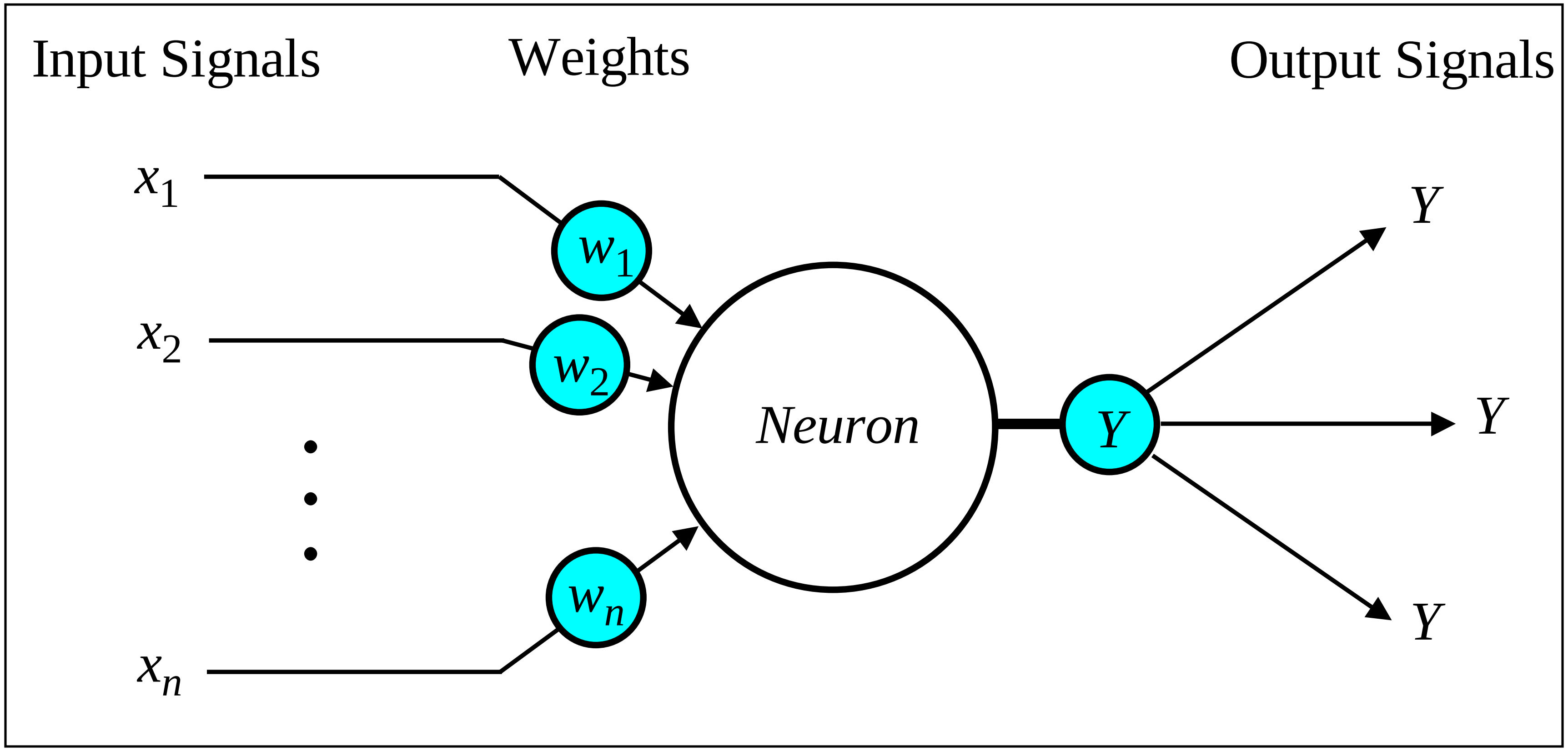
The ANNs 'learn' by adjusting the weights.

## Building an ANN

To build an artificial neural network, we must

- decide on network topology
- choose learning algorithm
- train the network

# The neuron


Input Signals
Weights
Output Signals
$x_1$
$x_2$
$x_n$
$w_1$
$w_2$
$w_n$
*Neuron*
$Y$
$Y$
$Y$
$Y$

## Computing the output

McCulloch&Pitts (1943)

The neuron computes the weighted sum of the input signals and compares the result with a threshold value, θ.

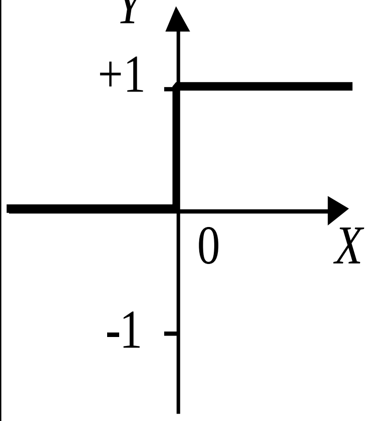
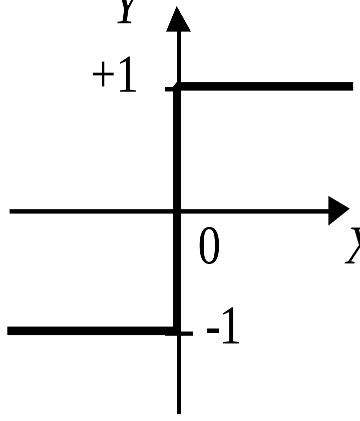
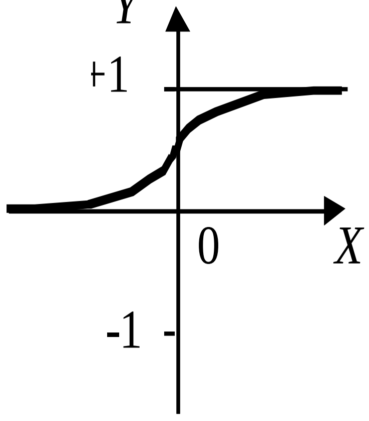
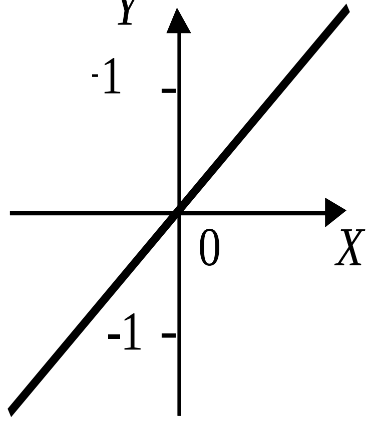
- If the net input is less than the threshold, the neuron output is -1.
- If the net input is greater than or equal to the threshold, the neuron becomes activated and its output attains a value +1.

The neuron uses the following transfer or **activation function**:

$$X = \sum_{i=1}^{n} x_i w_i \qquad Y = \begin{cases} +1, & \text{if } X \geq \theta \\ -1, & \text{if } X < \theta \end{cases}$$

This type of activation function is called a **sign** function.

# Activation functions of a neuron

| *Step function* | *Sign function* | *Sigmoid function* | *Linear function* |
|---|---|---|---|
|  |  |  |  |
| $Y^{step} = \begin{cases} 1, \text{ if } X \geq 0 \\ 0, \text{if } X < 0 \end{cases}$ | $Y^{sign} = \begin{cases} +1, \text{if } X \geq 0 \\ -1, \text{if } X < 0 \end{cases}$ | $Y^{sigmoid} = \dfrac{1}{1+e^{-X}}$ | $Y^{linear} = X$ |

## Activation functions

Step and sign functions (hard-limit functions)

- Classification, pattern recognition

Sigmoid function

- Output [0,1]
- Back-propagation networks

Linear function

- Output = weighted input
- Linear approximation

# Can a single neuron learn a task?

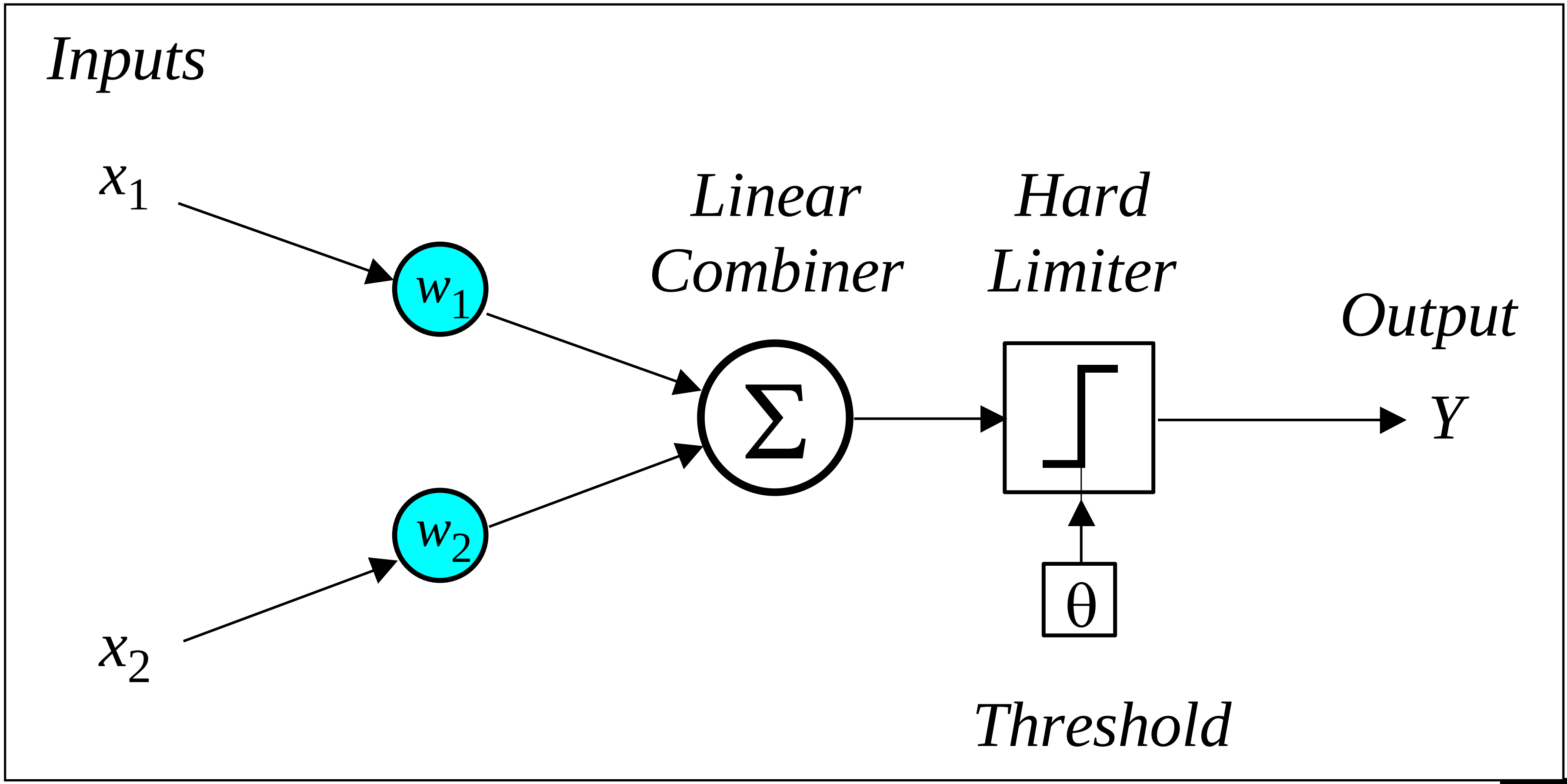
The **perceptron**

- 1958, Frank Rosenblatt
- based on McCulloch-Pitts neural model
- a training algorithm that provided the first procedure for training a simple ANN

The simplest form of a neural network.

Consists of a single neuron with adjustable synaptic weights and a hard limiter (sign or step function).

# Single-layer two-input perceptron



## Rosenblatt's perceptron

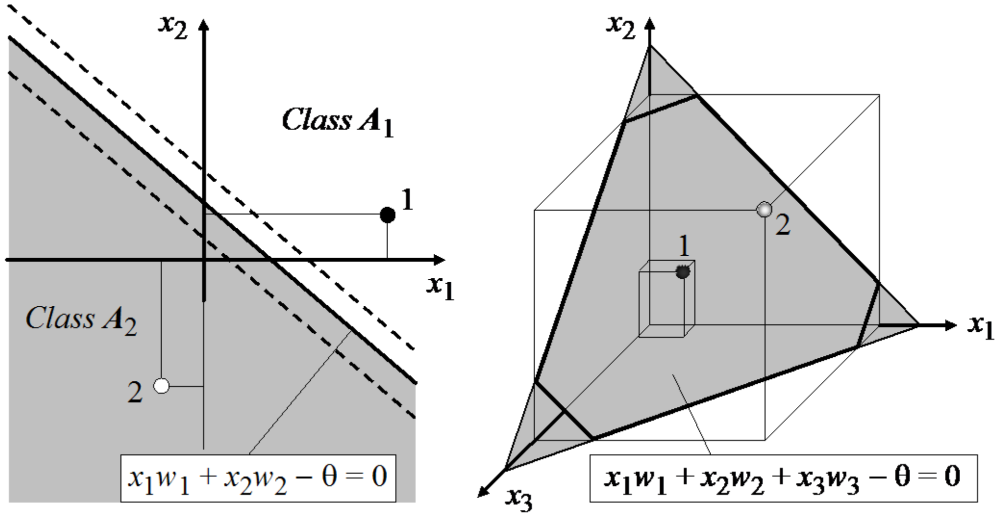
The weighted sum of the inputs is applied to the hard limiter (sign function) which calculates the output

- +1 if the input was positive
- -1 if the input was negative

The aim is to classify inputs, in other words externally applied stimuli, $x_1$, $x_2$ ….$x_{n,}$ into one of two classes A1 and A2

# Linear separability

In the case of an elementary perceptron, the n-dimensional space is divided by a *hyperplane* into two decision regions.



$x_2$
Class $A_1$
1
$x_1$
Class $A_2$
2
$x_1w_1 + x_2w_2 - \theta = 0$
$x_2$
2
1
$x_1$
$x_3$
$x_1w_1 + x_2w_2 + x_3w_3 - \theta = 0$

(*a*) Two-input perceptron. (*b*) Three-input perceptron.

# How does the perceptron learn classification

Uses a form of supervised learning

Small adjustments are made in the weights to reduce the difference between the actual and desired outputs of the perceptron.

Initial weights are randomly assigned, usually in the range [–0.5, 0.5]

After an attempt to solve a training problem the output is compared to the correct output.

The weights then updated to reduce the error.

## Perceptron learning

If at iteration p, the actual output is Y(p) and the desired output is $Y_d(p)$, then the error is given by:

$$e(p) = Y_d(p) - Y(p) \quad \text{where } p = 1, 2, 3, \ldots$$

Iteration p here refers to the pth training example presented to the perceptron.

If the error e(p) is positive, we need to increase perceptron output Y(p), and if it is negative, we need to decrease Y(p).

Each input $x_i$ contributes $x_i w_i$ to the total input

## The Perceptron learning rule

$$w_i(p+1) = w_i(p) + \alpha \cdot x_i(p) \cdot e(p)$$

where

- p = 1, 2, 3, . . .
- $\alpha$ is the **learning rate**, a positive constant less than unity.

Using this rule we can derive the perceptron training algorithm for classification tasks (**Rosenblatt,**1960)

## Perceptron’s training algorithm

Step 1: Initialisation

- Set initial weights w1, w2,…, wn and threshold θ to random numbers in the range [–0.5, 0.5].

- If the error, e(p), is positive, we need to increase perceptron output Y(p), but if it is negative, we need to decrease Y(p).

## Perceptron’s training algorithm

Step 2: Activation

- Activate the perceptron by applying inputs x1(p), x2(p),…, xn(p) and desired output $Y_d(p)$. Calculate the actual output at iteration p = 1

$$Y(p) = step\left[\sum_{i=1}^{n} x_i(p)\, w_i(p) - \theta\right]$$

where n is the number of the perceptron inputs, and *step* is the step activation function.

## Perceptron's training algorithm

Step 3: Weight training

- Update the weights of the perceptron

$$w_i(p+1) = w_i(p) + \Delta w_i(p)$$

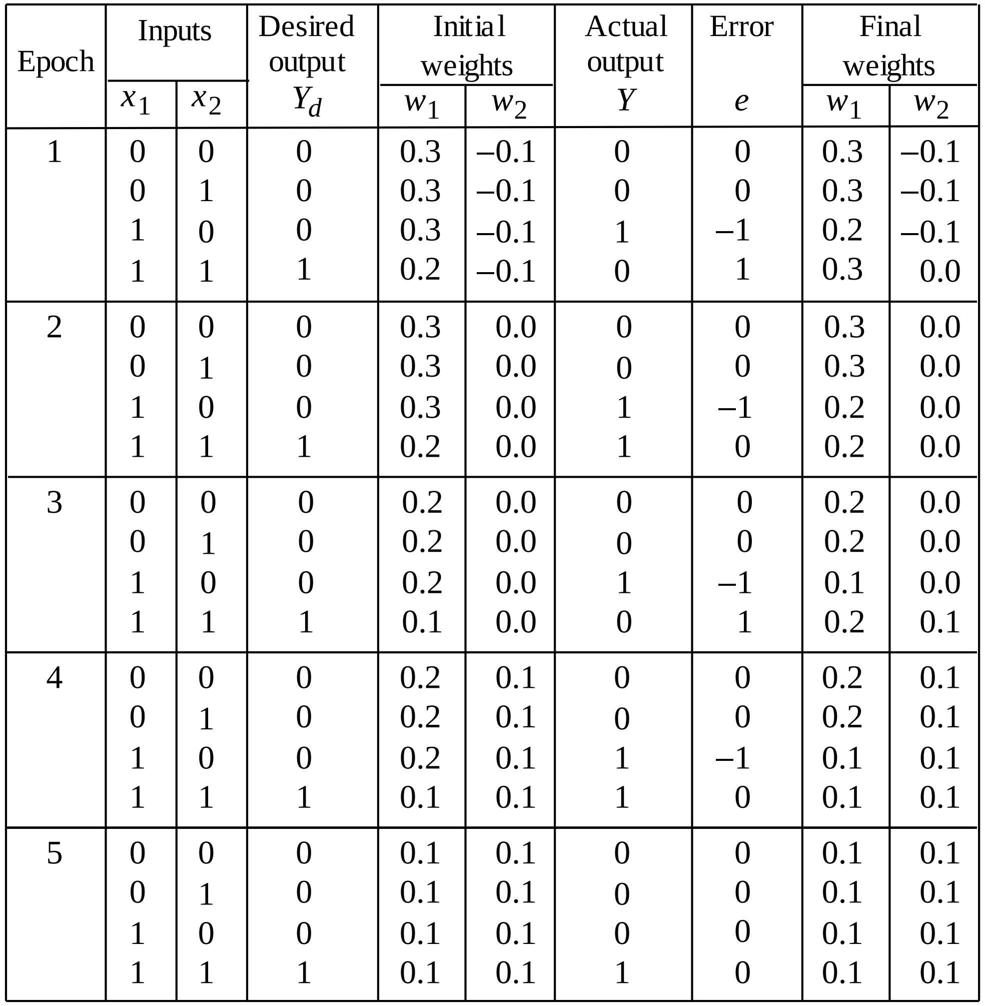
where  is the weight correction at iteration p.

- The weight correction is computed by the delta rule:

$$\Delta w_i(p) = \alpha \cdot x_i(p) \cdot e(p)$$

Step4: Increase iteration p by 1, go back to step 2 and repeat the process until we achieve the desired output.

# Example of perceptron learning - AND



| Epoch | Inputs | | Desired output | Initial weights | | Actual output | Error | Final weights | |
|---|---|---|---|---|---|---|---|---|---|
| | $x_1$ | $x_2$ | $Y_d$ | $w_1$ | $w_2$ | $Y$ | $e$ | $w_1$ | $w_2$ |
| 1 | 0 | 0 | 0 | 0.3 | –0.1 | 0 | 0 | 0.3 | –0.1 |
| | 0 | 1 | 0 | 0.3 | –0.1 | 0 | 0 | 0.3 | –0.1 |
| | 1 | 0 | 0 | 0.3 | –0.1 | 1 | –1 | 0.2 | –0.1 |
| | 1 | 1 | 1 | 0.2 | –0.1 | 0 | 1 | 0.3 | 0.0 |
| 2 | 0 | 0 | 0 | 0.3 | 0.0 | 0 | 0 | 0.3 | 0.0 |
| | 0 | 1 | 0 | 0.3 | 0.0 | 0 | 0 | 0.3 | 0.0 |
| | 1 | 0 | 0 | 0.3 | 0.0 | 1 | –1 | 0.2 | 0.0 |
| | 1 | 1 | 1 | 0.2 | 0.0 | 1 | 0 | 0.2 | 0.0 |
| 3 | 0 | 0 | 0 | 0.2 | 0.0 | 0 | 0 | 0.2 | 0.0 |
| | 0 | 1 | 0 | 0.2 | 0.0 | 0 | 0 | 0.2 | 0.0 |
| | 1 | 0 | 0 | 0.2 | 0.0 | 1 | –1 | 0.1 | 0.0 |
| | 1 | 1 | 1 | 0.1 | 0.0 | 0 | 1 | 0.2 | 0.1 |
| 4 | 0 | 0 | 0 | 0.2 | 0.1 | 0 | 0 | 0.2 | 0.1 |
| | 0 | 1 | 0 | 0.2 | 0.1 | 0 | 0 | 0.2 | 0.1 |
| | 1 | 0 | 0 | 0.2 | 0.1 | 1 | –1 | 0.1 | 0.1 |
| | 1 | 1 | 1 | 0.1 | 0.1 | 1 | 0 | 0.1 | 0.1 |
| 5 | 0 | 0 | 0 | 0.1 | 0.1 | 0 | 0 | 0.1 | 0.1 |
| | 0 | 1 | 0 | 0.1 | 0.1 | 0 | 0 | 0.1 | 0.1 |
| | 1 | 0 | 0 | 0.1 | 0.1 | 0 | 0 | 0.1 | 0.1 |
| | 1 | 1 | 1 | 0.1 | 0.1 | 1 | 0 | 0.1 | 0.1 |

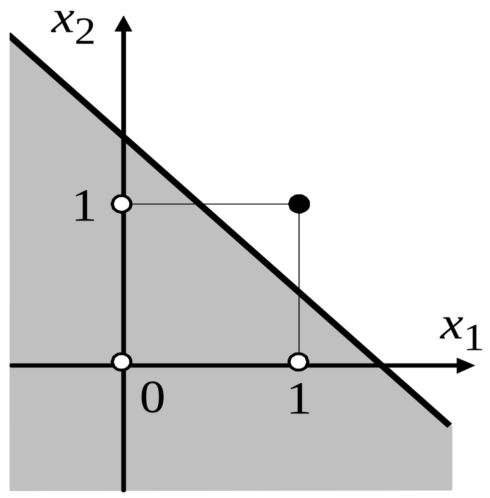
Threshold: $\theta = 0.2$; learning rate: $\alpha = 0.1$

# Basic logical operations

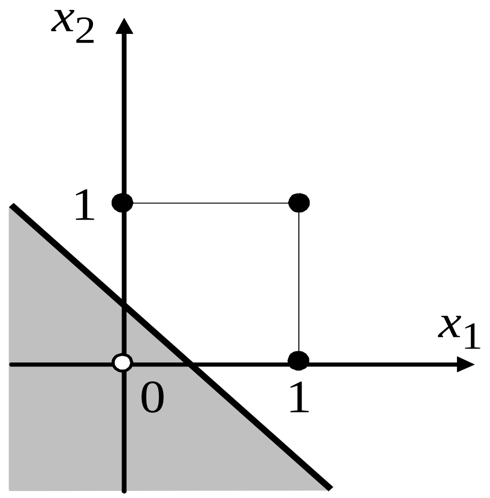
Two-dimensional plots of basic logical operations

A perceptron can learn the operations AND and OR, but not Exclusive-OR.

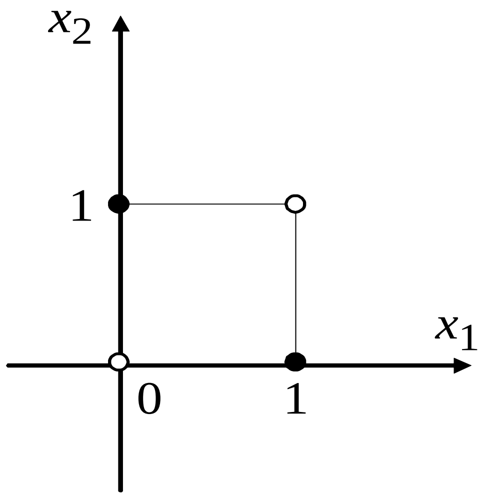
Linearly-separable problems



$(a)$ *AND* $(x_1 \cap x_2)$



$(b)$ *OR* $(x_1 \cup x_2)$



$(c)$ *Exclusive-OR* $(x_1 \oplus x_2)$

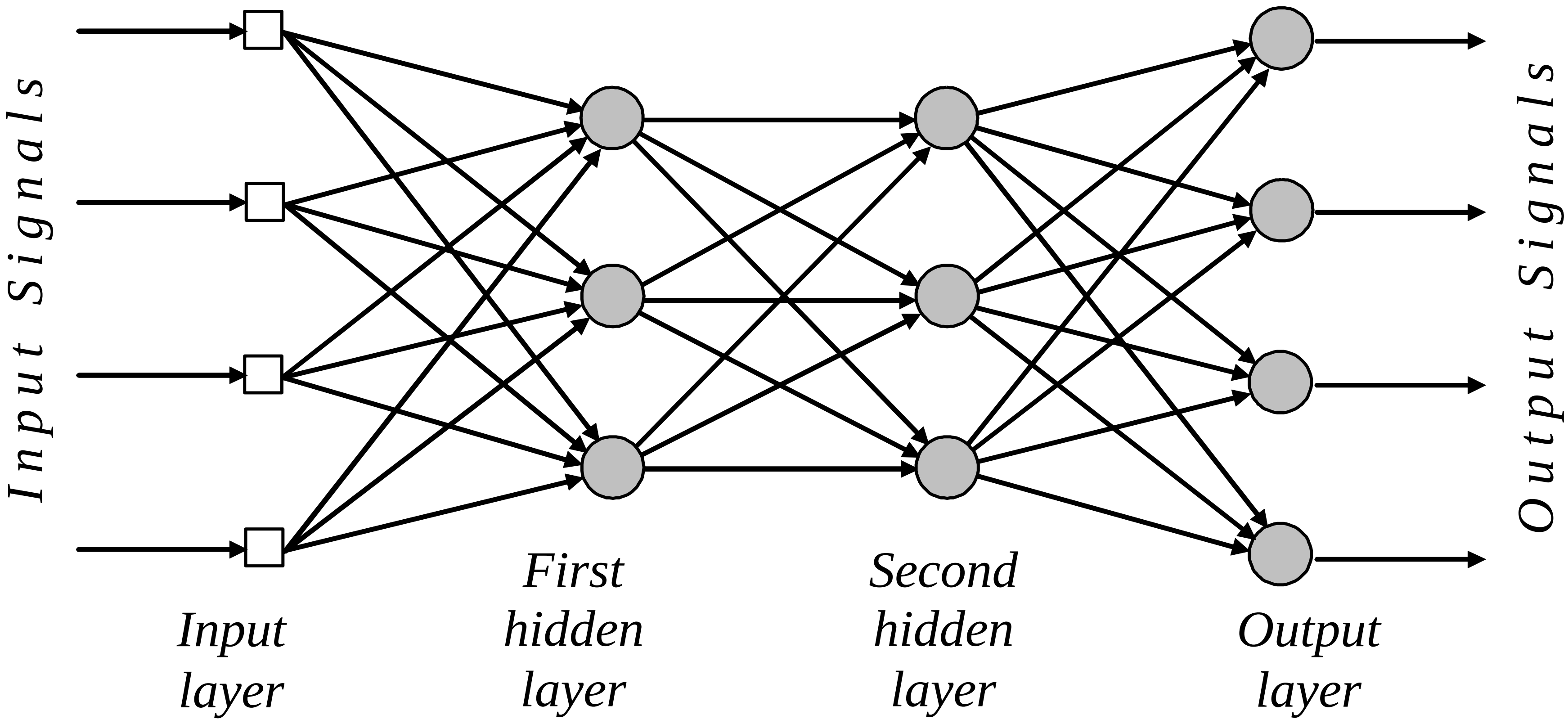
## Multilayer neural networks

A multilayer perceptron is a feed-forward neural network with one or more hidden layers.

The network consists of an input layer of source neurons, at least one middle or hidden layer of computational neurons, and an output layer of computational neurons.

The input signals are propagated in a forward direction on a layer-by-layer basis.

# ANN with two hidden layers



## Hidden layers

A hidden layer "hides" its desired output.

- Neurons in the hidden layer cannot be observed through the input/output behaviour of the network.

Commercial ANNs

- three and sometimes four layers (1-2 hidden)
- each layer 10 to 1000 neurons.

Experimental neural networks may have five or even six layers, including three or four hidden layers, and utilise millions of neurons.

## Back-propagation ANN

Learning in a multilayer network proceeds the same way as for a perceptron.

A training set of input patterns is presented to the network.

The network computes its output pattern, and if there is an error – or in other words a difference between actual and desired output patterns – the weights are adjusted to reduce this error.

## Back-propagation ANN

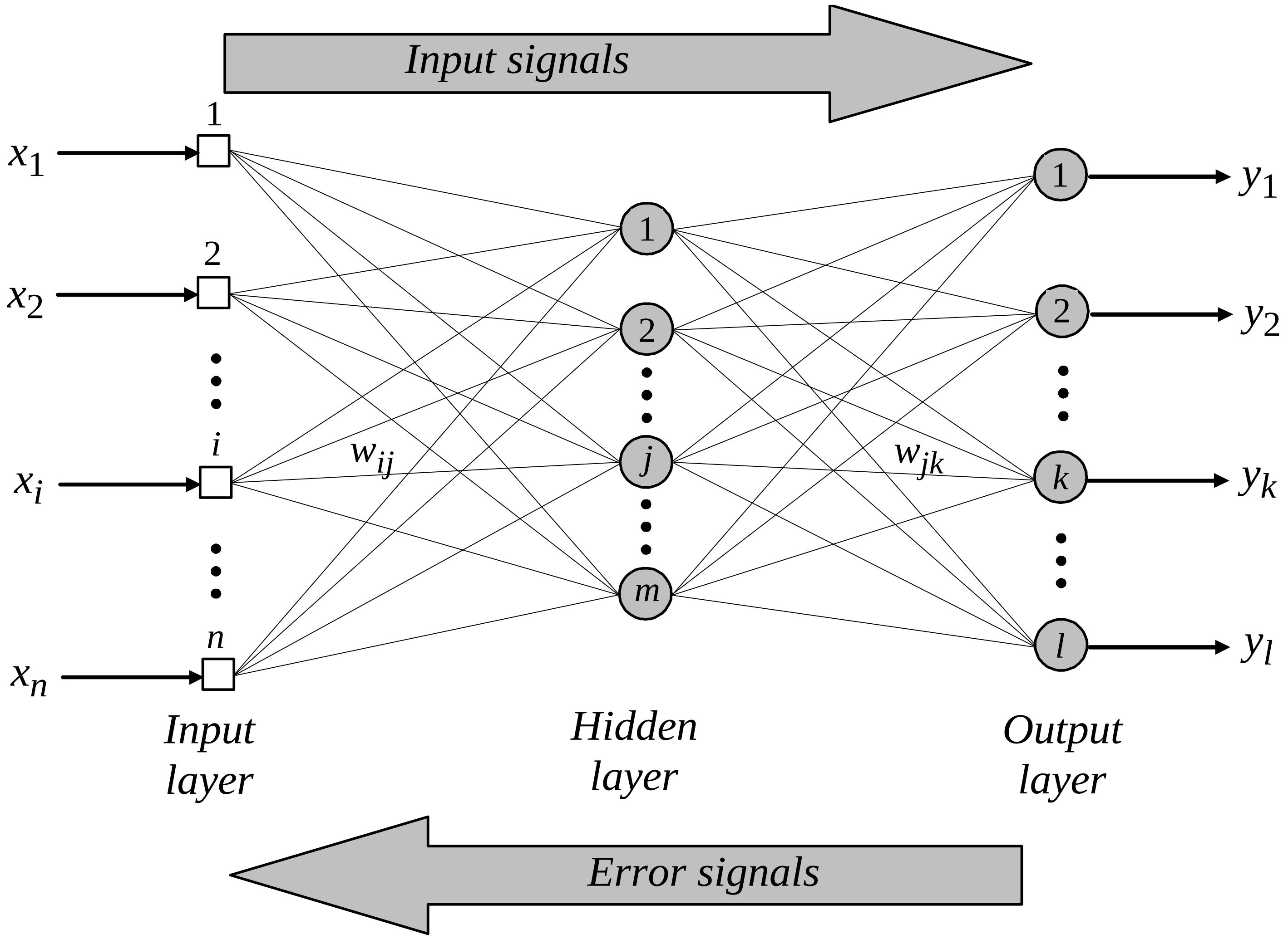
In a back-propagation neural network, the learning algorithm has two phases.

First, a training input pattern is presented to the network input layer.  The network propagates the input pattern from layer to layer until the output pattern is generated by the output layer.

If this pattern is different from the desired output, an error is calculated and then propagated backwards through the network from the output layer to the input layer.

The weights are modified as the error is propagated.

# Three-layer back-porpagation ANN



# Back-propagation training algorithm

**Step 1: Initialisation**

Set all the weights and threshold levels of the network to random numbers uniformly distributed inside a small range:

$$\left( -\frac{2.4}{F_i}, \quad +\frac{2.4}{F_i} \right)$$

where $F_i$ is the total number of inputs of neuron $i$ in the network. The weight initialisation is done on a neuron-by-neuron basis.

# Back-propagation training algorithm

**Step 2: Activation**

Activate the back-propagation neural network by applying inputs $x_1(p), x_2(p),\ldots, x_n(p)$ and desired outputs $y_{d,1}(p), y_{d,2}(p),\ldots, y_{d,n}(p)$.

(*a*) Calculate the actual outputs of the neurons in the hidden layer:

$$y_j(p) = sigmoid\left[\sum_{i=1}^{n} x_i(p)\cdot w_{ij}(p) - \theta_j\right]$$

where *n* is the number of inputs of neuron *j* in the hidden layer, and *sigmoid* is the *sigmoid* activation function.

# Back-propagation training algorithm

**Step 2: Activation (continued)**

(*b*) Calculate the actual outputs of the neurons in the output layer:

$$y_k(p) = sigmoid\left[\sum_{j=1}^{m} x_{jk}(p) \cdot w_{jk}(p) - \theta_k\right]$$

where *m* is the number of inputs of neuron *k* in the output layer.

## Back-propagation training algorithm

**Step 3: Weight training**

Update the weights in the back-propagation network propagating backward the errors associated with output neurons.

(*a*) Calculate the error gradient for the neurons in the output layer:

$$\delta_k(p) = y_k(p) \cdot [1 - y_k(p)] \cdot e_k(p)$$

where

$$e_k(p) = y_{d,k}(p) - y_k(p)$$

Calculate the weight corrections:

$$\Delta w_{jk}(p) = \alpha \cdot y_j(p) \cdot \delta_k(p)$$

Update the weights at the output neurons:

$$w_{jk}(p+1) = w_{jk}(p) + \Delta w_{jk}(p)$$

# Back-propagation training algorithm

**Step 3: Weight training (continued)**

(*b*) Calculate the error gradient for the neurons in the hidden layer:

$$\delta_j(p) = y_j(p) \cdot [1 - y_j(p)] \cdot \sum_{k=1}^{l} \delta_k(p)\, w_{jk}(p)$$

Calculate the weight corrections:

$$\Delta w_{ij}(p) = \alpha \cdot x_i(p) \cdot \delta_j(p)$$

Update the weights at the hidden neurons:

$$w_{ij}(p+1) = w_{ij}(p) + \Delta w_{ij}(p)$$

## Back-propagation training algorithm

**Step 4: Iteration**

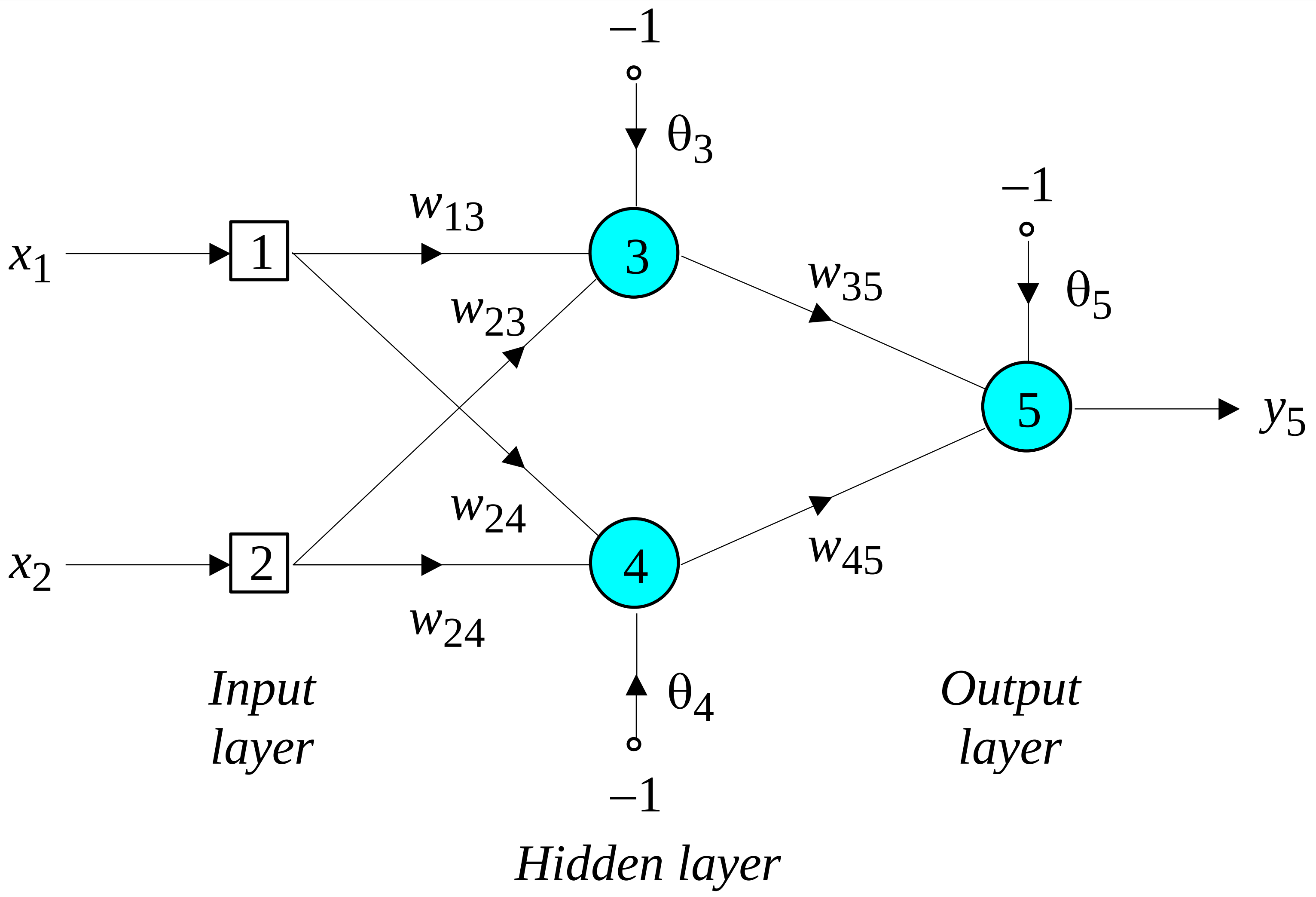
Increase iteration *p* by one, go back to *Step 2* and repeat the process until the selected error criterion is satisfied.

## Example: X-OR

As an example, we may consider the three-layer back-propagation network.

Suppose that the network is required to perform logical operation *Exclusive-OR*. Recall that a single-layer perceptron could not do this operation. Now we will apply the three-layer net.

# Example: three-layer network for solving the Exclusive-OR operation



## Example (cont.)

The effect of the threshold applied to a neuron in the hidden or output layer is represented by its weight, $\theta$, connected to a fixed input equal to –1.

The initial weights and threshold levels are set randomly as follows:

$w_{13} = 0.5$, $w_{14} = 0.9$, $w_{23} = 0.4$, $w_{24} = 1.0$,
$w_{35} = -1.2$, $w_{45} = 1.1$, $\theta_3 = 0.8$, $\theta_4 = -0.1$ and $\theta_5 = 0.3$.

## Example (cont.)

- We consider a training set where inputs $x_1$ and $x_2$ are equal to 1 and desired output $y_{d,5}$ is 0. The actual outputs of neurons 3 and 4 in the hidden layer are calculated as

$$y_3 = sigmoid\ (x_1 w_{13} + x_2 w_{23} - \theta_3) = 1/\left[1 + e^{-(1\cdot 0.5 + 1\cdot 0.4 - 1\cdot 0.8)}\right] = 0.5250$$

$$y_4 = sigmoid\ (x_1 w_{14} + x_2 w_{24} - \theta_4) = 1/\left[1 + e^{-(1\cdot 0.9 + 1\cdot 1.0 + 1\cdot 0.1)}\right] = 0.8808$$

- Now the actual output of neuron 5 in the output layer is determined as:

$$y_5 = sigmoid\ (y_3 w_{35} + y_4 w_{45} - \theta_5) = 1/\left[1 + e^{-(-0.5250\cdot 1.2 + 0.8808\cdot 1.1 - 1\cdot 0.3)}\right] = 0.5097$$

- Thus, the following error is obtained:

$$e = y_{d,5} - y_5 = 0 - 0.5097 = -0.5097$$

# Example (cont.)

- The next step is weight training. To update the weights and threshold levels in our network, we propagate the error *e* from the output layer backward to the input layer.
- First, we calculate the error gradient for neuron 5 in the output layer:

$$\delta_5 = y_5\,(1 - y_5)\,e = 0.5097 \cdot (1 - 0.5097) \cdot (-0.5097) = -0.1274$$

- Then we determine the weight corrections assuming that the learning rate parameter, $\alpha$, is equal to 0.1:

$$\Delta w_{35} = \alpha \cdot y_3 \cdot \delta_5 = 0.1 \cdot 0.5250 \cdot (-0.1274) = -0.0067$$
$$\Delta w_{45} = \alpha \cdot y_4 \cdot \delta_5 = 0.1 \cdot 0.8808 \cdot (-0.1274) = -0.0112$$
$$\Delta \theta_5 = \alpha \cdot (-1) \cdot \delta_5 = 0.1 \cdot (-1) \cdot (-0.1274) = -0.0127$$

## Example (cont.)

- Next we calculate the error gradients for neurons 3 and 4 in the hidden layer:

$$\delta_3 = y_3(1 - y_3) \cdot \delta_5 \cdot w_{35} = 0.5250 \cdot (1 - 0.5250) \cdot (-0.1274) \cdot (-1.2) = 0.0381$$

$$\delta_4 = y_4(1 - y_4) \cdot \delta_5 \cdot w_{45} = 0.8808 \cdot (1 - 0.8808) \cdot (-0.1274) \cdot 1.1 = -0.0147$$

- We then determine the weight corrections:

$$\Delta w_{13} = \alpha \cdot x_1 \cdot \delta_3 = 0.1 \cdot 1 \cdot 0.0381 = 0.0038$$
$$\Delta w_{23} = \alpha \cdot x_2 \cdot \delta_3 = 0.1 \cdot 1 \cdot 0.0381 = 0.0038$$
$$\Delta \theta_3 = \alpha \cdot (-1) \cdot \delta_3 = 0.1 \cdot (-1) \cdot 0.0381 = -0.0038$$
$$\Delta w_{14} = \alpha \cdot x_1 \cdot \delta_4 = 0.1 \cdot 1 \cdot (-0.0147) = -0.0015$$
$$\Delta w_{24} = \alpha \cdot x_2 \cdot \delta_4 = 0.1 \cdot 1 \cdot (-0.0147) = -0.0015$$
$$\Delta \theta_4 = \alpha \cdot (-1) \cdot \delta_4 = 0.1 \cdot (-1) \cdot (-0.0147) = 0.0015$$

# Example (cont.)

At last, we update all weights and threshold:

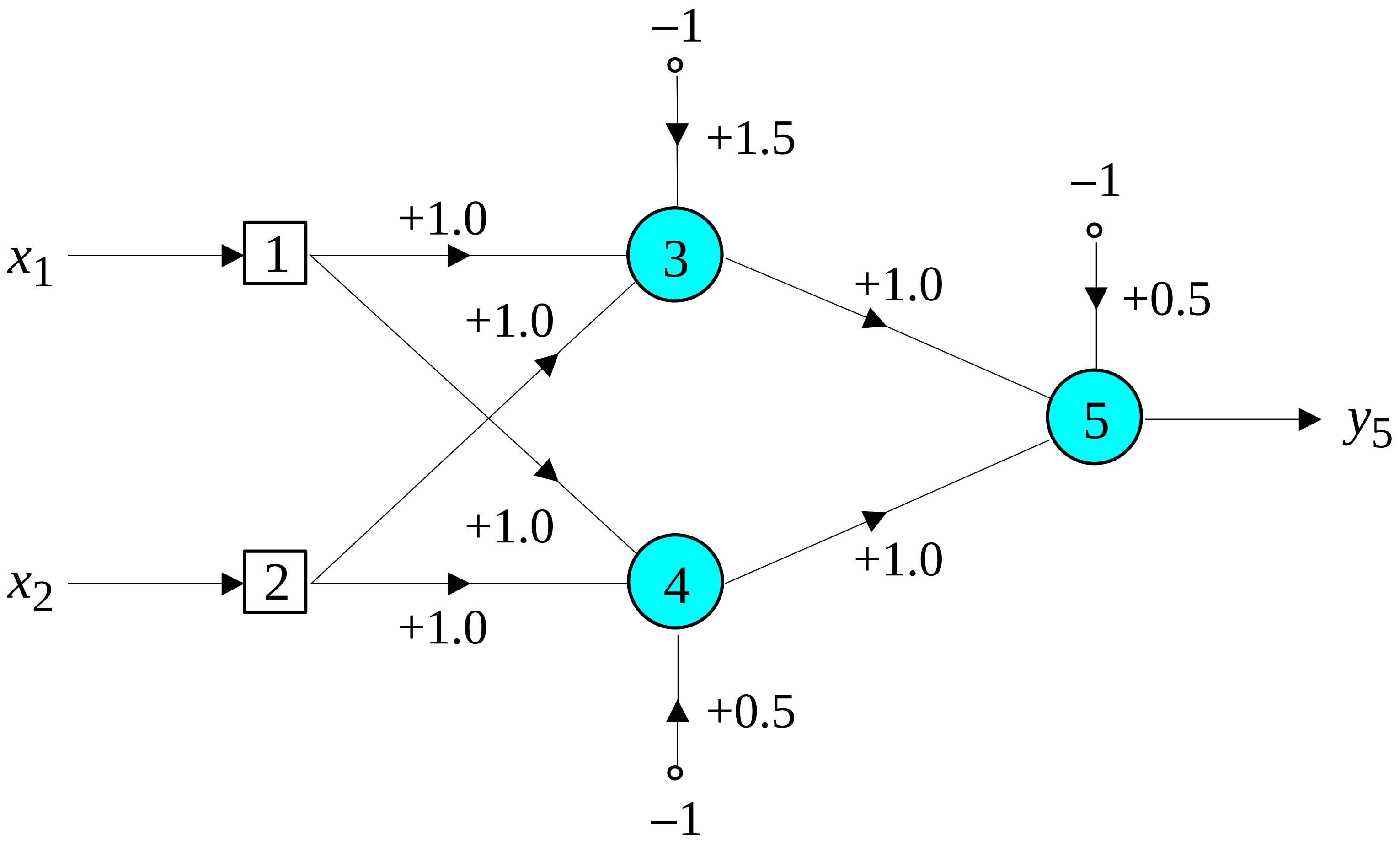
$$w_{13} = w_{13} + \Delta w_{13} = 0.5 + 0.0038 = 0.5038$$
$$w_{14} = w_{14} + \Delta w_{14} = 0.9 - 0.0015 = 0.8985$$
$$w_{23} = w_{23} + \Delta w_{23} = 0.4 + 0.0038 = 0.4038$$
$$w_{24} = w_{24} + \Delta w_{24} = 1.0 - 0.0015 = 0.9985$$
$$w_{35} = w_{35} + \Delta w_{35} = -1.2 - 0.0067 = -1.2067$$
$$w_{45} = w_{45} + \Delta w_{45} = 1.1 - 0.0112 = 1.0888$$
$$\theta_3 = \theta_3 + \Delta\theta_3 = 0.8 - 0.0038 = 0.7962$$
$$\theta_4 = \theta_4 + \Delta\theta_4 = -0.1 + 0.0015 = -0.0985$$
$$\theta_5 = \theta_5 + \Delta\theta_5 = 0.3 + 0.0127 = 0.3127$$

The training process is repeated until the sum of squared errors is less than 0.001.

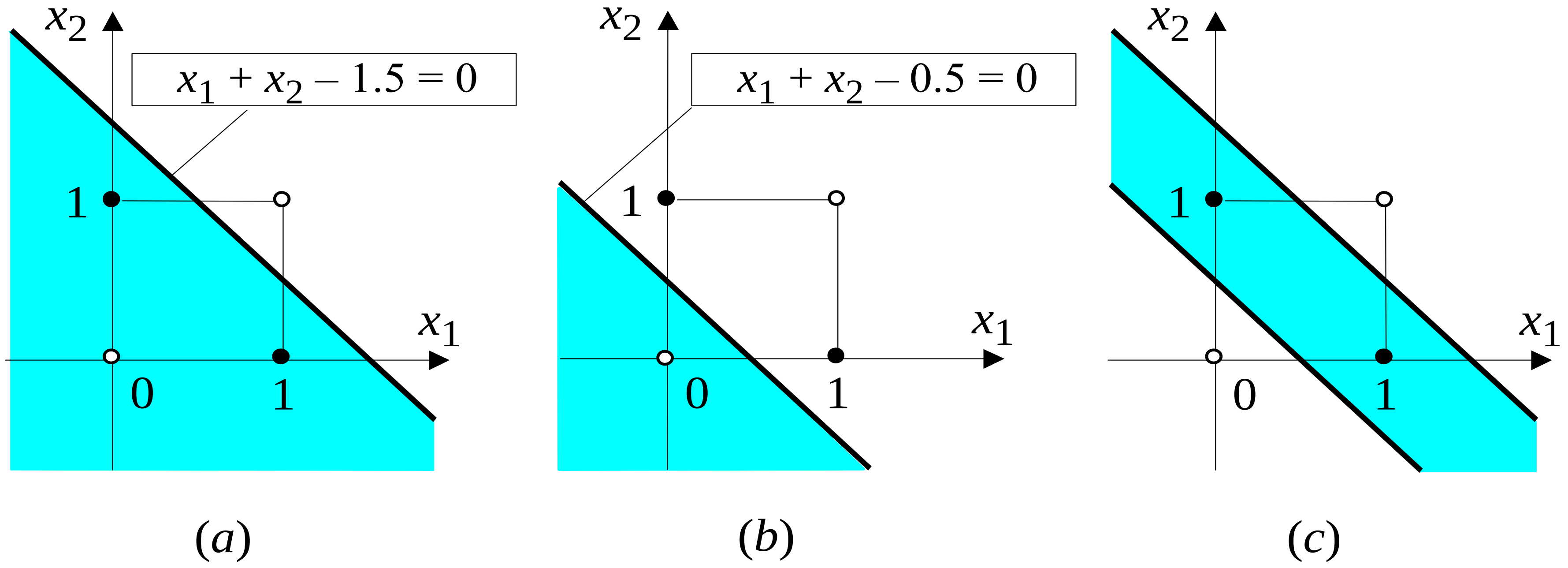
## Final results of three-layer network learning

| Inputs | | Desired output | Actual output | Error | Sum of squared errors |
|---|---|---|---|---|---|
| $x_1$ | $x_2$ | $y_d$ | $y_5$ | $e$ | |
| 1 | 1 | 0 | 0.0155 | –0.0155 | 0.0010 |
| 0 | 1 | 1 | 0.9849 | 0.0151 | |
| 1 | 0 | 1 | 0.9849 | 0.0151 | |
| 0 | 0 | 0 | 0.0175 | –0.0175 | |

# ANN for solving the exclusive-OR

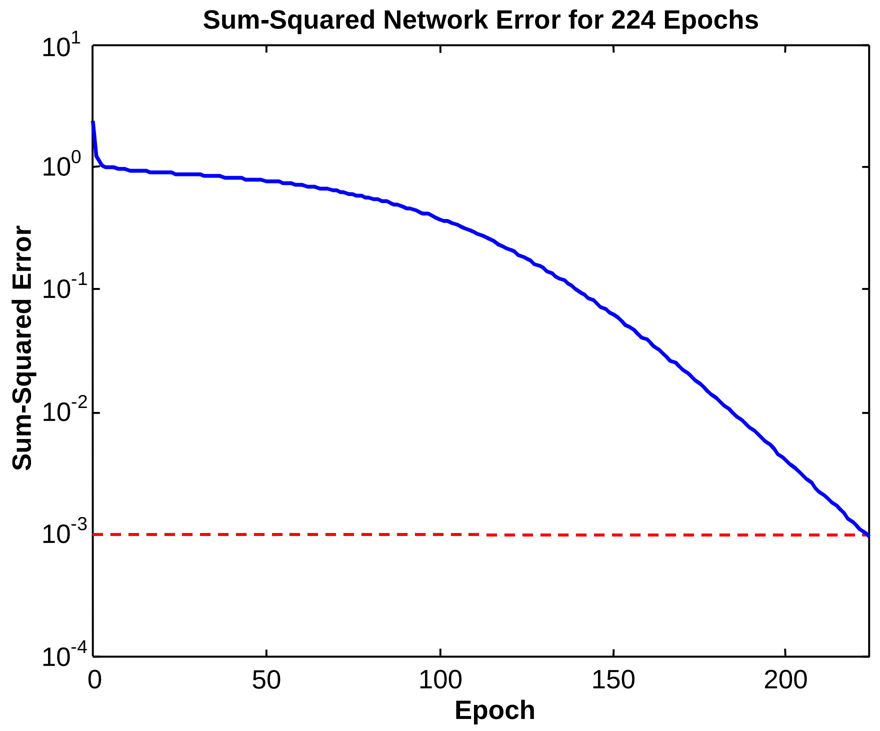


# Decision boundaries

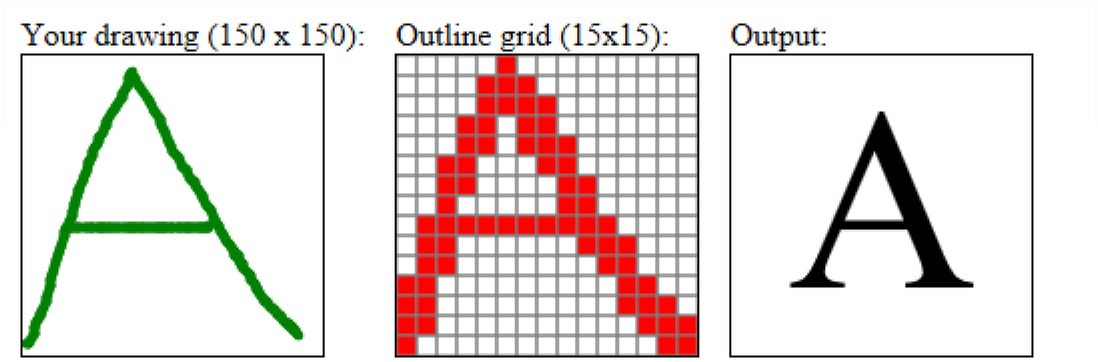


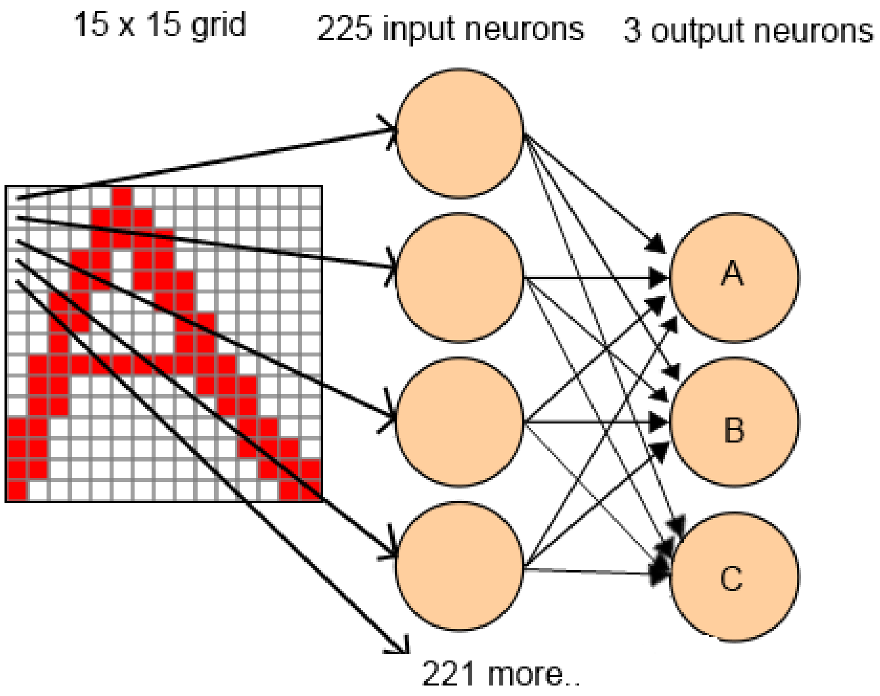
(*a*) Decision boundary constructed by hidden neuron 3;
(*b*) Decision boundary constructed by hidden neuron 4;
(*c*) Decision boundaries constructed by the complete three-layer network

# Learning curve for Exclusive-OR



# Example: OCR





https://robertbeisicht.wordpress.com/2014/07/04/feed-forward-neural-network-in-javascript/

# Accelerated learning in multilayer ANN

A multilayer network learns much faster when the sigmoidal activation function is represented by a hyperbolic tangent:

$$Y^{tanh} = \frac{2a}{1+e^{-bX}} - a$$

where a and b are constants.

Suitable values for a and b would be:

a = 1.716 and b = 0.667

# Accelerated learning in multilayer ANN

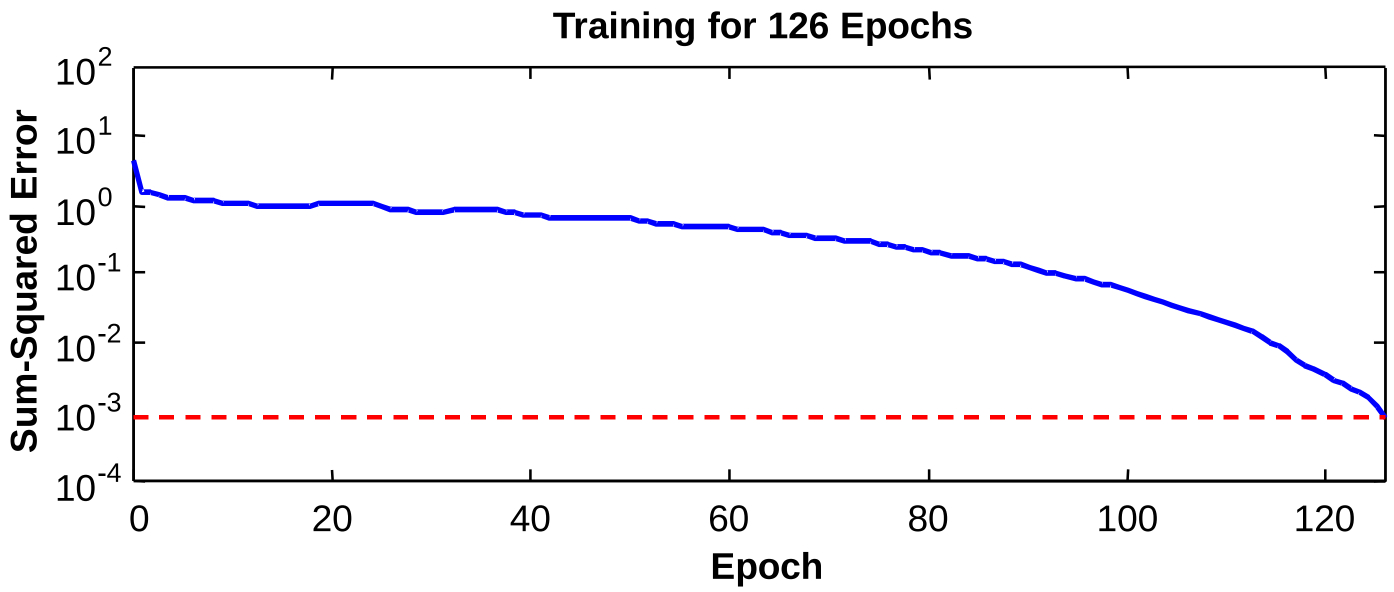
We also can accelerate training by including a momentum term in the delta rule (generalised delta rule):

$$\Delta w_{jk}(p) = \beta \cdot \Delta w_{jk}(p-1) + \alpha \cdot y_j(p) \cdot \delta_k(p)$$

where β is a positive number ($0 \leq \beta < 1$) called the momentum constant. Typically, the momentum constant is set to 0.95.

The presence of momentum tends to have stabilising effect on the training – it accelerates the movement downhill and slows down the process when peaks and valleys are reached.

# Learning with momentum for Exclusive-OR



## Learning with adaptive learning rate

To accelerate the convergence and yet avoid the danger of instability, we can apply two heuristics:

**Heuristic 1**

If the change of the sum of squared errors has the same algebraic sign for several consequent epochs, then the learning rate parameter, $\alpha$, should be increased.

**Heuristic 2**

If the algebraic sign of the change of the sum of squared errors alternates for several consequent epochs, then the learning rate parameter, $\alpha$, should be decreased.

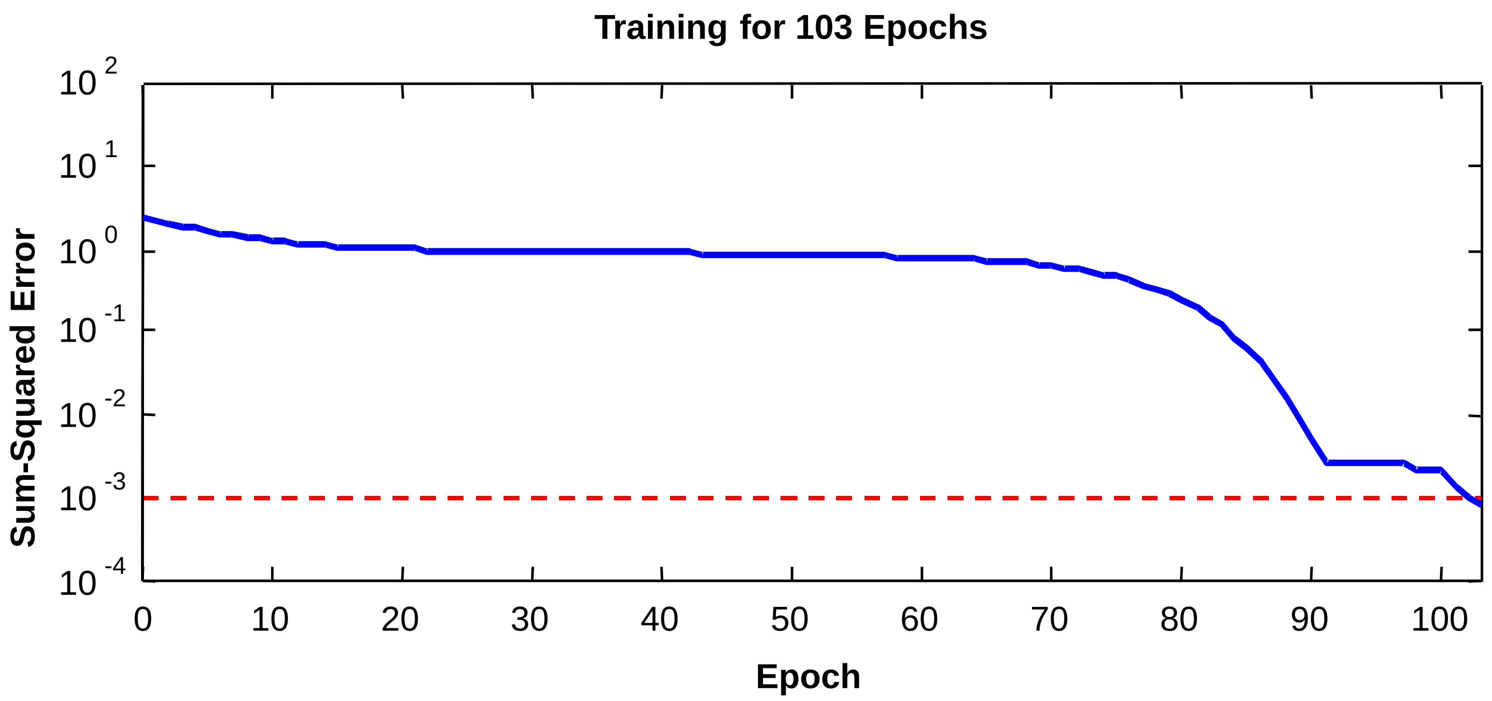
# Learning with adaptive learning rate

Adapting the learning rate requires some changes in the back-propagation algorithm.

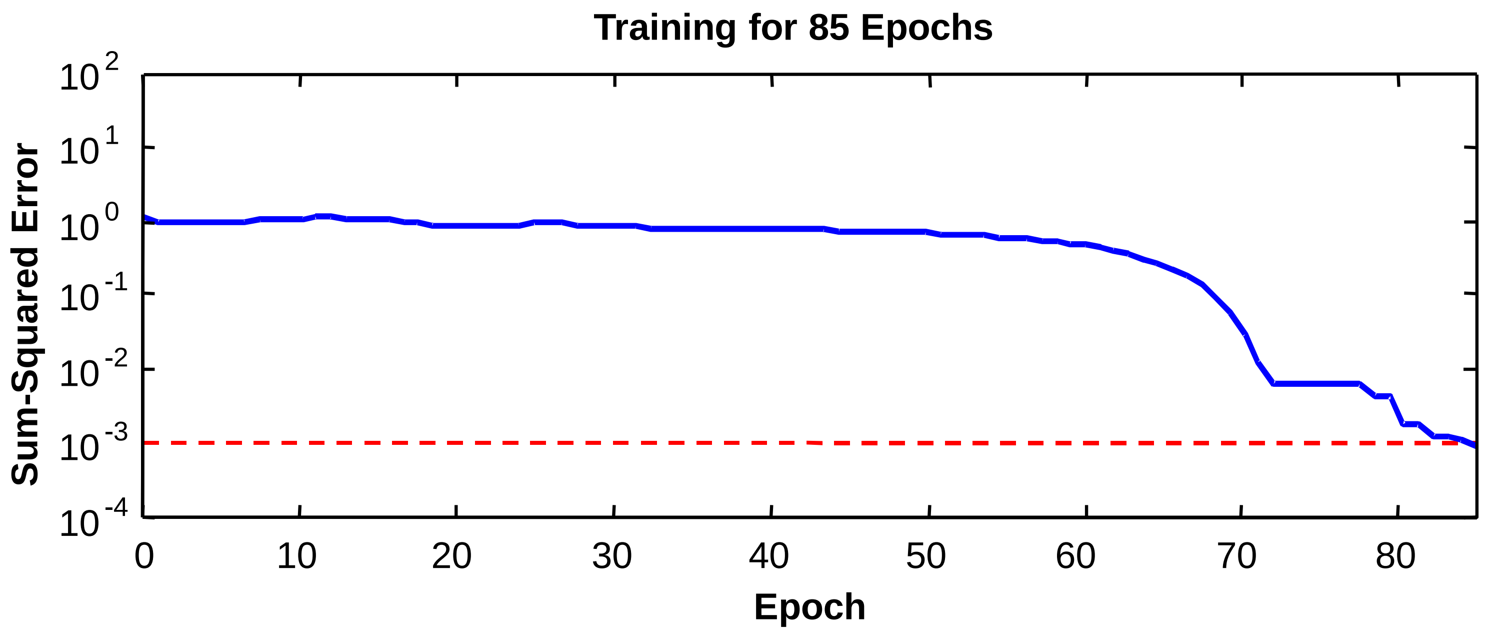
If the sum of squared errors at the current epoch exceeds the previous value by more than a predefined ratio (typically 1.04), the learning rate parameter is decreased (typically by multiplying by 0.7) and new weights and thresholds are calculated.

If the error is less than the previous one, the learning rate is increased (typically by multiplying by 1.05).

# Learning with adaptive learning rate



# Learning with momentum and adaptive learning rate



## Conclusions

Very useful approach for machine learning

Perceptron

Multi-layered ANN

Accelerated learning and learning with adaptive rate