

① Find eigenvalues and vectors for

$$A = \begin{bmatrix} \phi & -1 \\ 2 & 3 \end{bmatrix}$$

$$\det(A - \lambda I) = \phi$$

$$\begin{bmatrix} \phi & -1 \\ 2 & 3 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} -\lambda & -1 \\ 2 & 3-\lambda \end{bmatrix}$$

$$(-\lambda)(3-\lambda) - (-1 \cdot 2) = \phi$$

$$-3\lambda + \lambda^2 + 2 = \phi$$

$$\lambda = \frac{3 \pm \sqrt{(3)^2 - 4 \cdot 2}}{2} = \frac{3 \pm \sqrt{9 - 8}}{2}$$

$$V_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad V_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\lambda_1 = \frac{3+1}{2} = \frac{4}{2} = 2$$

$$\lambda_2 = \frac{3-1}{2} = \frac{2}{2} = 1$$

$V_2 =$

$$(A - \lambda_2 I) v = \phi$$

$$\begin{vmatrix} -2 & -1 \\ 2 & 1 \end{vmatrix} \begin{cases} V_1 \\ V_2 \end{cases} = \phi$$

$$\begin{array}{l} -2V_1 - V_2 = \phi \\ 2V_1 + V_2 = \phi \end{array} \left| \begin{array}{cc|c} -1 & -1 & V_1 \\ 2 & 1 & V_2 \end{array} \right. \begin{array}{l} V_1 = \phi \\ V_2 = \phi \end{array}$$

(1 cont)

python implementation

import numpy as np

A = np.array([[0, -1], [2, 3]])

evals, evecs = np.linalg.eig(A)

(2)

~~for c, v in evals.items():~~

~~print(c, v)~~

$$v = \sum_{i=1}^n c_i v_i \quad \text{if } c_1 v_1 + c_2 v_2 + \dots + c_n v_n = c \cdot v$$

$$\frac{c}{\sqrt{v}} = c$$

Next time it won't be

so ghetto.