

① Find eigenvalues and vectors for

$$A = \begin{bmatrix} \phi & -1 \\ 2 & 3 \end{bmatrix}$$

$$\det(A - \lambda I) = \phi$$

$$\begin{bmatrix} \phi & -1 \\ 2 & 3 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} -\lambda & -1 \\ 2 & 3-\lambda \end{bmatrix}$$

$$(-\lambda)(3-\lambda) - (-1 \cdot 2) = \phi$$

$$-3\lambda + \lambda^2 + 2 = \phi$$

$$\lambda = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{3 \pm \sqrt{9 - 8}}{2}$$

$$V_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, V_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \lambda_1 = \frac{3+1}{2} = \frac{4}{2} = 2$$

$$\lambda_2 = \frac{3-1}{2} = \frac{2}{2} = 1$$

$$V_2 =$$

$$(A - \lambda_1 I) v = \phi$$

$$\begin{array}{cc|c} -2 & -1 & V_1 \\ 2 & 1 & V_2 \end{array} = \phi$$

$$\begin{array}{l} -2V_1 - V_2 = \phi \\ 2V_1 + V_2 = \phi \end{array} \left| \begin{array}{cc|c} -2 & -1 & V_1 \\ 2 & 1 & V_2 \end{array} \right. \begin{array}{l} -V_1 = V_2 \\ -V_1 + V_2 = \phi \\ 2V_2 + V_1 = \phi \end{array}$$

(1 contd) Python implementation

```
import numpy as np  
A = np.array([[0, -1], [2, 3]])  
evals, evecs = np.linalg.eig(A)
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(2)

$$\sum_i c_i v_i$$

$$v = c_1 v_1 + c_2 v_2 + \dots + c_n v_n$$

$$\delta = (\lambda_1 - \lambda_2)(\lambda_1 - \lambda_3)$$

$$\delta = S + \lambda_1 \lambda_2$$

$$\frac{\delta - \lambda_1 \lambda_2}{S} = 1$$

$$\frac{S - \delta + \lambda_1 \lambda_2}{S} = 1$$

$$\delta = v(L - A)$$