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Design of full electric power steering with enhanced performance over that of hydraulic power-assisted steering

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Abstract

This paper presents a method of designing a full electrical power steering system to replace a hydraulic power-assisted steering system with improved performance and benefits including energy saving, improved steering 'feel', simpler construction and environmental gain. The designed performance of the electrical power steering system represented an ideal hydraulic power-assisted steering power boost curve which was mathematically modelled to provide the required control characteristic for the electrical power steering system, including variation in the perceived power assistance with the vehicle's forward speed. A full electrical power steering system provides all the torque necessary to steer the wheels, and the steering feel is artificially generated by an electric 'feedback' motor which provides resistance to the driver's input. The performance of the electrical power steering system described in this paper was enhanced by manipulating the reactive torque to the driver's input at the steering wheel so that it depended upon the driving conditions. Full-vehicle software models were generated using ADAMS/car software based on an actual car fitted with hydraulic power-assisted steering and full electrical power steering. The simulation results from both models were compared, and it is concluded that the steering performances of both systems were similar but the steering feel of the full electrical power steering system could be tuned to provide improved feedback to the driver in use. The performance of the full electrical power steering system could be further improved with the introduction of a controller to manipulate the steering feel during undesired conditions.

Keywords

Power assisted, steering, automotive, hydraulic, electric, feel, design, performance, control, algorithm

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Introduction

The invention of power-assisted steering (PAS) by Davis, a consulting engineer who published the results of his work in 1945, has enabled heavy-duty vehicles to be steered easily when manual steering was found to be impractical.[1] Advances in PAS technology and reduction in unit manufacturing costs coupled with greater consumer demand for speed, safety, comfort and performance have meant that PAS has now become a standard fitment for almost every modern passenger car.[2] The introduction of electrical power steering (EPS) has more recently provided an alternative to hydraulic power-assisted steering (HPAS), as it is more energy efficient, less complicated and more environmental friendly because it avoids the use of hydraulic fluid and the power assistance is operational only when a steering input is made.[3]

This paper proposes a design of a full EPS system to replace and improve on a conventional steering system such as an HPAS system, when fitted to a passenger car. The main objectives were to develop a power steering system which responds to the driver's input like a conventional HPAS system does, provides all the required power assistance and offers improved steering 'feel'.

The difference between the 'full' EPS system and the HPAS system is that the HPAS system gives power

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assistance to the muscular action of the driver through the steering wheel and the mechanical connection to the steering rack pinion, while the full EPS system uses none of the driver's muscular action. The power assistance is fully provided by the EPS system. The reaction or feel motor opposes the driver input in order to create the driver's steering feel; hence the system consumes power to oppose the driver's muscular action. A modern HPAS system requires only very low muscular input, and so the power required by the full EPS system to oppose the muscular action of the driver is correspondingly very low.

Over the years, an 'ideal' power boost characteristic curve for an HPAS system has been established.[4] An HPAS system with this characteristic would have a good performance in terms of power assistance and steering feel, and such an ideal characteristic could usefully be incorporated into the design of a full EPS system. The power boost characteristics of most current EPS systems have been designed using computer-aided vehicle dynamics analysis[5] but the benefits of an ideal EPS system have not been fully realized.

An ideal HPAS power boost characteristic curve

There are many types of power boost characteristic used by different manufacturers in the automotive industry for their HPAS systems. The differences between these characteristics mainly arise from the different designs of the hydraulic valves produced by different manufacturers. As suggested by Adams,[4] an ideal power boost curve for an HPAS system should have the following advantages.

1. At low vehicle speeds and while parking, the driver needs to apply less steering-wheel torque but the power assistance is high. This behaviour is very good since quick action is required during parking or manoeuvring at low vehicle speeds.
2. At high vehicle speeds, the driver needs to apply an increased steering-wheel torque for steering assistance to take effect. For specific vehicle speeds, the power assistance will be activated only after the driver exceeds a certain amount of torque or deflection angle. This is to ensure that the driver will have sensitivity when handling a high-speed vehicle and avoids any human error that might cause the vehicle to be difficult to control as a result of a small change in the steering-wheel rotation.

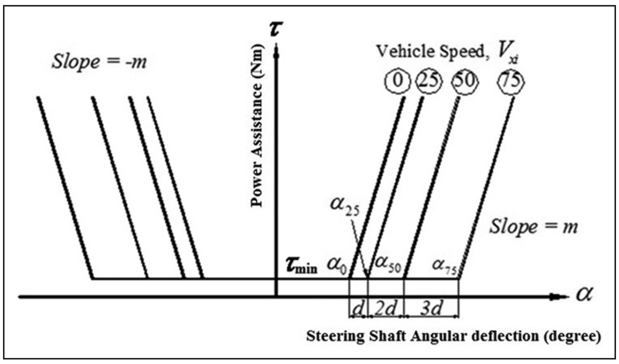

Figure 1. Mathematical representation of the boost curve in EPS.

The above characteristics were also used by Ursolino et al.[6] for the design of their EPS system for emergent markets.

Full EPS system

A conventional EPS system provides steering power assistance through an electric servomotor drive.[3] The actuation of the servomotor corresponds to a specified design curve, determined by the steering-wheel torque and the vehicle speed. The steering system presented in this paper is a full EPS, which means that, unlike a conventional HPAS, all the required power during a steering operation is provided by the system, and the reactive torque at the steering wheel for steering feel purposes is artificially generated by an electric motor. Full EPS has all the advantages of conventional EPAS plus better steering feel for the driver, which can be customised to create the required characteristic, as explained here.

Modelling the characteristics of steering power assistance

The power boost characteristic of an HPAS system presented by Adams[4] was modelled mathematically as shown in Figure 1 to provide the necessary equations for an EPS control algorithm.

The characteristics shown in Figure 1 relate the steering power assistance τ (N m) to the applied steering torque as indicated by α (deg), which is the twist of the steering shaft and is proportional to the steering torque applied by the driver and the vehicle's road speed V_x (km/h).

The relationship between τ and α is linear with the same slope but different offsets depending on the vehicle's road speed, as indicated in Figure 1. The offset (designated by α_i, where i represents the road speed) is where power assistance is introduced; Figure 1 shows examples of this at 0 km/h, 25 km/h, 50 km/h and 75 km/h. It is proposed here that the offset is not linearly related to α and that, at low speeds, the required steering-shaft deflection angle α for power assistance is smaller than at high vehicle speeds so that the steering feels 'lighter' at low vehicle speeds and 'heavier' at high vehicle speeds. The example in Figure 1 proposes that between 0 km/h and 25 km/h the increase is one unit (namely d), between 25 km/h and 50 km/h it is two units (namely $2d$), between 50 km/h and 75km/h it is three units (namely $3d$) and so on. This proposal of a different increase in d was designed to enable the

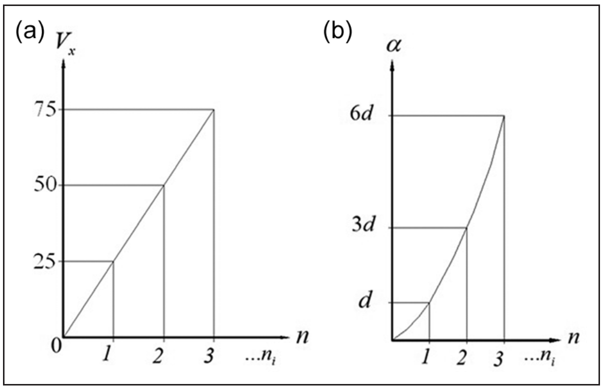

Figure 2. Representation of transformations: (a) calculate n at a specified value of V_x; (b) compute α_i at the calculated n.

steering feel characteristics to be 'tuned', as explained later in this paper.

A mathematical formulation was used to predict α_i at a given speed V_{xi}. Two transformations were required to enable the formula to be derived. First, a relationship between the speed and the index of counting n was made linear so that, given a speed V_{xi}, the linear value of n could be calculated. The variable n is a temporary parameter used to compute the multiple of the first offset d. Then the linear value of n was transformed to obtain the actual value of α_i. The linear relationship was derived from Figure 2.

From Figure 2(a), $V_{xi} = 25n_i$, which means that

$$n_i = \frac{V_{xi}}{25} \tag{1}$$

Note that $n_0 = 0, n_{25} = 1, n_{50} = 2, n_{75} = 3, \ldots$. The corresponding sequence can be represented as an arithmetic summation series (see Figure 2(b)) according to

$$\alpha_i = \frac{n_i(n_i + 1)d}{2} \Rightarrow \alpha_i = \frac{1}{2}\frac{V_{xi}}{25}\left(\frac{V_{xi}}{25} + 1\right)d + \alpha_0 \tag{2}$$

The constant α_0 is added to above expression to represent steering backlash, the minimum steering-shaft deflection for activation of power assistance. At $\alpha = \alpha_i$, the corresponding torque is $\tau = \tau_{\min}$, where $\tau_{\min}$ in the minimum power assistance (torque). Therefore the linear equation passing through these points can be represented by

$$\tau - \tau_{min} = \pm m(\alpha \pm \alpha_i); \Rightarrow \tau = m\alpha + C\,\{\text{right-hand side}\};$$
$$\Rightarrow \tau = -m\alpha + C\,\{\text{left-hand side}\}$$

where $C = \tau_{min} - m\alpha_i$.

The values of d, α_0 and the slope m were selected to match the power boost characteristic of the EPS to the HPAS characteristic as much as possible. The graphical representation of the derivation is presented in Figure 3.

The formula relating the deflection angle to the EPS steering boost torque which was previously derived can be programmed in an electric motor drive.

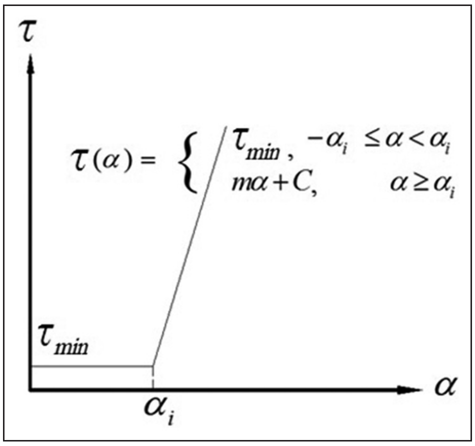

Figure 3. Mathematical representation of the power boost curve.

Control algorithm for full EPS

A full EPS system with electric motor feedback to provide steering feel to the driver will generate minimal torque in the steering shaft during operation because EPS provides the full steering actuation power.[7] The basic configuration of the proposed steering system is shown in Figure 4. The system requires either a flexible mechanical connection or completely no mechanical connection in order to receive signals representing the deflection angle of the steering shaft. It is therefore suggested that the system be applicable for the application of steer-by-wire and semi-active steering, a newly proposed system that uses a flexible steering shaft which can be routed to any desired location and the steering wheel can be placed on either side of the vehicle. A detailed proposal for semi-active steering has been given by Baharom et al.[8] A flexible steering shaft will also buckle during frontal collision, thus preventing driver injury. The stiffness of the flexible shaft in the twist direction has to be carefully selected so that vehicle stability and safety can be achieved in the event of system failure.[9] The shaft can also be designed to conform to the limits of the corrected steer angle as specified by steering regulations in applicable countries. The full steer-by-wire system[10] without any mechanical back-up is currently not permissible in UK and EU regulations; however, it may be possible that the systems will be approved in the future owing to its advantages.

The power for the actuation of the EPS was provided by an electric motor (a power motor) based on the deflection angle, i.e. the difference between the steering-wheel angle and the steering rack pinion rotation angle. The amount of power assistance was determined from the power boost characteristic curve as explained previously and was included in the steering feel algorithm by means of a reaction motor installed close to the steering wheel. This served to provide steering-wheel torque feedback to the driver.

The reactive torque τ_{feel} acting on the steering column for steering feel is represented by

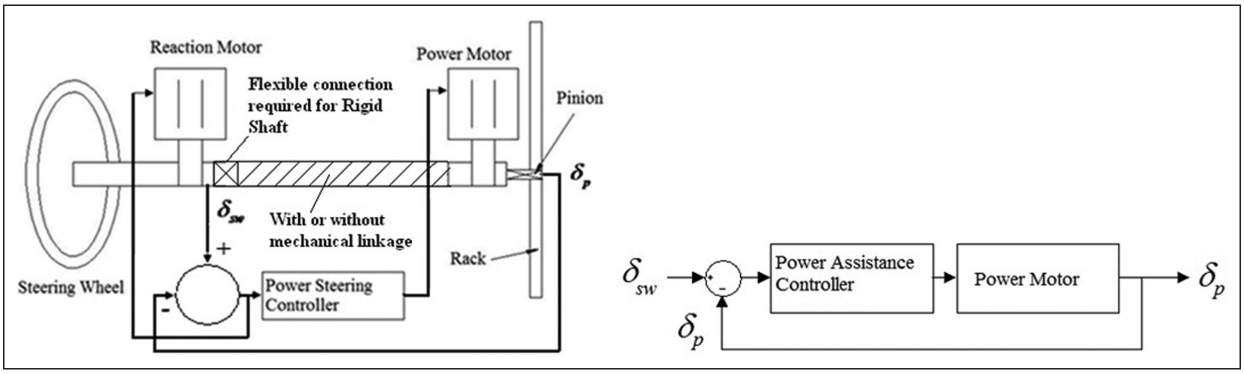

Figure 4. Schematic and control block diagrams of full EPS.

Table 1. General data for the vehicle model.

Constant	Description	Value	Units
m	Total mass of the vehicle	1940	kg
L	Wheelbase	2.7	m
a	Distance from the centre of gravity to the front contact patch	1.17	m
b	Distance from the centre of gravity to the rear contact patch	1.53	m
I_{zz}	Total yaw inertia	3134	kg m^2
T_f	Front track width	1.53	m

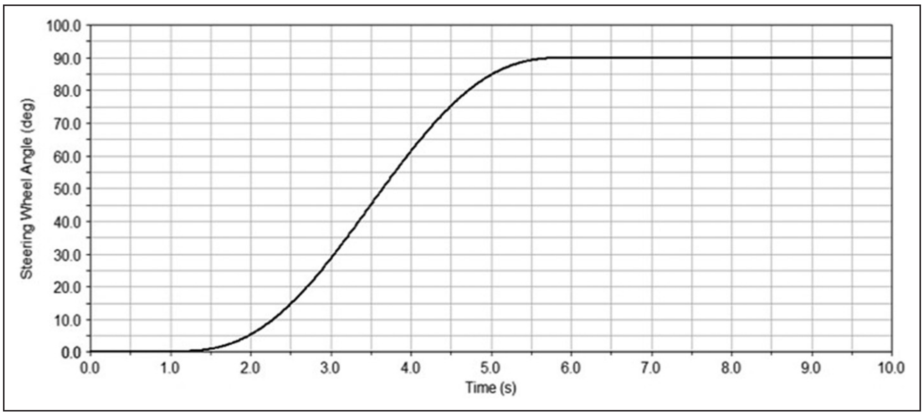

Figure 5. Characteristic of the steering-wheel angle used as the input for most analyses.

$$\tau_{feel} = -K_f(\delta_{sw} - \delta_p) \tag{3}$$

The value of the constant K_f was determined by calibrating the reactive torque of the full EPS system to be equal to the operating reactive torque of a conventional HPAS system at normal vehicle operating speeds based on the designer's choice. Here, the calibration was performed at a vehicle speed of 50 km/h.

In order to verify the effectiveness of the proposed full EPS system, two vehicle models based on data taken from a real car were developed using ADAMS/car software. The basic data for the car are given in Table 1. The first vehicle model was fitted with conventional HPAS, which represented the existing real vehicle. The second model was fitted with full EPS, which represented a modified vehicle.

Analysis and calibration of the reactive torque

The selected event for all the analyses was cornering with the steering-wheel angle characteristic shown in Figure 5. In this case, a vehicle started from a straight line and began cornering after 1 s from a straight-ahead position to a steering-wheel angle of 90°. The cornering process took 5 s to complete, at a forward speed of 50 km/h. This vehicle's forward speed was selected to be representative of a driving speed on urban roads in many countries.

The steering reactive torque for full EPS was artificially generated by the reaction motor in order to inform the driver about what was happening at the road wheels. The calibration process to determine the

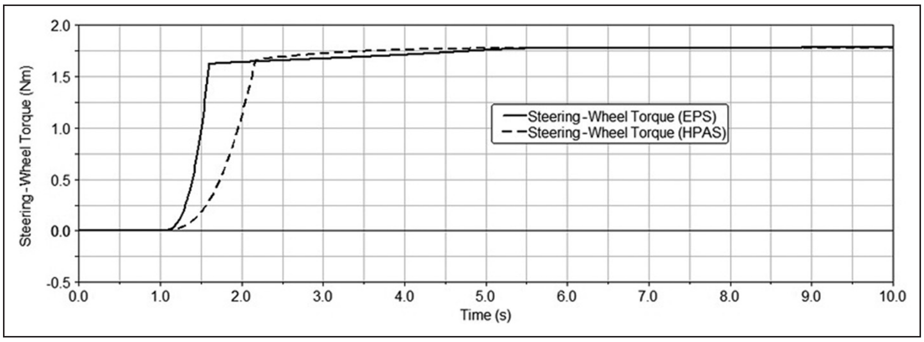

Figure 6. Reactive torque calibration for EPS to match the characteristics of HPAS at $V_x = 50$ km/h: steering-wheel torque versus time.
EPS: electrical power steering; HPAS: hydraulic power-assisted steering.

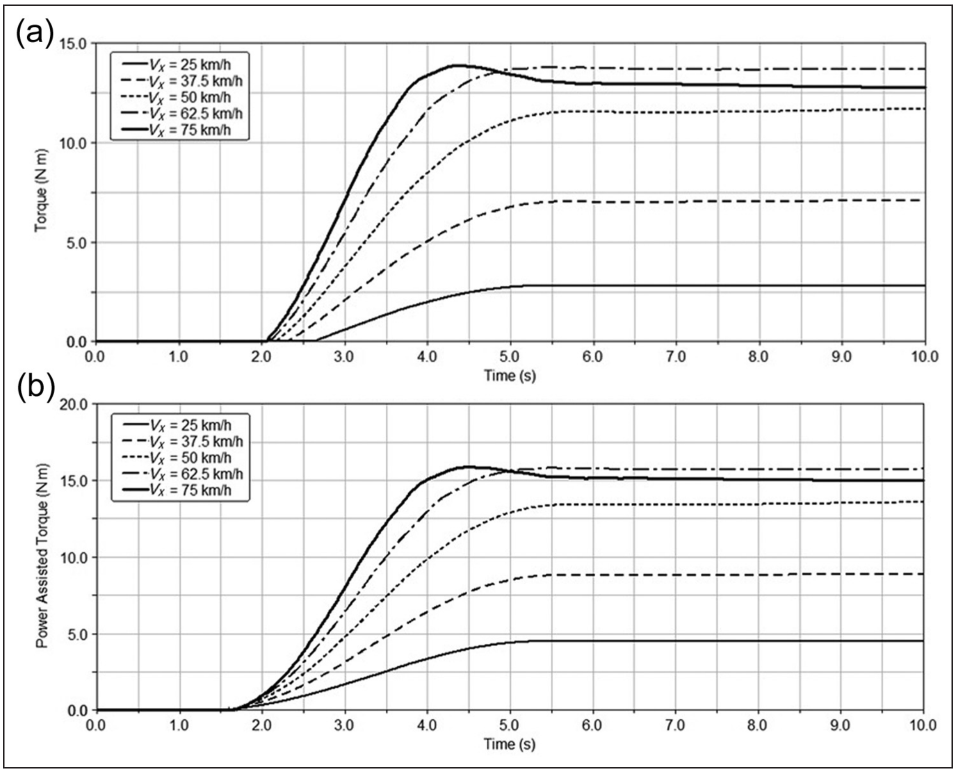

Figure 7. Comparisons of the power-assisted torques versus time for (a) HPAS ($\alpha_0 = 0.5°$; $d = 0.125$; $M = 115.74$) and (b) full EPS.

value of K_f was performed by simulating the software model of full EPS and varying the values of K_f until the desired value was found. The final result is presented in Figure 6.

The computed value of K_f was then used to represent the artificial 'stiffness' of the flexible shaft of the full EPS system for all future analysis cases.

Results and discussion

The performance of a full EPS system in terms of power assistance and reactive feel torque was evaluated by comparing simulation results from full EPS and conventional HPAS. Each was simulated under several different speeds from 25 km/h to 75 km/h with an incremental value of 12.5 km/h. The output results for comparison were the power-assisted torque and the steering-wheel torque. For the case of HPAS, the power-assisted torque is the torque applied at the pinion due to the hydraulic pressure acting on the hydraulic piston faces during power assistance. The steering-wheel torque for HPAS is applied by the driver at the steering wheel. On the other hand, the power-assisted torque for EPS is provided by the power motor as shown in Figure 4 in order to turn a pinion. The steering-wheel torque for this system is artificially generated by the feel motor (Figure 4) which opposes the driver's effort to turn the steering wheel. The results for the power-assisted torques are shown in Figure 7 and the results for the steering-wheel torques are illustrated in Figure 8. These are discussed below.

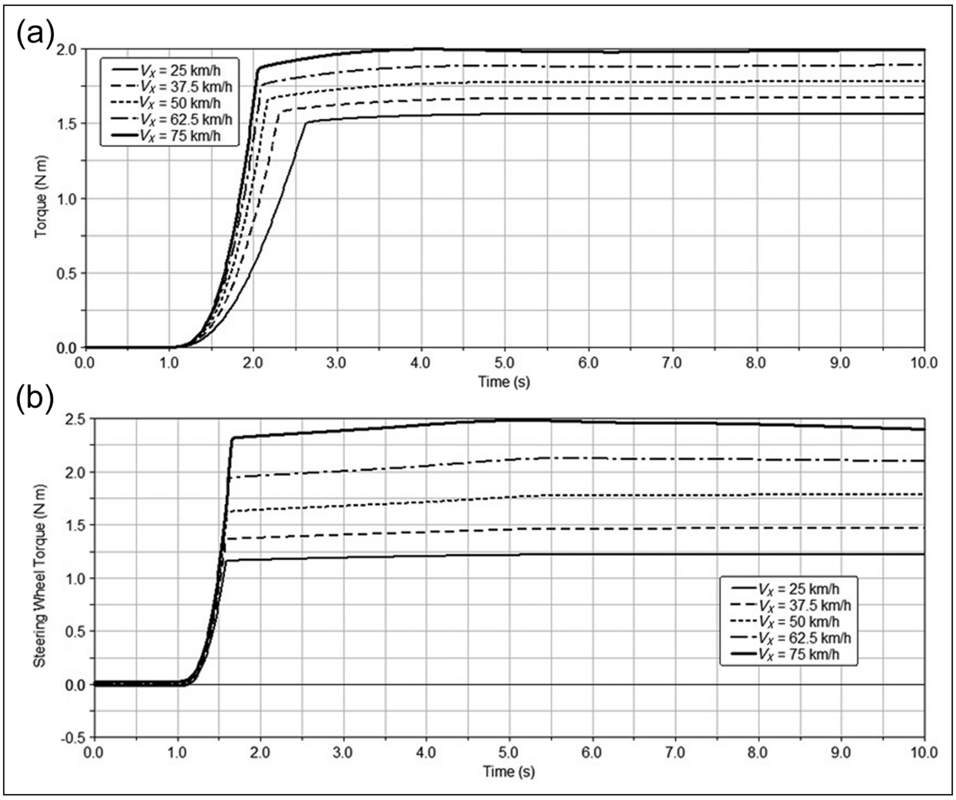

Figure 8. Comparisons of the steering-wheel torques versus time for (a) HPAS ($\alpha_0 = 0.5°$; $d = 0.125$; $M = 115.74$) and (b) full EPS.

It can be observed from Figure 7 that, at each vehicle speed, the actuation of power assistance for the HPAS occurs at a later time than for the full EPS. As the vehicle speed increases, the actuation time for the power assistance decreases. On the other hand, the actuation of power assistance in full EPS occurs at the same time for all cases. The explanation of this relates to the design of the power boost curve and the total torque of the HPAS, which is the sum of its power-assisted torque and steering-wheel torque. Based on the design of the power boost curve, power assistance for each specific vehicle speed would only be actuated after the deflection angle exceeded a certain value. Prior to exceeding this specific deflection angle, the required torque to steer the front wheels would be provided by the driver directly by muscular action through mechanical connection between the steering wheel and the rack pinion. It takes some time for the torsion bar to reach the required deflection for power assistance since the front road wheels also move during the time when the steering wheel is turned. However, as the vehicle speed increases, the self-aligning moment also increases. As a result, more resistance is generated at the road wheels to resist the steering-wheel torque. Therefore, as the vehicle speed increases, the development of the required deflection angle for power activation must also increase.

Since both systems make use of the same power boost curve, the actuation of power assistance in full EPS also occurs when the difference between the steering-wheel angle and the pinion rotation angle exceeds a certain value. Before reaching the specified difference in the angles, the steering wheel is being turned by the driver who feels the reactive torque at the steering wheel, but the front road wheels do not move. The times taken to reach the specified difference in the angles for each vehicle speed are the same for all cases because all of them make use of the same steering-wheel input.

The steering-wheel torque for HPAS from Figure 8(a) can be observed to originate from a single point while the duration time for each case to reach a specific steady state varies depending on the vehicle speed. As the speed increases, the time taken for the flexible shaft (torsion bar) to deflect in order to generate the steering-wheel torque also increases. Similarly, the steering-wheel torque for full EPS also originates from a single point but the system activates almost immediately for all the cases.

The single point origin for the steering-wheel torque for an HPAS system is explained by the fact that all analyses make use of the same steering-wheel input. The time taken for the steering-wheel torque of the HPAS system to reach steady state depends on the vehicle speed and has the same explanation of why its power-assisted torque starts at different times. This is because it takes some time for the torsion bar to reach the required deflection for power assistance since the front road wheels also move during the time when the steering wheel is turned. The increase in self-aligning

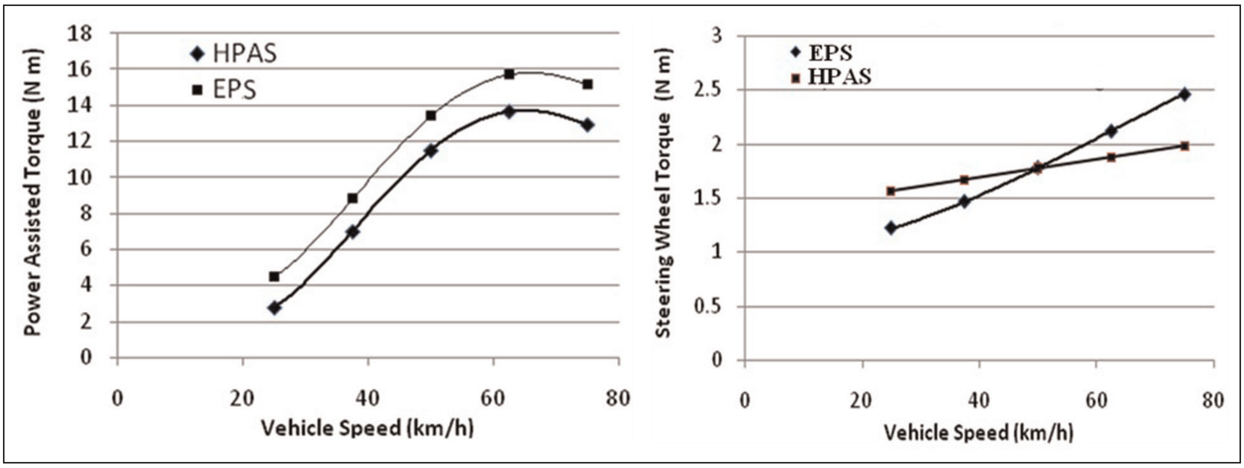

Figure 9. Characteristic plots of (a) the power-assisted torque and (b) the steering-wheel torque as functions of the vehicle speed. EPS: electrical power steering; HPAS: hydraulic power-assisted steering.

moment due to the increase in the vehicle speed causes more resistance for the road wheels to turn and hence leads to more deflection of the torsion bar.

The reason that there is a single starting point for the steering-wheel torque in a full EPS system is because all analyses were performed using the same steering-wheel input. The steady state value for each case started almost immediately because the reaction motor immediately applied a reactive torque due to the difference in the rotation angles of the steering wheel and pinion, and this caused a rapid rise in the steady state steering-wheel torque. The delay for each vehicle speed was due to the time taken to reach a specific deflection angle. Since the deflection angles were very small, the response time difference in each case was also small.

In order to understand the characteristics of the power-assisted torque and the steering-wheel torque with variation in the vehicle's forward speed, detailed plots were obtained from Figures 7 and 8. The data for all the plots were taken at the simulation time $t = 7$ s, where steady state values started to settle down. The results are shown in Figure 9.

From Figure 9(a), it can be observed for both cases that initially the power-assisted torque increases at an increasing rate until the vehicle speed reaches about 50 km/h. The rate of torque increase then decreases until the vehicle speed reaches about 65 km/h. The power-assisted torque then starts to decrease at an increasing rate until the vehicle speed reaches 75 km/h.

The explanation for the first portion of the graphs could be that the region is within the linear range of the cornering stiffnesses. As the vehicle speed increases, the cornering stiffness also increases and therefore the vehicle demands more power assistance. The second portion of the graphs is where non-linearity of the cornering stiffness starts to occur. Within this portion, the contact between the tyre and the ground starts to deteriorate as the vehicle speed increases. The last part of the graph is where slip starts to occur; as the tyre loses grip on the road, less power assistance is required as the self-aligning torque is reduced.

It can be noted from Figure 9(a) that the amount of power assistance provided by full EPS was more than that by HPAS. This is because some of the required torque in an HPAS system was provided by the driver, unlike the full EPS system which provides all the actuation power for steering. Based on these explanations, it could perhaps be viewed that a full EPS system would be less economical than an HPAS system since it provides all the steering actuation power (with none from the driver). However, in operation, the full EPS system would be expected to be more economical than the HPAS system because the HPAS system is always in operation when a vehicle is running. The full EPS system operates only when the steering is actuated, and therefore energy can be saved with less involvement in cornering manoeuvres.

From Figure 9(b), it can be observed that the steering-wheel torque for a full EPS system increases at an increasing rate while the steering-wheel torque for an HPAS system increases linearly. The required steering-wheel torque for a full EPS system was lower than for an HPAS system when the vehicle speeds were below 50 km/h and higher when the vehicle speeds were above 50 km/h.

The steering-wheel torque for a full EPS system increases at an increasing rate with increasing vehicle speed because of the design of the power boost characteristic curve. The increasing trend is similar to the characteristic of the arithmetic function which was used to construct the power-boost curve. The steering-wheel torque for an HPAS system increased linearly with increasing vehicle speed because the vehicle speed varied linearly with the self-aligning torque within a certain range. The linear increase in the self-aligning torque also caused an increase in the reactive torque in a linear fashion. The two graphs intercept when the vehicle speed is 50 km/h because that is the point where the full EPS reactive torque was calibrated.

Based on Figure 9(b), the performance of the full EPS system was predicted to be better than that of an HPAS system because the system provides a non-linear variable steering-wheel torque based on the vehicle speed. At low vehicle speeds, the driver's response to the steering-wheel input should be fast, especially during parking. At high vehicle speeds, it is very sensitive to the steering-wheel input; therefore the reactive torque of the steering wheel should be high in order to avoid any mistakes by the driver.

The overlaid plots of power assistance characteristics for both HPAS and full EPS are shown in Figure 10.

From Figure 10, it can be clearly seen that the sum of the steering-wheel torque and the power-assisted torque of the HPAS system is almost equal to the power-assisted torque of the full EPS system. The slight difference during the cornering event (between 1 s and 5 s) was because the torsion bar of the HPAS system was modelled with a damping value (as found in a real vehicle). During steady state cornering, the amounts of power assistance for both systems were identical. The results verified that the full EPS system provided all the actuation power during cornering.

Performance enhancement of the steering-wheel torque

Performance enhancement of the reactive torque for steering feel can be achieved by adding active control. Since full EPS uses electric motors in operation, it is much easier to implement active control than with conventional HPAS. Active control of the reactive torque from the electric motor can be achieved by controlling the input current. On the other hand, the reactive torque for an HPAS system can be varied by changing the properties of the torsion bar.[4]

Active control of the reactive torque can enhance the performance of a steering system during extreme conditions or to respond to a specific driver's needs. Some cases are illustrated as follows.

1. During an emergency, e.g. collision avoidance, it is desirable that the steering-wheel torque is lighter even though the vehicle may be moving at a high speed.
2. During parking or moving off, it is sometimes desirable to turn the steering wheel as fast as possible.
3. When a vehicle is yawing or skidding, it is desirable to have a correct feel about what is actually happening at the road wheels depending on the environmental conditions.

It should be noted that, depending on the driving situation, it is desirable that the steering-wheel torque varies with the rotational speed of the steering wheel, the yaw velocity and the lateral acceleration. The reactive torque can be varied with some modifications to the original formula according to

$$\tau_{feel} = -K_f(\delta_{sw} - \delta_p) f(\dot{\delta}_{sw}, r, a_y) \tag{4}$$

The function above can be derived using an empirical formula which is a function of the rotational speed of the steering wheel, the yaw rate and the lateral acceleration depending on the design and the requirements.

An example of the cases previously presented was analysed in detail. When a driver sees an obstacle in front while driving at a high speed, he or she will take immediate (steering) action to avoid the obstacle. However, at high vehicle speeds, it is recommended that the steering-wheel torque be high to provide the safety related to the vehicle's sensitivity. These two cases conflict with each other because the vehicle cannot be steered sufficiently quickly to avoid an obstacle if the reactive torque of the steering wheel is very heavy.

It is possible to solve this conflict by implementing active control on the reactive torque of the full EPS system by adding a term to the existing reactive torque which is a function of the rotational speed of the steering wheel according to

$$\tau_{feel} = -K_f(\delta_{sw} - \delta_{rm}) \frac{C}{C \pm \dot{\delta}_{sw}} \tag{5}$$

This formula was implemented on the software model and the simulation results were compared with those from a conventional HPAS vehicle. The input angle characteristic is similar to Figure 6 but the time taken for the steering action was less than 1 s, which represented a collision avoidance event. The results are presented in Figure 11. The constant C was obtained by using an iteration technique to obtain the desired characteristics.

From Figure 11, at an early stage in the steering action, the reactive torque for the full EPS system was higher. However, as the driver applies more effort to turn the steering wheel, the rotational speed of the

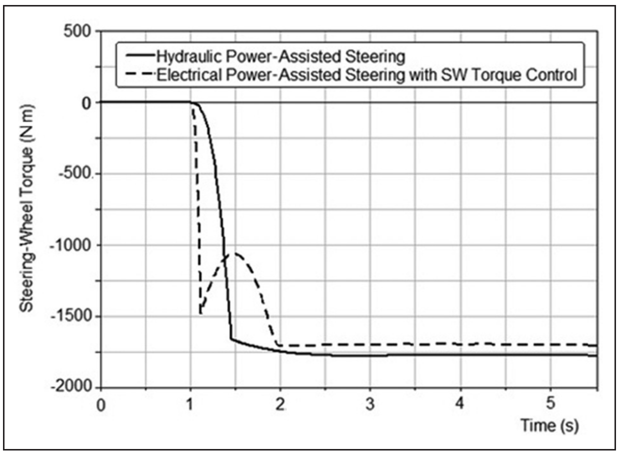

Figure 11. Active control of the reactive torque during an emergency.
SW: steering-wheel.

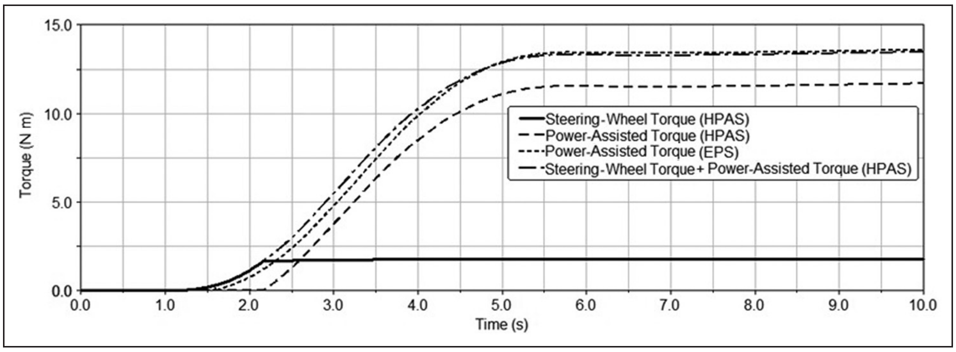

Figure 10. Comparison of the amounts of total power assistance as shown by a plot of the torque versus time.
HPAS: hydraulic power-assisted steering; EPS: electrical power steering.

steering wheel increases and the reactive torque for the full EPS system becomes lower than for the HPAS system in order to allow fast steering action. This characteristic can be obtained because the added term is a function of the rotational speed of the steering wheel; as the function value increases, the term approaches a value much less than 1 while, at low vehicle speeds, the term value becomes approximately equal to 1.

Conclusions

A method of designing a full EPS system to replace and improve on a conventional HPAS system was presented which also introduced a new steering system with improved performance. The simulation results show that the characteristics of power assistance of a full EPS system were similar to those of an HPAS system but the full EPS system provides all the steering actuation power. The full EPS system offers energy-saving advantages because it operates only while steering action is being applied and also offers other benefits such as packaging and customised steering feel.

The reactive torque of the full EPS system had a better characteristic in terms of steering requirements; it was found to be non-linear in comparison with the linear behaviour of the HPAS system. At low vehicle speeds, the steering becomes lighter in a non-linear fashion to allow fast manoeuvring. In contrast, the steering becomes heavier at high vehicle speeds also in a non-linear fashion to prevent unintentional steering input where the vehicle's sensitivity is high.

The reactive torque could also be improved by adding active control such as manipulating the input current. This could be achieved by introducing some kind of empirical formula which might be a function of the rotational speed of the steering wheel, the yaw velocity and/or the lateral acceleration. By varying the steering-wheel torque in relation to the rotational speed of the steering wheel, it was shown from the simulation results that a high-speed vehicle could be manoeuvred quickly during collision avoidance even though the steering becomes heavier at high vehicle speeds.

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Appendix

Notation

a_y	total lateral acceleration (m/s^2)
C	general constant
d	angular unit to represent classification of the vehicle speed (deg)

i	subscript counting index
K_f	reactive torque constant for electrical power steering (deg)
m	slope
n	temporary counting index
r	yaw velocity of the vehicle (deg/s)
V	road speed of the vehicle (km/h)
α	angular deflection of the steering shaft (deg)
δ_p	pinion rotation angle (deg)
δ_{sw}	angle of the steering wheel (deg)
$\dot{\delta}_{sw}$	rotational speed of the steering wheel (deg/s)
τ	power assistance torque (N m)
τ_{feel}	reactive torque for steering feel (N m)
τ_{min}	minimum torque (N m)