Logarithms

1 Introduction

The function $y = a^x$ has an inverse: $x = \log_a y$ (where a is called the base).

Logarithms can be used to find a power:

$$\begin{aligned} 10^3 &= 1000 \\ \log_{10} 1000 &= 3 \end{aligned}$$

Find x in the following examples by converting the logarithm into indices and using the laws of indices learned previously:

$$\begin{aligned} \log_2 8 &= x \\ \log_3 \frac{1}{27} &= x \\ \ln 1 &= x \\ \log_{25} 5\frac{1}{27} &= x \\ \log_{0.5} 16 &= x \\ \log_8 16 &= x \\ \log_{10} 10^0.2 &= x \\ \log_a 625 &= 4 \\ \log_x 4 &= \frac{2}{3} \end{aligned}$$

2 Laws of Logs

2.1 Addition of Logs

$$\log_b x + \log_b y = \log_b xy$$
$$\log_2 8 + \log_2 4 = \log_2 32$$

2.2 Subtraction of Logs

$$\log_b x - \log_b y = \log_b \frac{x}{y}$$
$$\log_2 8 + \log_2 4 = \log_2 2$$

2.3 Log of a Power

$$\log_b x^a = a \log_b x$$
$$\log_2 2^3 = 3 \log_2 2$$

2.4 Log of 1

$$\log_b 1 = 0$$
$$\log_2 0 = 1$$

3 Examples

$$\begin{aligned}
\log_4 3 + 2\log_4 x &= 3\log_4 9 \\
2 &= 2\log_4 x - log_4 2 \\
\log_4 y &= \log_4 x - \log_4 5 + \log_4 3 \quad \text{solve y in terms of x} \\
\log_b y &= 3\log_b \sqrt{x} + 2\log_b 10 \quad \text{solve y in terms of x} \\
\log_2 x + log_2(x+2) &= 3
\end{aligned}$$

The resistance R of an electric conductor at temperature θ degrees is given by: $R = R_0 e^{\alpha\theta}$, where α is a constant and $R_0 = 5k\Omega$. Determine the value α correct to 4 significant figures when $R = 6k\Omega$ and $\theta = 1500C$