

**SAE TECHNICAL
PAPER SERIES**

981116

Control of Electric Power Steering Systems

J. S. Chen

Hermes Technologies

**Reprinted From: Developments in Tire, Wheel, Steering, and Suspension Technology
(SP-1338)**

SAE *The Engineering Society
For Advancing Mobility
Land Sea Air and Space®*
I N T E R N A T I O N A L

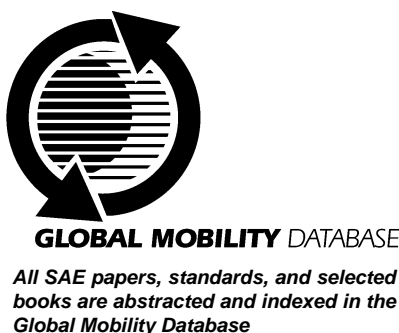
**International Congress and Exposition
Detroit, Michigan
February 23-26, 1998**

The appearance of this ISSN code at the bottom of this page indicates SAE's consent that copies of the paper may be made for personal or internal use of specific clients. This consent is given on the condition, however, that the copier pay a \$7.00 per article copy fee through the Copyright Clearance Center, Inc. Operations Center, 222 Rosewood Drive, Danvers, MA 01923 for copying beyond that permitted by Sections 107 or 108 of the U.S. Copyright Law. This consent does not extend to other kinds of copying such as copying for general distribution, for advertising or promotional purposes, for creating new collective works, or for resale.

SAE routinely stocks printed papers for a period of three years following date of publication. Direct your orders to SAE Customer Sales and Satisfaction Department.

Quantity reprint rates can be obtained from the Customer Sales and Satisfaction Department.

To request permission to reprint a technical paper or permission to use copyrighted SAE publications in other works, contact the SAE Publications Group.



No part of this publication may be reproduced in any form, in an electronic retrieval system or otherwise, without the prior written permission of the publisher.

ISSN 0148-7191

Copyright 1998 Society of Automotive Engineers, Inc.

Positions and opinions advanced in this paper are those of the author(s) and not necessarily those of SAE. The author is solely responsible for the content of the paper. A process is available by which discussions will be printed with the paper if it is published in SAE Transactions. For permission to publish this paper in full or in part, contact the SAE Publications Group.

Persons wishing to submit papers to be considered for presentation or publication through SAE should send the manuscript or a 300 word abstract of a proposed manuscript to: Secretary, Engineering Meetings Board, SAE.

Printed in USA

Control of Electric Power Steering Systems

J. S. Chen

Hermes Technologies

ABSTRACT

The dynamic characteristics of electric power steering (EPS) systems can be characterized by the transmissibility from rack load to steering wheel torque. The transmissibility can be studied by fixing the steering wheel and calculating the frequency response from the rack load to the torque needed to hold the steering wheel. A proportion-plus-derivative control is needed for EPS systems to generate desired static torque boost and avoid high transmissibility at the mid-frequency range. A pure proportion control can't satisfy both requirements at the same time.

INTRODUCTION

Power steering systems for automobiles are becoming ever more popular because they reduce steering efforts of the drivers, especially during parking lot maneuver. Hydraulic power steering has existed for many years and is widely used. However, the efficiency of such systems is low because of an engine driven hydraulic pump which runs all the time. The pump and associated piping also take up space and assembly time.

Electric power steering (EPS), which use an electric motor with an electronic controller, came to the market few years ago^[1]. It solves the problems associated with the hydraulic power steering. The motor only operates when steering assistance is needed. Hydraulic pump and piping are eliminated. The EPS also allows us to adjust static torque boost curves by modifying software in the electronic controllers without changing the torsion bars. The static torque boost curves can be alternated according to vehicle speed to improve steering feel.

The dynamic characteristics of EPS systems can be characterized by the transmissibility from rack load to steering wheel torque. The transmissibility shouldn't start to roll off too early to generate "dull" steering feel. Neither should the transmissibility stay high at high frequency such that high frequency road disturbance is transmitted to the steering wheel, and energy is wasted at the motor^[2]. Therefore, the transmissibility versus frequency relationship needs to be tailored carefully. The dynamic characteristics will change when the static characteristics are adjusted. The following paragraphs discuss the effects of motor feedback control on the transmissibility. A control design is included to show achievable performance.

SYSTEM FORMULATION

An EPS is shown in Figure 1. The steering wheel is connected to one end of a torque sensor, which senses torque and acts like a torsional spring. The other end of the torque sensor is connected to a gear box that is subsequently connected to a pinion. The rotation of the pinion moves a rack that is connected to wheels through tie-rods. The commands from the control unit to the motor are a function of torque sensor output and vehicle speed.

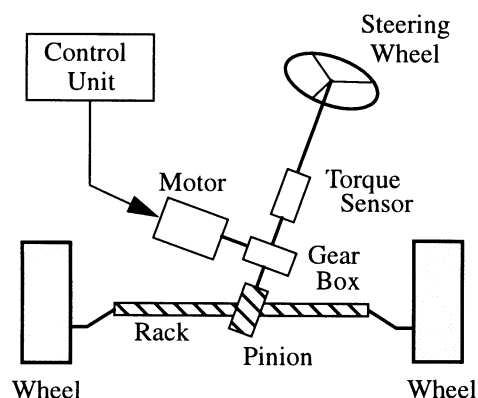


Figure 1. EPS system

The transmissibility from the rack load to the steering wheel torque can be studied by fixing the steering wheel and calculating the frequency response from the rack load to the torque needed to hold the steering wheel. When the steering wheel is fixed, the governing equation for the pinion is:

$$J_{eq} \ddot{\theta}_p = T_m - B_1 \dot{\theta}_p - K_s \theta_p + T_{ext}$$

where J_{eq} is the total inertias, which includes motor, gear train, pinion, and rack, reflected to the pinion axis. B_1 is the effective damping coefficient reflected to the pinion axis. K_s represents the stiffness of the torsion bar in the torque sensor. T_m and T_{ext} are the motor and external torque at the pinion axis. θ_p represents pinion angle.

Let's assume a DC motor is used in the system. Similar equations exist for other types of motors. The motor torque at the pinion is (after speed reduction):

$$T_m = \frac{N_1 K_a}{R} (u - K_b N_1 \dot{\theta}_p)$$

where K_a , R , and K_b are the torque constant, armature winding resistance, and motor back electromagnetic force constant; N_1 is gear box gear ratio; u is the motor input voltage

Combining above two equations, we get

$$J_p \ddot{\theta}_p + \left(B_1 + \frac{K_a K_b N_1^2}{R} \right) \dot{\theta}_p + K_s \theta_p = \frac{N_1 K_a}{R} u + T_{ext} \quad (\text{Eq. } \bullet)$$

The frequency response from the external torque applied at the rack to the torque needed to hold the steering wheel (T_{sw}) can be calculated from the above equation since T_{sw} is equal to

$$T_{sw} = K_s \theta_p$$

When the motor input voltage is proportional to the torque sensor output, u can be expressed as follows:

$$u = -K_p(\theta_p - \theta_{sw})$$

where K_p is the combination of the torque sensor stiffness and output gain. Note that θ_{sw} is zero when the steering wheel is fixed. Substituting above function into Equation [•], we get

$$J_p \ddot{\theta}_p + \left(B_1 + \frac{K_a K_b N_1^2}{R} \right) \dot{\theta}_p + \left(K_s + \frac{N_1 K_a K_p}{R} \right) \theta_p = T_{ext}$$

This is a standard second order equation. The natural frequency (ω_n) and damping ratio (ζ) of the system are equal to

$$\omega_n = \sqrt{\frac{K}{J_p}} \quad \zeta = \frac{B}{2\sqrt{J_p K}}$$

where

$$K = K_s + \frac{N_1 K_a K_p}{R} \quad B = B_1 + \frac{K_a K_b N_1^2}{R}$$

It is obvious that the natural frequency, which is related to the band width, increases with the proportional gain and the damping ratio decreases when the proportional gain increases. Higher band width and low damping ratio create harsh steering feel. But higher gain is needed for higher steering assistance. Potential conflicts exist when a proportional controller is used in the feedback loop.

The transmissibility curve (frequency response) of a proportional feedback system is shown in Figure 2. The parameters used in the calculation are listed in Table 1. The dash-dotted curve is the frequency response of a system with a very small proportional gain. The motor provides little assistance. It does add certain amount of damping to the system due to the back electromagnetic force. The solid curve represents a system with a higher

proportional gain. The gain was chosen to make the transmissibility at low frequencies equal to one fifth of the dash-dotted curve. Note that the closed-loop transmissibility at 90 rad/sec (14 Hz) is higher, which is undesirable. It allows high frequency torque disturbance to pass through. The time responses of the steering wheel torque to a unit torque impulse at the pinion are shown in Figure 3. For the system with a higher gain, drivers will feel the high frequency oscillation when the rack endures an impact.

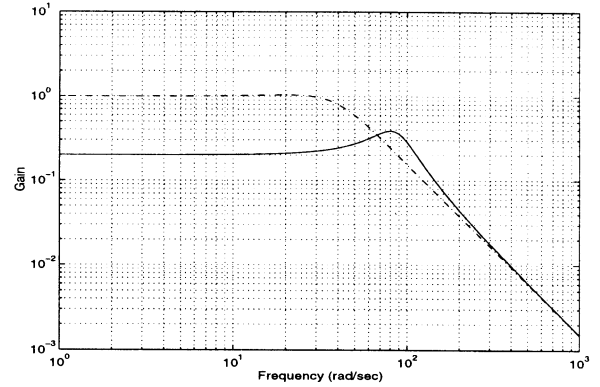


Figure 2. Transmissibility for systems with low (dash-dot) and high (solid) proportional gain

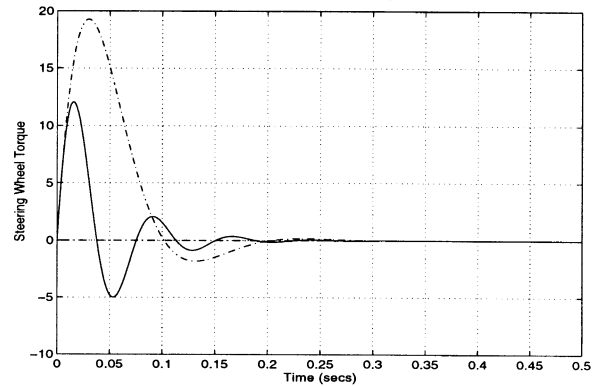


Figure 3. Impulse responses to pinion torque for systems with low (dash-dot) and high (solid) proportional gain

Table 1. System parameters

$J_{eq} = 0.06 \text{ kg-m}^2$	$B_1 = 0.3 \text{ Nm-sec}$
$K_s = 1.57 \text{ Nm/Deg}$	$N_1 = 25$
$K_b = 0.02 \text{ V-sec}$	$K_a = 0.02 \text{ Nm/A}$
$R = 0.1 \Omega$	

PROPORTION-PLUS-DERIVATIVE CONTROL

The dilemma faced by the proportion control can be eliminated by using a proportion-plus-derivative (PD) control. The control equation is

$$u = -K_p(\theta_p - \theta_{sw}) - K_d(\dot{\theta}_p - \dot{\theta}_{sw})$$

Substituting above function into Equation [•], we get

$$J_p \ddot{\theta}_p + \left(B_1 + \frac{K_a K_b N_1^2}{R} + \frac{N_1 K_a K_d}{R} \right) \dot{\theta}_p + \left(K_s + \frac{N_1 K_a K_p}{R} \right) \theta_p = T_{ext}$$

We can choose an appropriate proportional gain (K_p) to achieve desired steering assistance level and use derivative gain (K_d) to reach required damping ratio.

Figure 4 shows the transmissibility of a system with a PD control. The transmissibility is lower than the low gain system at all frequencies. Note that the transmissibilities are the same for low gain and PD systems at very high frequencies. In fact, the transmissibilities are equal to the open-loop system at high frequencies. The high frequency transmissibility is determined by rack, pinion, motor, and gear train inertias and won't be changed by a feedback control. The PD control closed-loop impulse response of the steering wheel torque is shown in Figure 5. Comparing with Figure 3, the response is much improved; peak amplitude is lower, and high frequency oscillation no longer exists.

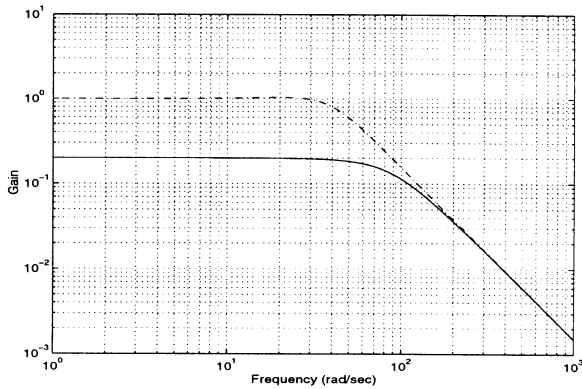


Figure 4. Closed-loop transmissibility (solid) with improved controller. (Transmissibility for systems with low gain system is also shown for comparison)

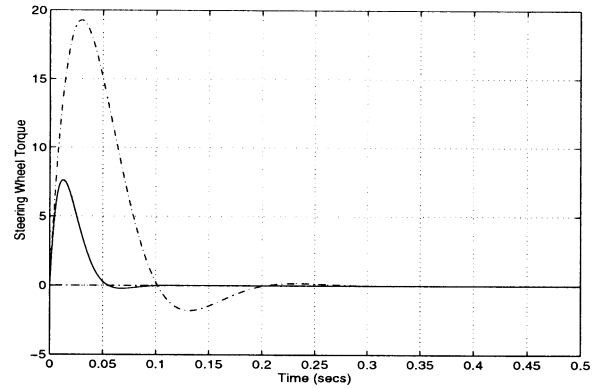


Figure 5. Closed-loop impulse response to pinion torque

Noise may be a problem with a PD controller. We can always approximate a PD controller with a lead compensator. The performances will deteriorate by a small amount.

Although a linear model is used in the study, data from system identification experiments can be used to design the control if the linear model is inadequate. The basic tuning principles will remain the same.

CONCLUSIONS

In summary, a proportion-plus-derivative control is needed for an EPS system to generate desired static torque boost and avoid the high transmissibility at the mid-frequency range generated by the feedback. A pure proportion control can't satisfy both requirements at the same time. The inertias of the rack, pinion, and gear train should also be selected carefully to suppress high-frequency transmissibility.

REFERENCES

1. Nakayama, T. and Suda, E., "The Present and Future of Electric Power Steering", Int. J. of Vehicle Design, pp. 243-254, Vol. 15, 1994.
2. Sugitani, N., Fujiwara, Y., Uchida, K., and Fujita, M., "Electric Power Steering with H-infinity Control Design to Obtain Road Information", Proc. American Control Conference, pp. 2935-2939, 1997.