Basic Statistical Descriptions of Data

- Motivation
  - To better understand the data: central tendency, variation and spread
- Data dispersion characteristics
  - median, max, min, quantiles, outliers, variance, etc.
- Numerical dimensions correspond to sorted intervals
  - Data dispersion: analyzed with multiple granularities of precision
  - Boxplot or quantile analysis on sorted intervals

Measuring the Central Tendency

- Mean (algebraic measure) (sample vs. population):

  $$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i \qquad \mu = \frac{\sum x}{N}$$

  Note: n is sample size and N is population size.

  - Weighted arithmetic mean:

  $$\bar{x} = \frac{\sum_{i=1}^{n} w_i x_i}{\sum_{i=1}^{n} w_i}$$

- Median:
  - Middle value if odd number of values, or average of the middle two values otherwise
  - Estimated by interpolation (for *grouped data*):

  $$median = L_1 + \left(\frac{n/2 - (\sum freq)l}{freq_{median}}\right) width$$

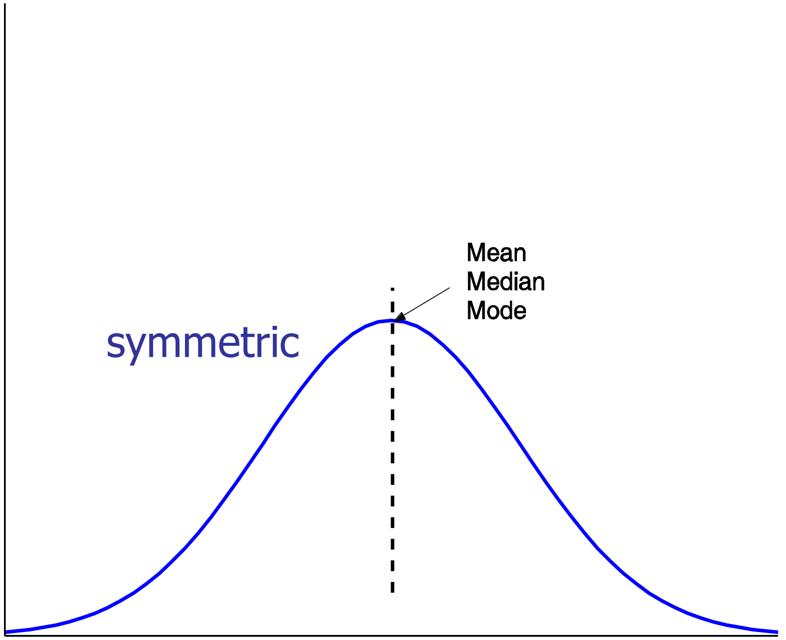
age	frequency
1–5	200
6–15	450
16–20	300
21–50	1500
51–80	700
81–110	44

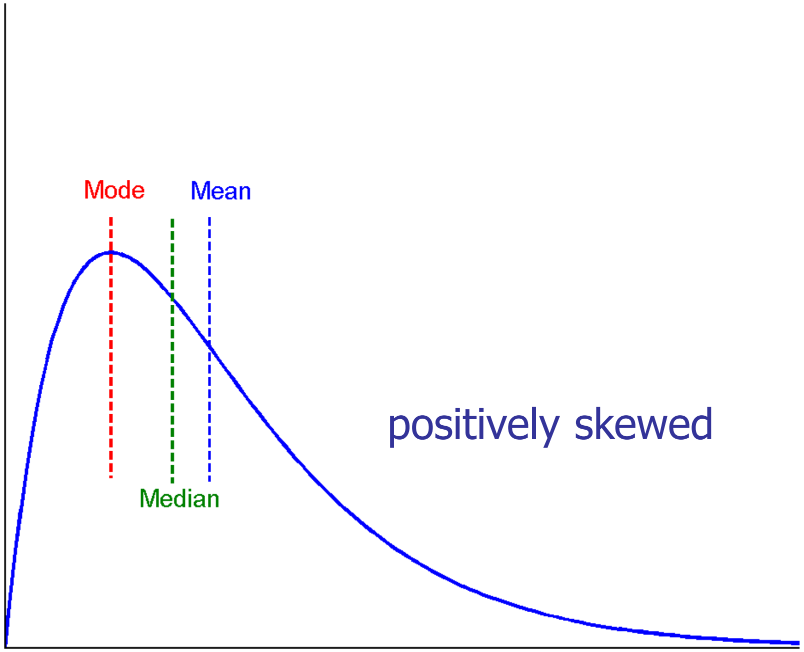
- Mode
  - Value that occurs most frequently in the data
  - Unimodal, bimodal, trimodal
  - Empirical formula:

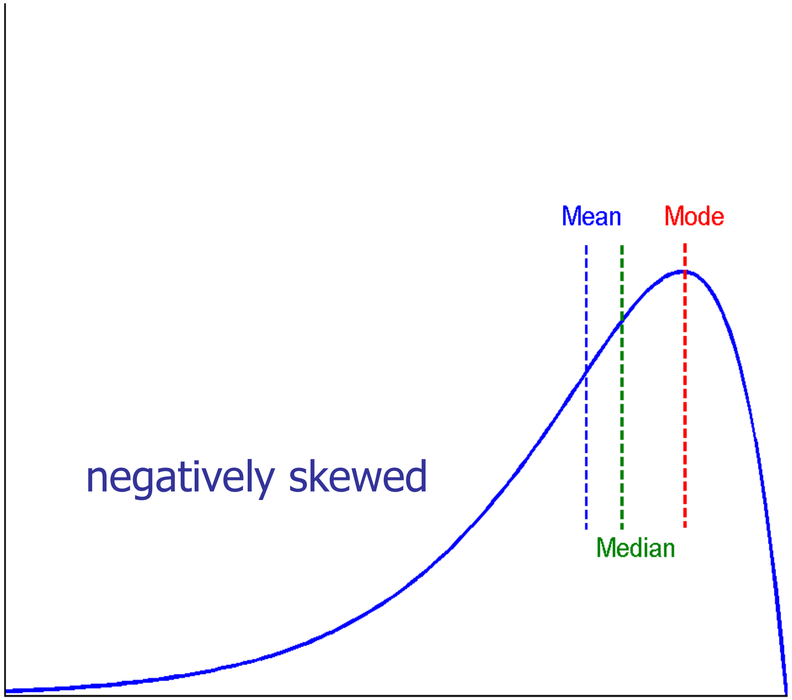
  $$mean - mode = 3 \times (mean - median)$$

Symmetric vs. Skewed Da

- Median, mean and mode of symmetric, positively and negatively skewed data







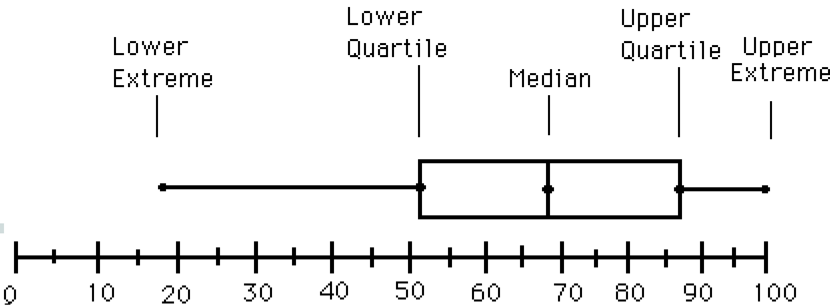
Measuring the Dispersion of Data

- Quartiles, outliers and boxplots
  - **Quartiles**: Q_1 (25th percentile), Q_3 (75th percentile)
  - **Inter-quartile range**: IQR = $Q_3 - Q_1$
  - **Five number summary**: min, Q_1, median, Q_3, max
  - **Boxplot**: ends of the box are the quartiles; median is marked; add whiskers, and plot outliers individually
  - **Outlier**: usually, a value higher/lower than 1.5 x IQR
- Variance and standard deviation (*sample: s, population: σ)*
  - **Variance**: (algebraic, scalable computation)

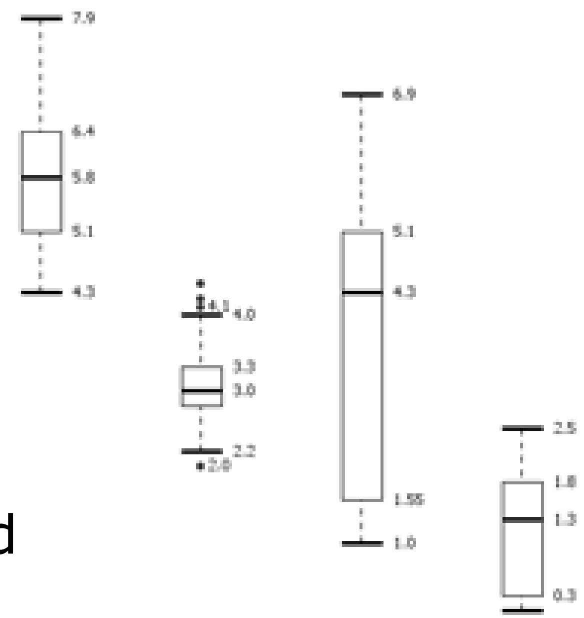
$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \overline{x})^2 = \frac{1}{n-1}[\sum_{i=1}^{n}x_i^{\,2} - \frac{1}{n}(\sum_{i=1}^{n}x_i)^2] \qquad \sigma^2 = \frac{1}{N}\sum_{i=1}^{n}(x_i - \mu)^2 = \frac{1}{N}\sum_{i=1}^{n}x_i^{\,2} - \mu^2$$

  - **Standard deviation** *s (or σ)* is the square root of variance *s^2 (or σ^2)*

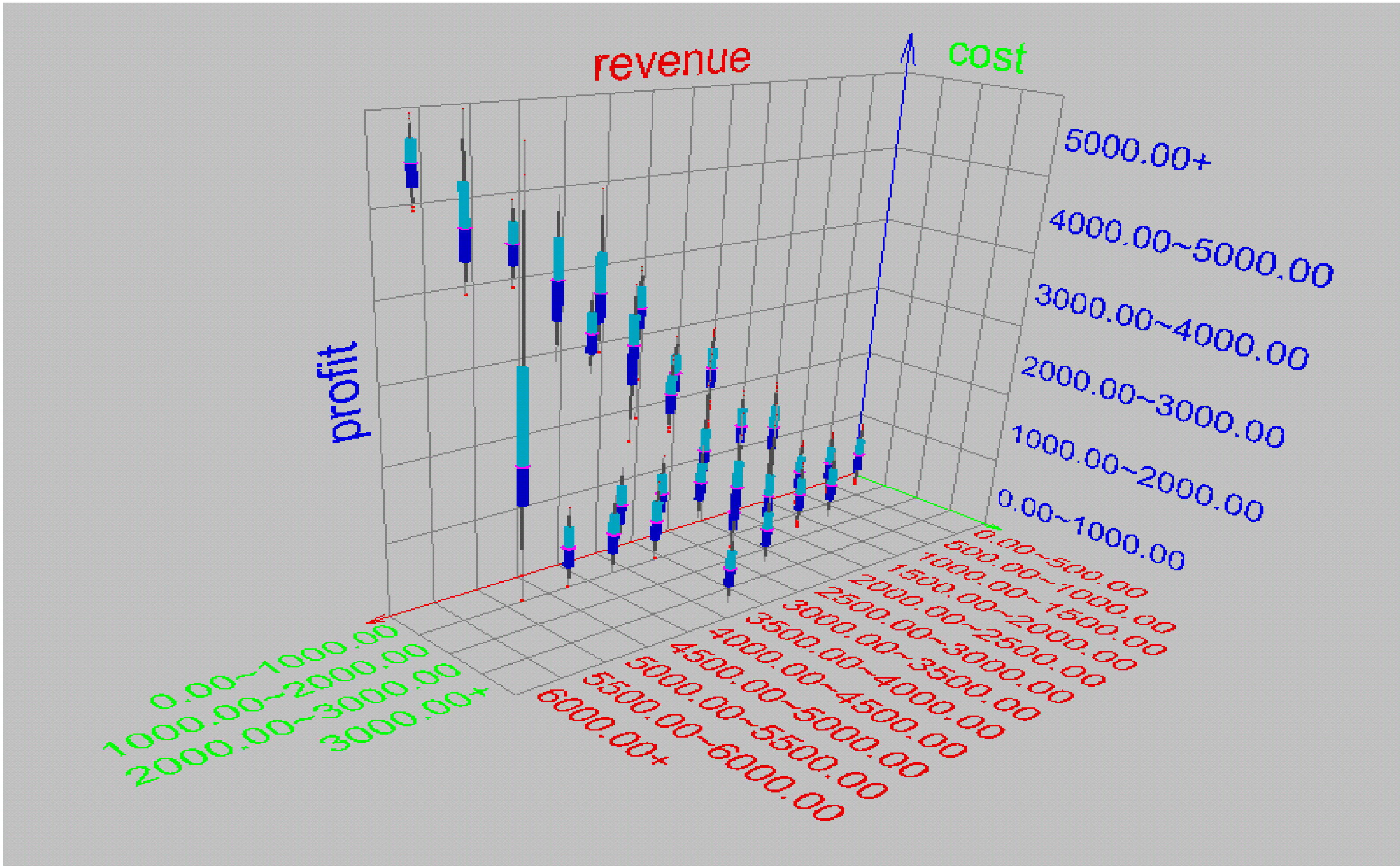
Boxplot Analysis



- **Five-number summary** of a distribution
  - Minimum, Q1, Median, Q3, Maximum
- **Boxplot**
  - Data is represented with a box
  - The ends of the box are at the first and third quartiles, i.e., the height of the box is IQR
  - The median is marked by a line within the box
  - Whiskers: two lines outside the box extended to Minimum and Maximum
  - Outliers: points beyond a specified outlier threshold, plotted individually



Visualization of Data Dispersion: 3-D Boxplots



Properties of Normal Distribution Curve

- The normal (distribution) curve
  - From $\mu-\sigma$ to $\mu+\sigma$: contains about 68% of the measurements (μ: mean, σ: standard deviation)
  - From $\mu-2\sigma$ to $\mu+2\sigma$: contains about 95% of it
  - From $\mu-3\sigma$ to $\mu+3\sigma$: contains about 99.7% of it