Data Analysis:

Confidence Intervals and Hypothesis Testing

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Statistical inference

- **statistical inference** includes
  - drawing conclusions about the properties of a population based on a relevant sample of data from that population
  - estimating the level of confidence in those conclusions
- inference can be valid only if the sample is **representative** of the population
- inference is possible only in contexts where the probability distribution is **stable**

Confidence intervals (CIs)

What is a confidence interval?

- A confidence interval is a measure of how well a statistic, calculated on a data sample, represents a population parameter.
- The parameter for which a confidence interval is most commonly stated is the **mean**.

- A confidence interval is expressed in terms of a percentage-based confidence level (e.g. 95%) and a range within which the *actual parameter* is expected to be found with that level of confidence. For example, the confidence interval

  $$\mathbf{455.5 \pm 5.4}, \text{ with a confidence level of } \mathbf{99\%}$$

  states that we can be 99% confident that the parameter at hand is between 450.1 and 460.9. $\mathbf{455.5}$ is the value of the statistic (calculated on a sample).

The sampling distribution

- Any statistic, being calculated from a sample, will vary between samples and consequently will have a *distribution* - **this is what allows us to define confidence intervals**.
- Take the mean: if it is calculated repeatedly for different samples drawn from a population, these values of the mean will vary and will be distributed in some way.
- The distribution of a statistic
  - is called a **sampling distribution**
  - has a **standard error** (corresponding to the standard deviation of a value distribution)
  - has an **expected value** (corresponding to the mean of a value distribution)

The sampling distribution of the mean

Now we focus on the **mean** - the statistic that is most commonly associated with confidence intervals. For the mean of a numeric variable:

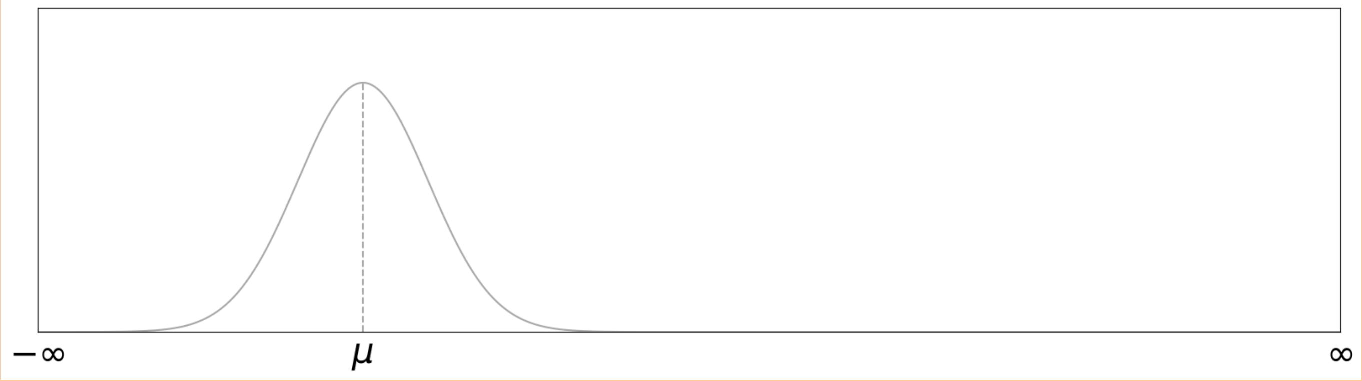
- the **sampling distribution** is *normal* in many cases:
  - when the value distribution is normal
  - if the value distribution is not normal but the sample size is greater than 30 (central limit theorem)

- the **standard error** is: $\boxed{\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}}$

  where $\sigma_{\bar{x}}$ is the standard error, σ is the standard deviation of the variable x and n is the sample size

- the **expected value** is equal to the population mean: $\boxed{E(\bar{x}) = \mu}$

  where $E(\bar{x})$ is the expected value for the mean and μ is the population mean.

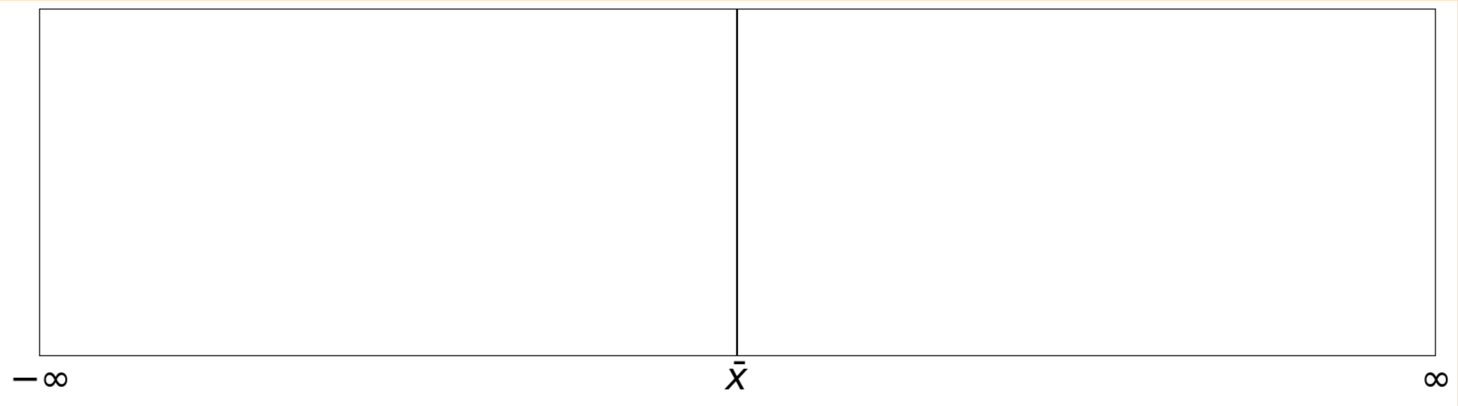

−∞
μ
∞

The concepts behind confidence intervals

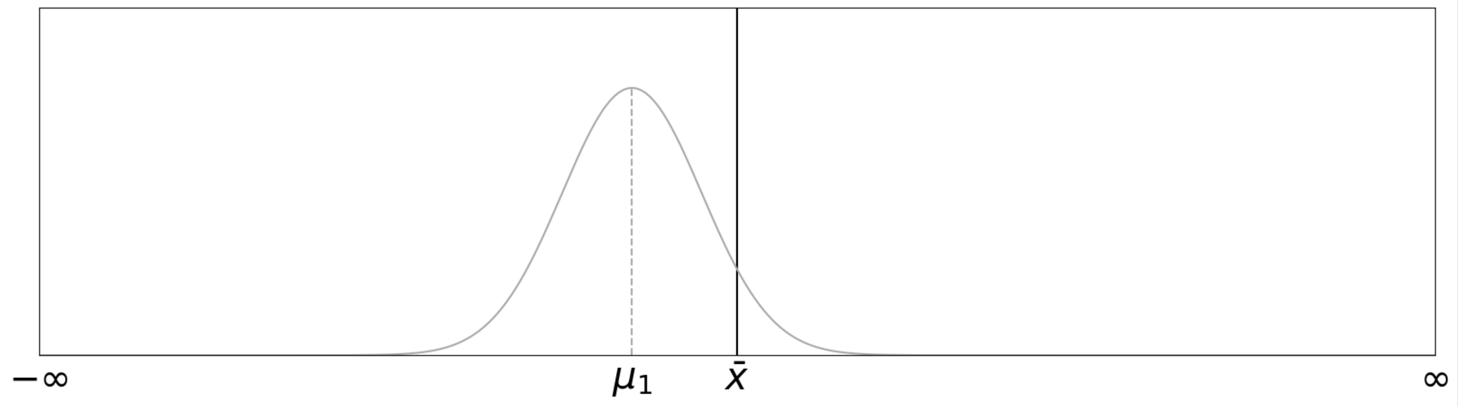
- Let's look at the situation where we know the standard deviation of our variable but do not know the mean. We have taken a sample and calculated the sample mean, $\bar{x}$:

$-\infty$
$\bar{x}$
∞

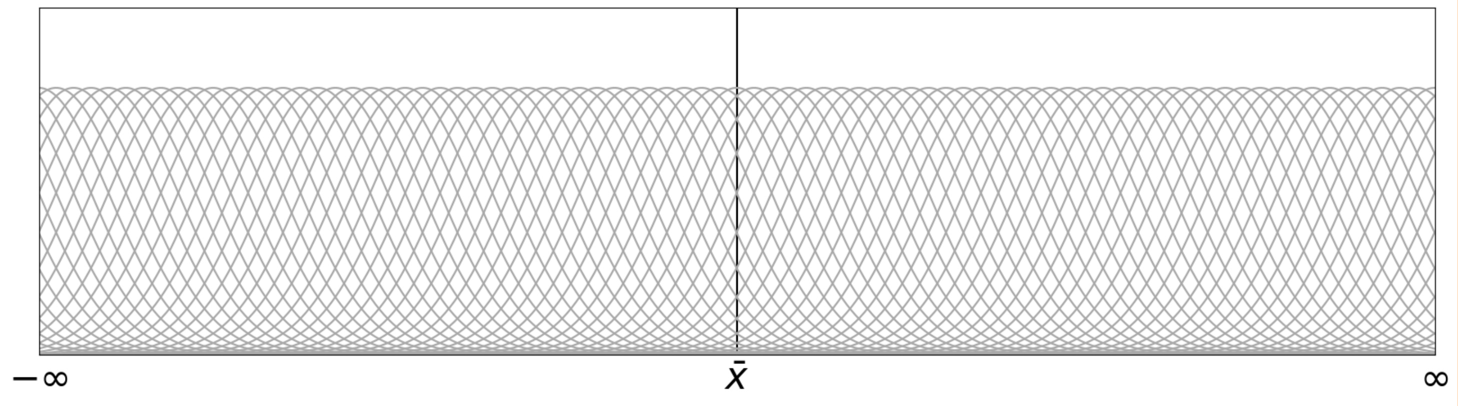
- We will place this sample mean in the middle of the picture, without loss of generality:



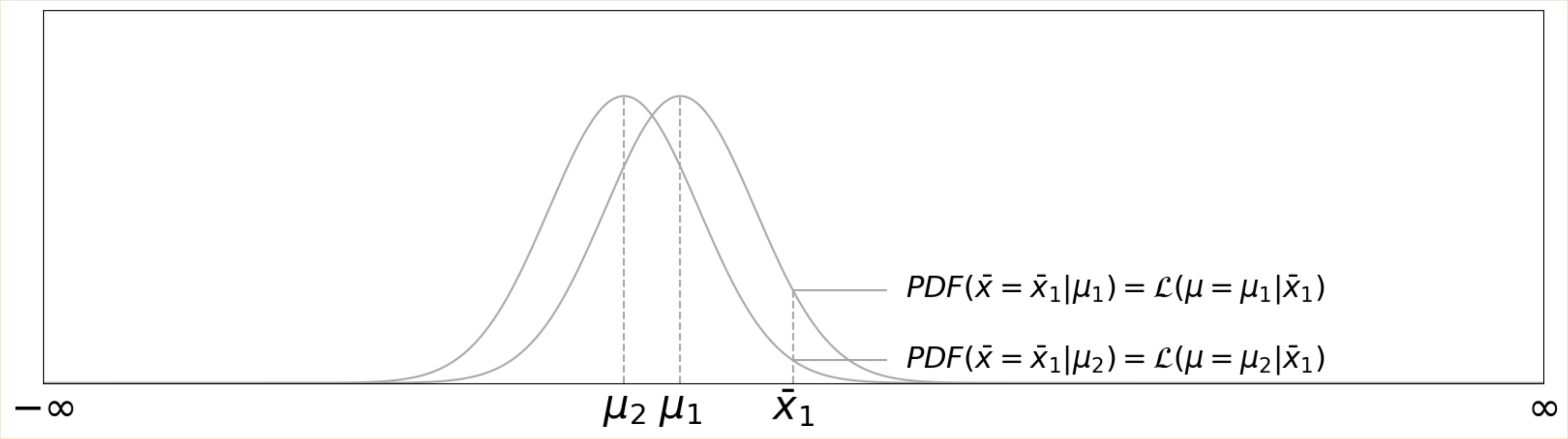
- This sample mean may have come from a distribution with expected value μ_1:



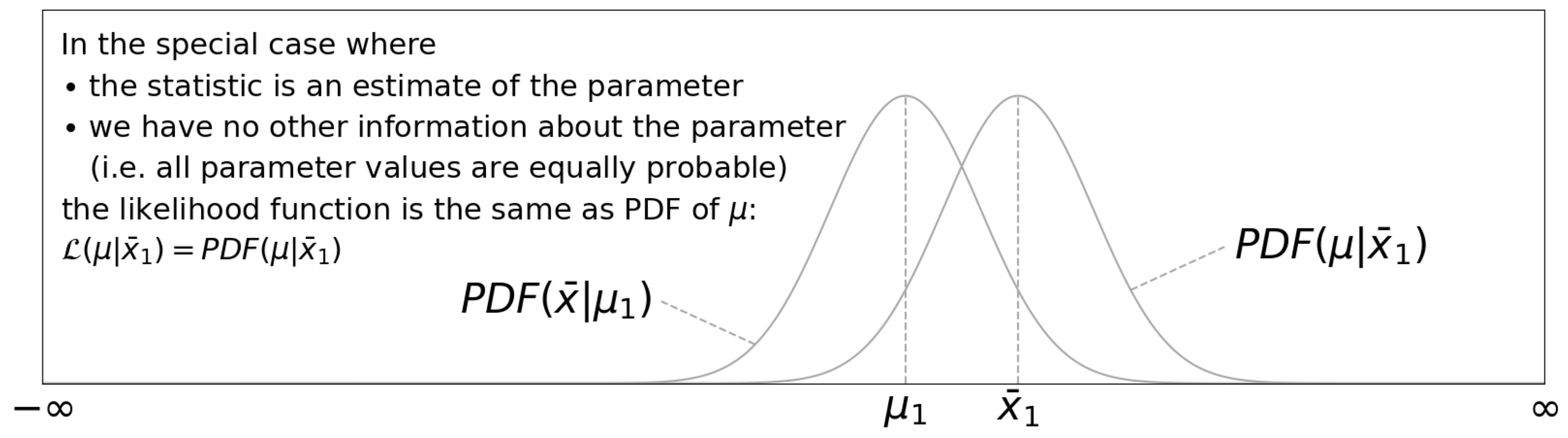
- But, it also may have come from any of an infinite set of equally probable distributions:



- Let us suppose that the real mean is some value μ_1. Its sampling distribution probability density function (PDF) is shown in the picture. Also shown is our sample mean $\bar{x}$ and the PDF value for $\bar{x}$ in that distribution. This PDF value is also the value of the **likelihood** function, which represents a measure of how likely the parameter (μ) is, given the statistic $(\bar{x})$ and is written as $\mathcal{L}(\mu|\bar{x})$.



- **If we assume that all means between $-\infty$ and $+\infty$ are equally probable**, the likelihood plot based on the experimentally obtained value of $\bar{x}$ can be treated as a PDF over μ and used to calculate the probability of μ falling in a particular range of values (by means of integration).



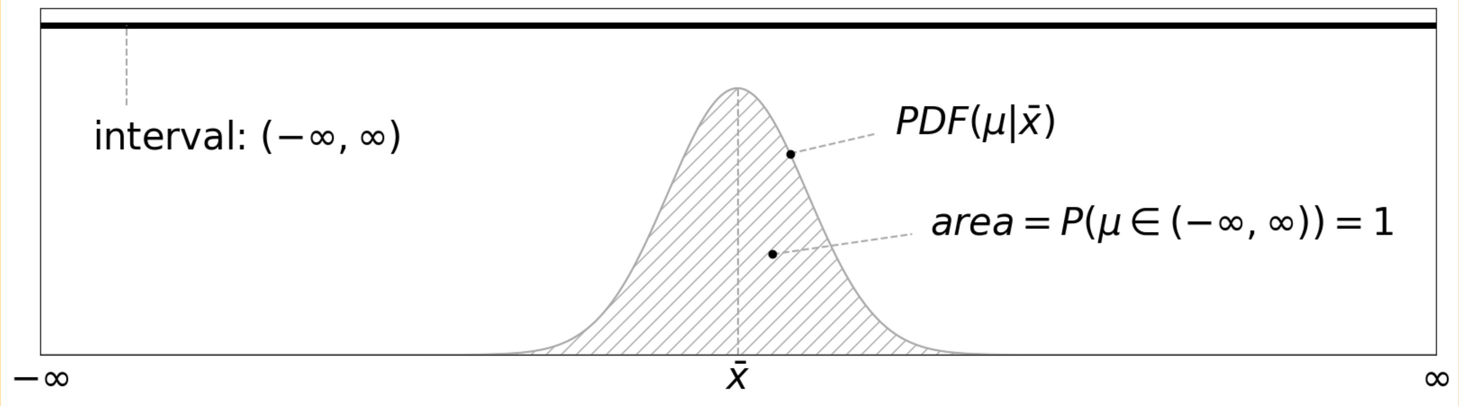
In the special case where

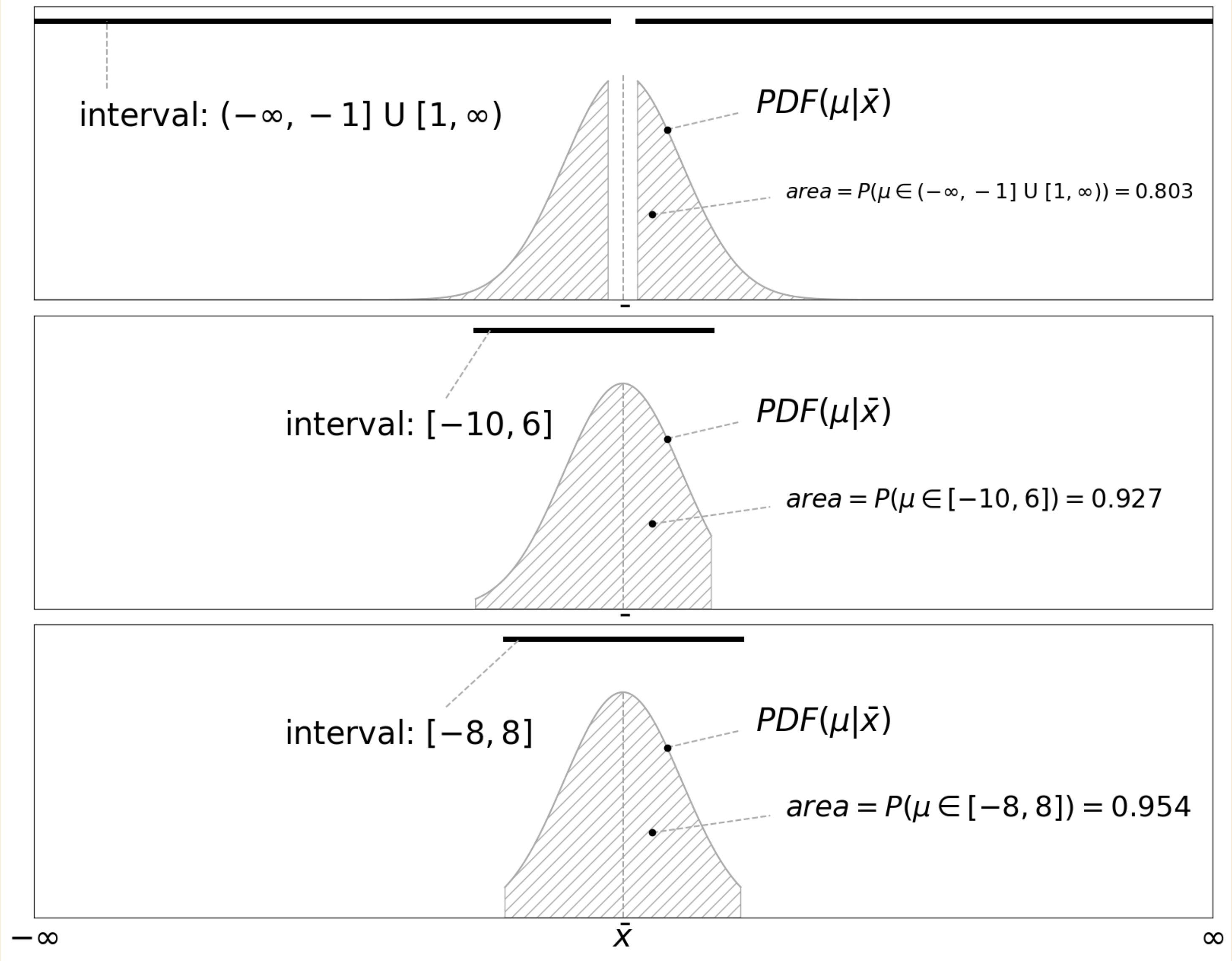
- the statistic is an estimate of the parameter
- we have no other information about the parameter (i.e. all parameter values are equally probable)

the likelihood function is the same as PDF of μ:
$\mathcal{L}(\mu|\bar{x}_1) = PDF(\mu|\bar{x}_1)$

- The following pictures show some ranges of possible values of μ. Each picture shows:
  - the range as a thick line running at the top of the picture, with sections that are outside of the range cut out
  - the μ value likelihood PDF, hatched in the areas included in the range
  - the likelihood of μ belonging to the range, $P(\mu \in RANGE)$, which is calculated as the area under the PDF curve and which is 1 when all values are included (range $(-\infty, \infty)$)

  Click on the pictures for videos demonstrating the accumulation of probability 'slices' into confidence intervals.




interval: $(-\infty, -1] \cup [1, \infty)$
$PDF(\mu|\bar{x})$
$area = P(\mu \in (-\infty, -1] \cup [1, \infty)) = 0.803$
interval: $[-10, 6]$
$PDF(\mu|\bar{x})$
$area = P(\mu \in [-10, 6]) = 0.927$
interval: $[-8, 8]$
$PDF(\mu|\bar{x})$
$area = P(\mu \in [-8, 8]) = 0.954$
$-\infty$
$\bar{x}$
∞

The meaning of a confidence interval

- A confidence interval states, with a certain level of confidence, that the real mean is in a particular range. The four pictures of ranges and likelihoods lead to the following statements:

  1. The mean is somewhere between $-\infty$ and ∞, with a confidence level of 100%.
  2. The mean is somewhere between $-\infty$ and $\bar{x} - 1$ or between $\bar{x} + 1$ and ∞, with a confidence level of 80%.
  3. The mean is somewhere between $\bar{x} - 10$ and $\bar{x} + 6$, with a confidence level of 93%.
  4. The mean is somewhere between $\bar{x} - 8$ and $\bar{x} + 8$, with a confidence level of 95%.

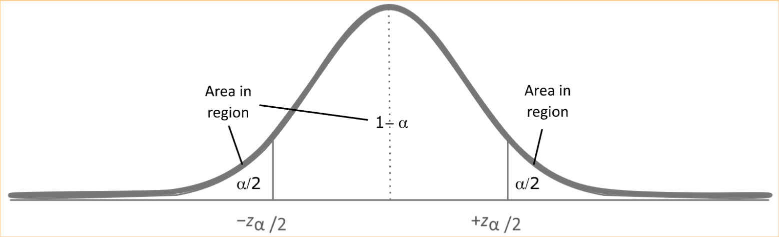
Confidence interval as used in statistics

- Most of the intervals listed above are not very useful ($-\infty$ to ∞?!)
- In statistics the interval used has the following properties:
  - is symmetrical around the point of highest likelihood (the sample mean, $\bar{x}$)
  - maximises the likelihood among all intervals of the same size
- It can be stated as:

  $\bar{x}\pm$ <half-interval for LoC>, with level of confidence <LoC>

Deriving a confidence interval in practice

- In practice, the distribution that is used is the z-distribution (a normal distribution with standard deviation 1 and mean 0):



Source: [MSD]

- If we define $\alpha = 1 - \frac{LoC}{100}$, where LoC is the percentual value of the level of confidence (e.g. if the required level of confidence is 95%, $LoC = 95$), then we can find two values on the x-axis, $-z_{\alpha/2}$ and $z_{\alpha/2}$, that 'fence off' an area under the distribution curve of $\frac{\alpha}{2}$ to the left and to the right, respectively.
- The above step is performed by looking up a table that maps $\frac{\alpha}{2}$ values to values for $z_{\alpha/2}$.
- The interval between $-z_{\alpha/2}$ and $z_{\alpha/2}$ is the z-distribution *confidence interval* for level of confidence $1 - \alpha$.
- The z-distribution confidence interval then needs to be converted to a confidence interval in the variable space: this comprises the use of the standard error as the scaling factor and the expected value as offset.

HOWTO

Deriving a confidence interval

Example scenario: We know the population standard deviation $(\sigma = 10)$ and the mean $(\bar{x} = 251)$ and size $(n = 100)$ of a sample.

1. Calculate $\alpha/2$: $\alpha/2 = \dfrac{1 - \dfrac{LoC}{100}}{2}$

   Example: For a level of confidence of 95% this is $\alpha/2 = \dfrac{1 - \dfrac{95}{100}}{2} = 0.025$

2. Lookup cut-off value $z_{\alpha/2}$

   If the table (as the one on the next page) contains upper-tail values, we look for $\alpha/2$ in the table. If the table is for two-tailed values, we look up α.

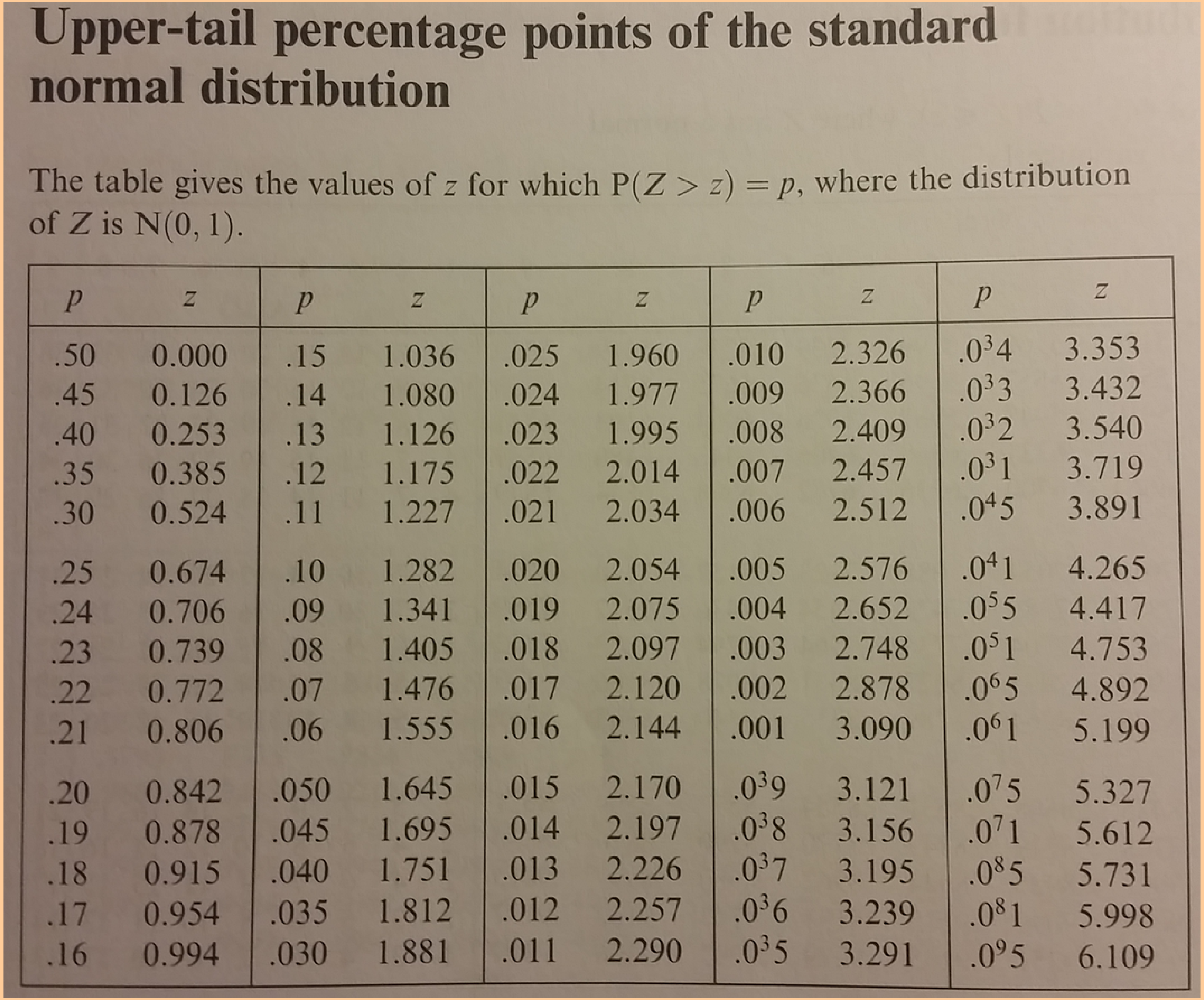
   Example: The z-value corresponding to $\alpha/2$ of 0.025 is 1.96

3. Convert the z-value to a value in the variable space using the following formula: $CI_h = z_{\alpha/2}\dfrac{\sigma}{\sqrt{n}}$, where CI_h is the half-interval.

   Example: $CI_h = 1.96 \times \dfrac{10}{\sqrt{100}} = 1.96$

4. State the confidence interval: $\bar{x} \pm z_{\alpha/2}(\dfrac{\sigma}{\sqrt{n}})$, with a confidence of LoC%

   Example: 251 ± 1.96, with 95% confidence



Upper-tail percentage points of the standard normal distribution

The table gives the values of z for which $P(Z > z) = p$, where the distribution of Z is $N(0, 1)$.

p	z	p	z	p	z	p	z	p	z
.50	0.000	.15	1.036	.025	1.960	.010	2.326	$.0^{3}4$	3.353
.45	0.126	.14	1.080	.024	1.977	.009	2.366	$.0^{3}3$	3.432
.40	0.253	.13	1.126	.023	1.995	.008	2.409	$.0^{3}2$	3.540
.35	0.385	.12	1.175	.022	2.014	.007	2.457	$.0^{3}1$	3.719
.30	0.524	.11	1.227	.021	2.034	.006	2.512	$.0^{4}5$	3.891
.25	0.674	.10	1.282	.020	2.054	.005	2.576	$.0^{4}1$	4.265
.24	0.706	.09	1.341	.019	2.075	.004	2.652	$.0^{5}5$	4.417
.23	0.739	.08	1.405	.018	2.097	.003	2.748	$.0^{5}1$	4.753
.22	0.772	.07	1.476	.017	2.120	.002	2.878	$.0^{6}5$	4.892
.21	0.806	.06	1.555	.016	2.144	.001	3.090	$.0^{6}1$	5.199
.20	0.842	.050	1.645	.015	2.170	$.0^{3}9$	3.121	$.0^{7}5$	5.327
.19	0.878	.045	1.695	.014	2.197	$.0^{3}8$	3.156	$.0^{7}1$	5.612
.18	0.915	.040	1.751	.013	2.226	$.0^{3}7$	3.195	$.0^{8}5$	5.731
.17	0.954	.035	1.812	.012	2.257	$.0^{3}6$	3.239	$.0^{8}1$	5.998
.16	0.994	.030	1.881	.011	2.290	$.0^{3}5$	3.291	$.0^{9}5$	6.109

Confidence interval for different distributions and sample sizes

While the way we define the confidence interval is in principle always the same, there are three different variants of the method, with applicability depending on:

- whether the standard deviation of the data is known or not
- the distribution of the data
- the size of the sample

The tables show the applicable **method variants (1, 2, 3)** for different combinations of the relevant factors. An **X** indicates a case where a confidence interval cannot be defined.

Standard deviation known (σ)

Sample size → **Distribution ↓**	large $(n \geq 30)$	small $(n < 30)$
Normal	1	1
Any other	1	X

Standard deviation unknown

Sample size → **Distribution ↓**	large $(n \geq 30)$	small $(n < 30)$
Normal	2	3
Any other	2	X

Variant 1 of method for determining confidence intervals

Variant 1 of the method for determining CIs equates to the steps described in the previous slides. This variant of the method is applicable when the following **conditions** are met:

- the standard deviation of the variable is known
- at least one of the conditions for normality of the sampling distribution is met (already mentioned on slide 4):
  - the variable distribution is normal
  - the variable distribution is not known but the sample size is large, i.e. greater than 30 (central limit theorem*)

In the case that the distribution is not known and the sample is small (< 30), the confidence interval cannot be defined, even if the standard deviation is known.

*The central limit theorem says that if the sample size is sufficiently large, the sampling distribution of the mean is approximately normal.

Variant 2 of method for determining confidence intervals

Variant 2 of the method for determining CIs is identical to variant 1, except that the standard deviation must be calculated using the formula for the standard deviation estimate calculated from a sample (S), and this statistic used instead of the population standard deviaion (σ). This variant is applicable when the following **conditions** are met:

- the standard deviation of the variable is not known
- the sample size is large ($>$30)

HOWTO

Deriving a confidence interval (variant 2)

Example scenario: We do not know the population standard deviation but we have a sample with standard deviation ($S = 10$), mean ($\bar{x} = 251$) and size of at least 30 ($n = 100$).

Proceed as in variant 1 but using S instead of σ (slide 15).

Variant 3 of method for determining confidence intervals

Variant 3 of the method for determining CIs is the same as variant 2, while using the **t-distribution** rather than the z-distribution. It is used when these **conditions** are met:

- the standard deviation of the variable is not known
- the sample size is small ($<$30)
- the variable distribution is known to be normal

The t-distribution is similar in shape to the normal distribution but has heavier tails, due to the additional uncertainty introduced by the small-sample estimate of the standard deviation. There are many t-distributions, one for every sample size (the smaller the sample size, the heavier the tails). As the sample size gets bigger, the t-distribution approaches the normal distribution. See a graph of some t-distribution PDFs on Wikipedia.

HOWTO

Deriving a confidence interval (variant 3)

Example scenario: We do not know the population standard deviation but we have a sample with standard deviation ($S = 10$), mean ($\bar{x} = 251$) and size smaller than 30 ($n = 16$). Because the sample is small we need to know that the data is normally distributed, otherwise a confidence interval cannot be specified.

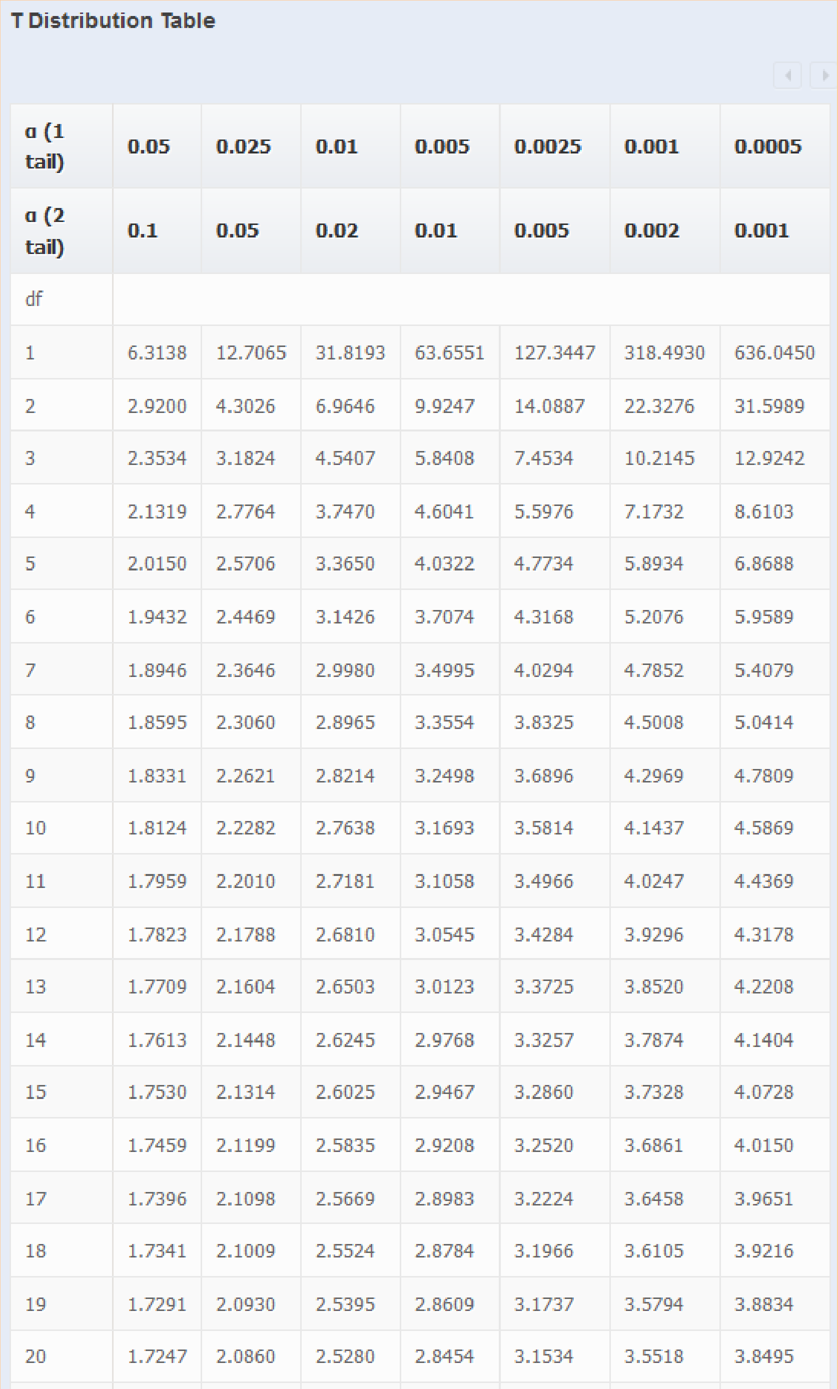
1. Calculate $\alpha/2$: $\alpha/2 = \dfrac{1 - 0.01LoC}{2}$

   Example: For a level of confidence of 95% this is $\alpha/2 = \dfrac{1 - 0.95}{2} = 0.025$
2. Lookup the t-value in the t-table, for α and degrees of freedom $df = n - 1$. Look for $\alpha/2$ among upper-tail values or for α among two-tailed values. The result will be the same.

   Example: The t-value corresponding to $df = 15$ and α of 0.05 two-tailed is $t = 2.13$
3. De-standardise using the following formula: $CI_h = t\dfrac{S}{\sqrt{n}}$, where CI_h is the CI half-interval.

   Example: $CI_h = 2.13 \times \dfrac{10}{\sqrt{16}} = 5.33$
4. State the confidence interval: $\bar{x} \pm t\dfrac{S}{\sqrt{n}}$, with a confidence of LoC%

   Example: 251 ± 5.33, with 95% confidence



T Distribution Table

α (1 tail)	0.05	0.025	0.01	0.005	0.0025	0.001	0.0005
α (2 tail)	0.1	0.05	0.02	0.01	0.005	0.002	0.001
df							
1	6.3138	12.7065	31.8193	63.6551	127.3447	318.4930	636.0450
2	2.9200	4.3026	6.9646	9.9247	14.0887	22.3276	31.5989
3	2.3534	3.1824	4.5407	5.8408	7.4534	10.2145	12.9242
4	2.1319	2.7764	3.7470	4.6041	5.5976	7.1732	8.6103
5	2.0150	2.5706	3.3650	4.0322	4.7734	5.8934	6.8688
6	1.9432	2.4469	3.1426	3.7074	4.3168	5.2076	5.9589
7	1.8946	2.3646	2.9980	3.4995	4.0294	4.7852	5.4079
8	1.8595	2.3060	2.8965	3.3554	3.8325	4.5008	5.0414
9	1.8331	2.2621	2.8214	3.2498	3.6896	4.2969	4.7809
10	1.8124	2.2282	2.7638	3.1693	3.5814	4.1437	4.5869
11	1.7959	2.2010	2.7181	3.1058	3.4966	4.0247	4.4369
12	1.7823	2.1788	2.6810	3.0545	3.4284	3.9296	4.3178
13	1.7709	2.1604	2.6503	3.0123	3.3725	3.8520	4.2208
14	1.7613	2.1448	2.6245	2.9768	3.3257	3.7874	4.1404
15	1.7530	2.1314	2.6025	2.9467	3.2860	3.7328	4.0728
16	1.7459	2.1199	2.5835	2.9208	3.2520	3.6861	4.0150
17	1.7396	2.1098	2.5669	2.8983	3.2224	3.6458	3.9651
18	1.7341	2.1009	2.5524	2.8784	3.1966	3.6105	3.9216
19	1.7291	2.0930	2.5395	2.8609	3.1737	3.5794	3.8834
20	1.7247	2.0860	2.5280	2.8454	3.1534	3.5518	3.8495

Hypothesis tests

What is a hypothesis test?

- A hypothesis test is used to check if a sample of data supports a particular hypothesis made about the population from which the sample was drawn.
- Specifically, a hypothesis test investigates whether a hypothesised parameter value falls within an appropriate confidence interval defined using a sample statistic.
- Intuitively, what we are interested in is whether the hypothesised parameter (the theory) and sample statistic (the experiment) are far enough apart to signal a low likelihood of the hypothesis, and for us to **reject** the null hypothesis. If they are not far enough apart, we still cannot be sure that the hypothesis is true, but we **fail to reject it**.
- We will demonstrate this by hypothesising a population mean and using a sample mean to test the hypothesis.

The p-value

The p-value is the central concept in statistical hypothesis testing.

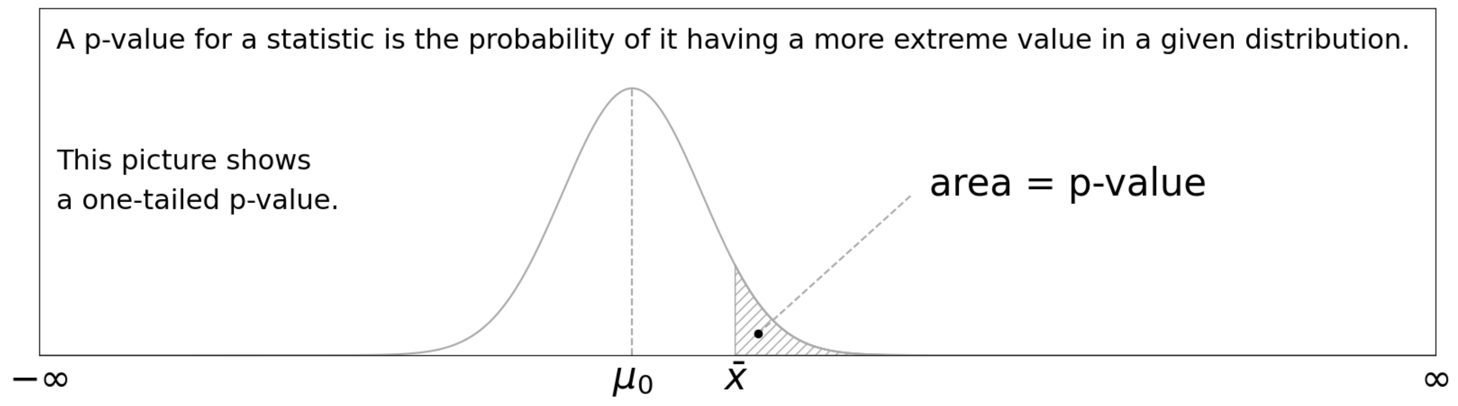
A p-value of a statistic's value, with respect to a hypothetical distribution for that statistic, is the probability of more extreme values of the statistic being drawn from that distribution.

$p(X, f_0, x_1) = P(X \geq x_1 \mid f_0)$ (upper tail extreme is of interest)

$p(X, f_0, x_1) = P(X \leq x_1 \mid f_0)$ (lower tail extreme is of interest)

where X is a variable with PDF f_0 and x_1 a particular value of X.

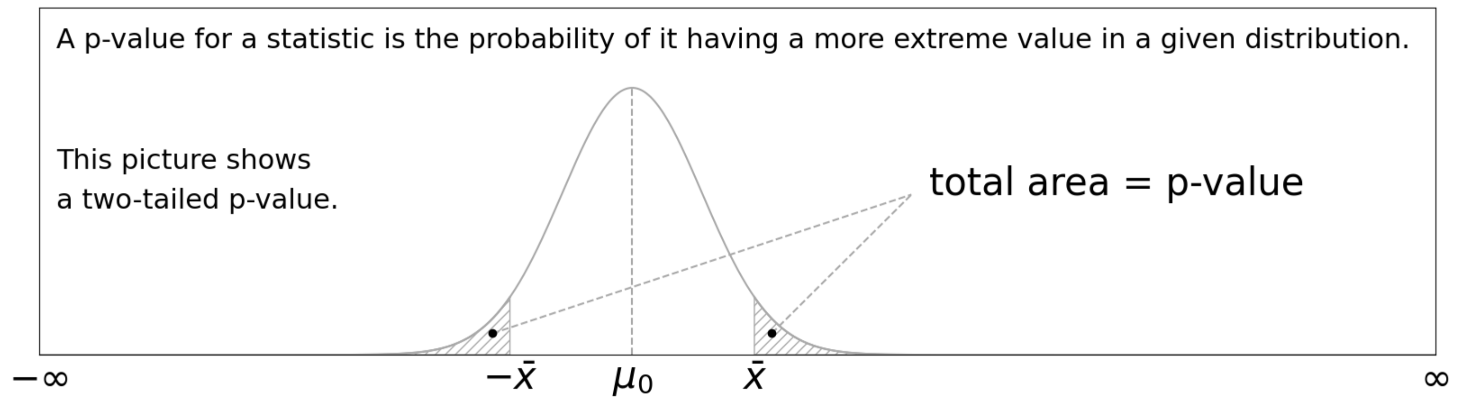
In the picture, the x-axis represents the variable (corresponding to X above), the mean $\bar{x}$ is the value of interest (corresponding to x_1 above) and the curve is the hypothetical probability density function (corresponding to f_0 above).


A p-value for a statistic is the probability of it having a more extreme value in a given distribution.
This picture shows
a one-tailed p-value.
area = p-value
$-\infty$
μ_0
$\bar{x}$
∞

Depending on how the variable is expected to behave or its meaning in the context of the analysis, its p-value may be defined as one-tailed or two-tailed. Two-tailed means that we are watching out for exreme values in both directions (very high and very low).

$p(X, f_0, x) = P(|X| \geq |x_1| \mid f_0)$, (both tail extremes are of interest)

where X is a variable with PDF f_0 and x_1 a particular value of X.


A p-value for a statistic is the probability of it having a more extreme value in a given distribution.
This picture shows
a two-tailed p-value.
total area = p-value
$-\infty$
$-\bar{x}$
μ_0
$\bar{x}$
∞

For example, a producer of orange juice may be analysing the amount of juice poured into cartons by a machine, with emphasis on not short-changing their customers and without concern for possible over-filling. In this case the test would be one-tailed. If the goal is to strike a perfect balance and check for both over- and under-filling, then the test should be two-tailed.

Hypothesis testing terminology

- The **threshold of probability** or **level of significance** is the probability corresponding to α in the description of the confidence interval (see previous slides) and common values used are 5% and 1% (0.05 and 0.01).
- A hypothesis test is stated through a null hypothesis and an alternative hypothesis.

Example of a two-tailed test hypothesis, stating that is that the mean is equal to 100:

$H_0 : \mu = 100$ **null hypothesis**

$H_a : \mu \neq 100$ **alternative hypothesis**

Example of an upper-tailed test hypothesis, stating that is that the mean equal to or less than 150:

$H_0 : \mu \leq 150$ **null hypothesis**

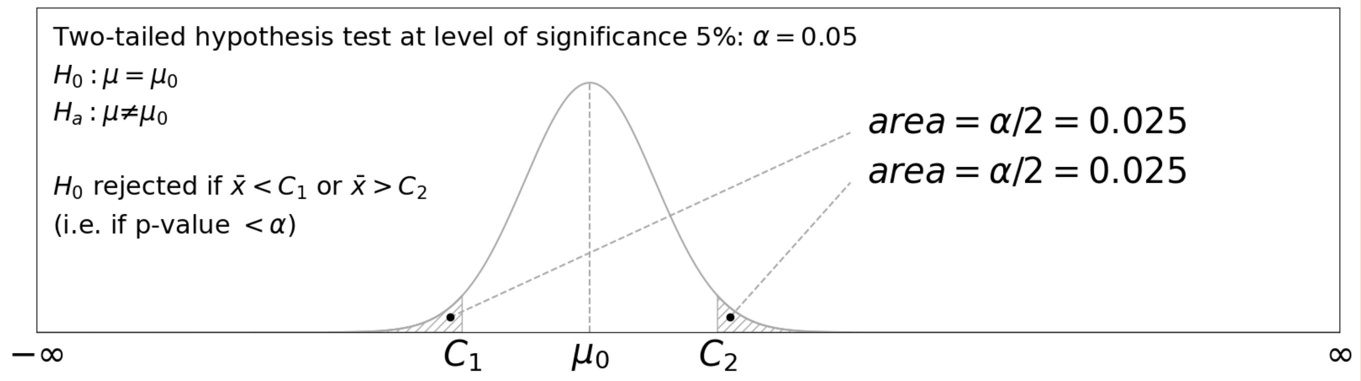
$H_a : \mu > 150$ **alternative hypothesis**

- The possible outcomes of a hypothesis test are:
  - the null hypothesis is **rejected**
  - **failure to reject** the null hypothesis

Performing a hypothesis test

Hypothesis testing steps:

- state the null and alternative hypotheses
- calculate the test statistic, which in the case of the mean is: $T = \dfrac{\bar{x} - \mu_0}{SE}$, where $\bar{x}$ is the sample mean, μ_0 is the mean specified in the null hypothesis and SE is the standard error ('standard deviation' of the sampling distribution) - T is a normalised value of the mean
- identify the **critical value** (C) by looking up the critical value corresponding to the required level of significance, in the table for the relevant sampling distribution
- compare the absolute value of the test statistic $(|T|)$ with the critical value
- decide the outcome:
  - **reject the null hypothesis** if $|T| > C$ (p-value $< \alpha$)
  - **fail to reject the null hypothesis** if $|T| \leq C$ (p-value $> \alpha$)



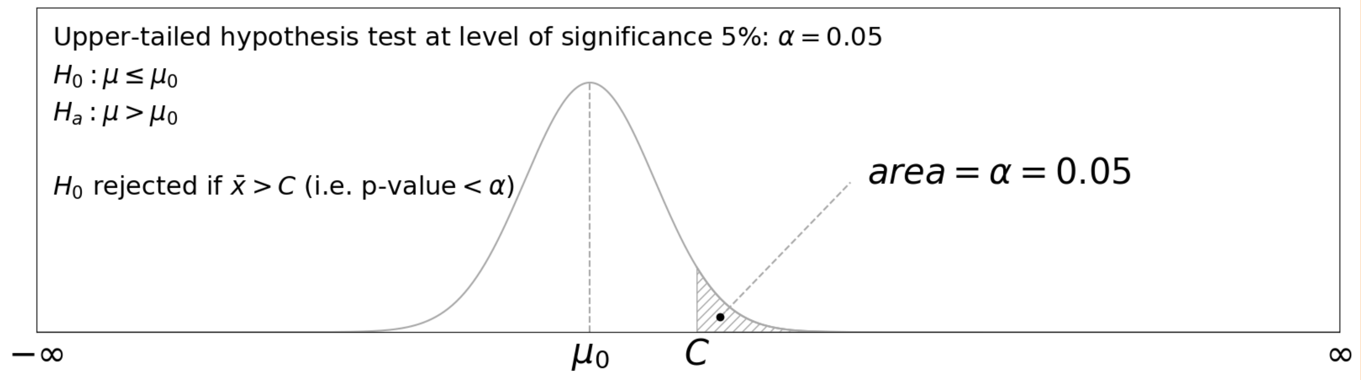
Two-tailed hypothesis test at level of significance 5%: $\alpha = 0.05$

$H_0 : \mu = \mu_0$

$H_a : \mu \neq \mu_0$

H_0 rejected if $\bar{x} < C_1$ or $\bar{x} > C_2$

(i.e. if p-value $< \alpha$)

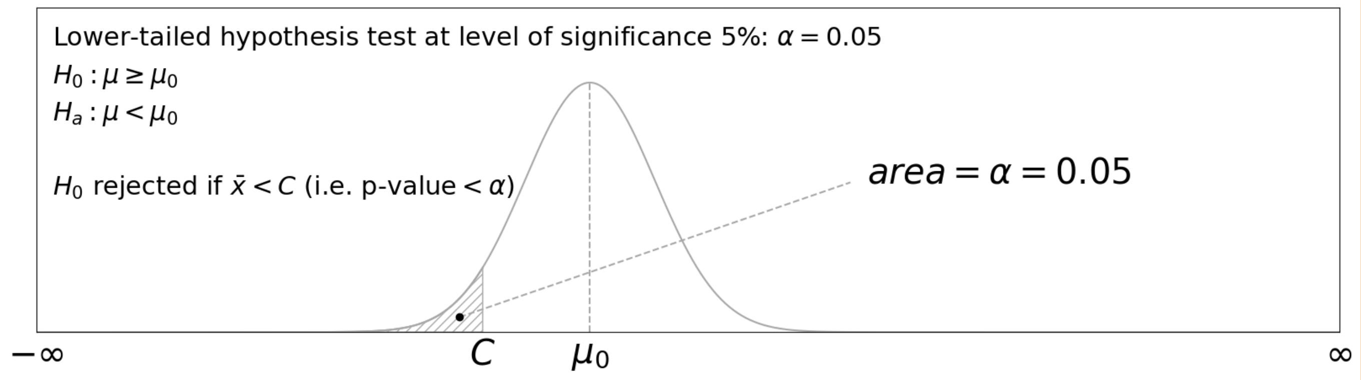


Upper-tailed hypothesis test at level of significance 5%: $\alpha = 0.05$

$H_0 : \mu \leq \mu_0$

$H_a : \mu > \mu_0$

H_0 rejected if $\bar{x} > C$ (i.e. p-value $< \alpha$)



Lower-tailed hypothesis test at level of significance 5%: $\alpha = 0.05$

$H_0 : \mu \geq \mu_0$

$H_a : \mu < \mu_0$

H_0 rejected if $\bar{x} < C$ (i.e. p-value $< \alpha$)

Hypothesis test variants

As with confidence intervals, there are 3 variants of the hypothesis testing method, differing by:

- how the standard error (SE) is calculated, which is either
  - from the population standard deviation (the parameter, σ) **or**
  - from the sample standard deviation (the statistic, S)
- which sampling distribution needs to be used:
  - z-distribution
  - t-distribution

HT variants by standard error calculation and sampling distribution

Standard error → **Sampling distribution ↓**	$SE = \frac{\sigma}{\sqrt{n}}$	$SE = \frac{S}{\sqrt{n}}$
z	1	2
t	X	3

The variants are applicable as follows:

Standard deviation known

Sample size → **Distribution ↓**	large $(n \geq 30)$	small $(n < 30)$
Normal	1	1
Any other	1	X

Standard deviation unknown

Sample size → **Distribution ↓**	large $(n \geq 30)$	small $(n < 30)$
Normal	2	3
Any other	2	X

When the data distribution is not known and the sample is small a hypothesis test cannot be performed reliably.

HOWTO

Hypothesis test

Example scenario: The standard deviation is not known but we have a sample of size $n = 100$, with calculated sample standard deviation of $S = 5.55$ and a mean of $\bar{x} = 64.32$.

1. Decide which variant of the hypothesis test is appropriate.

   Example: Variant 2 (σ not known, large sample as $100 > 30$) $\Rightarrow SE = \frac{S}{\sqrt{n}}$, sampling distribution is **z**

2. State the null and alternative hypotheses:

   Example: Assuming that the hypothesised mean is $\mu_0 = 65$

   $H_0 : \mu = 65$

   $H_a : \mu \neq 65$

3. Calculate the statistic $T = \frac{\bar{x} - \mu_0}{SE}$

   Example:

   $$T = \frac{64.32 - 65}{\frac{5.55}{\sqrt{100}}} = -1.225$$

4. Lookup critical value and compare

   Example: Assuming a level of significance of 5%, the critical value for the z-distribution is 1.96

   $|-1.225| = 1.225 < 1.96 \Rightarrow$ null hypothesis is **not rejected**

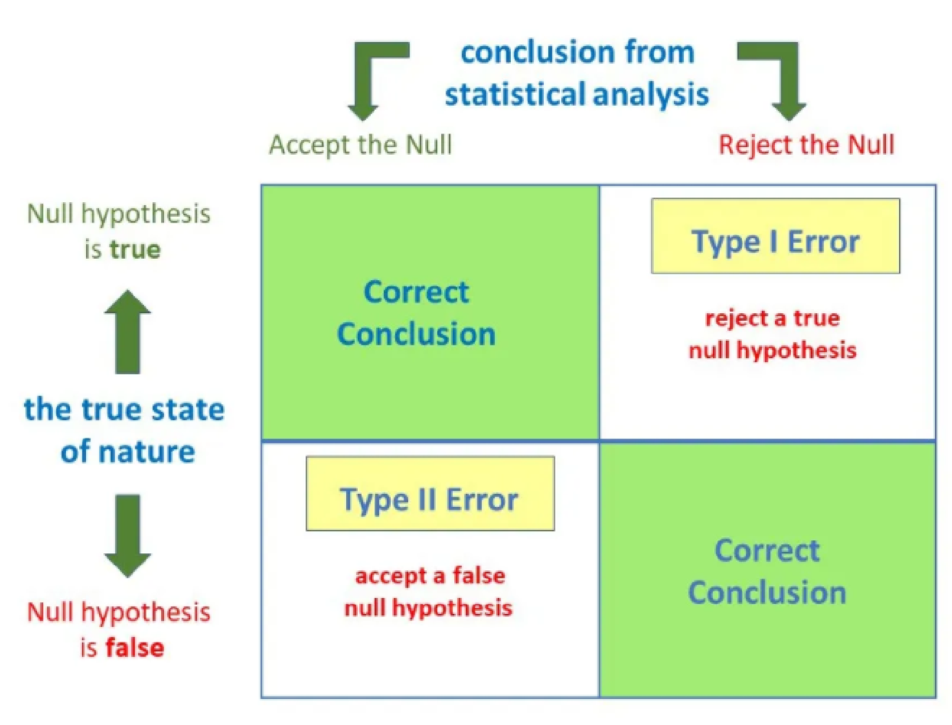
Hypothesis test errors

- Type I error - when the null hypothesis is rejected even though it is true

  The probability of a Type I error is exactly equal to the level of significance (α)

- Type II error - when the null hypothesis is not rejected even though it is not true

  The probability of a Type II error depends on the difference between the real and hypothesised value, the standard error and on the level of significance. It is denoted β and is often expressed in terms of power, which is calculated as $power = 1 - \beta$. The power of a test is the probability that it will detect an effect when one exists.



conclusion from statistical analysis

the true state of nature	Accept the Null	Reject the Null
Null hypothesis is true	Correct Conclusion	Type I Error reject a true null hypothesis
Null hypothesis is false	Type II Error accept a false null hypothesis	Correct Conclusion

Source: simplypsychology.org

- For many applications, it is a convention to accept type I error probabilities of under 0.05 (5%) and power values of over 0.8 (80%).

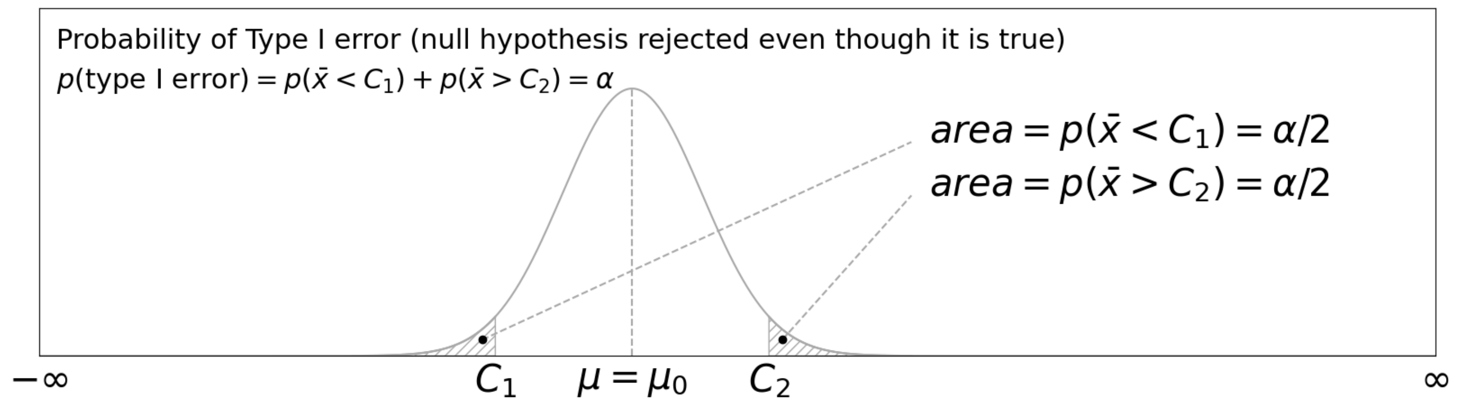
  However, there are applications where this balance of probabilities is unacceptable. For example, in medical screening, a type II error needs to be avoided, as it means a true condition not being detected, with potentially very negative consequences.

- The power of a test can be increased by:
  - reducing the test significance (increasing the probability of type I errors)
  - increasing the sample size
  - measuring variables more accurately
  - decreasing subject variability
  - increasing the difference between group means
  - using fewer groups or variables

- **Power analysis** is performed, with the use of the relationship that exists between
  - sample size
  - significance
  - effect size
  - power

  For a particular test, these four elements can be adjusted to reach the desired value in one particular element. For example
  - to increase the power, we might increase the sample size
  - we might calculate the required sample size based on the required power
  - we might calculate the required sample size given an expected effect (if the expected effect is smaller, we need a bigger sample size)
- Statistical power calculators can be found online, for example here.

[Hypothesis test errors]

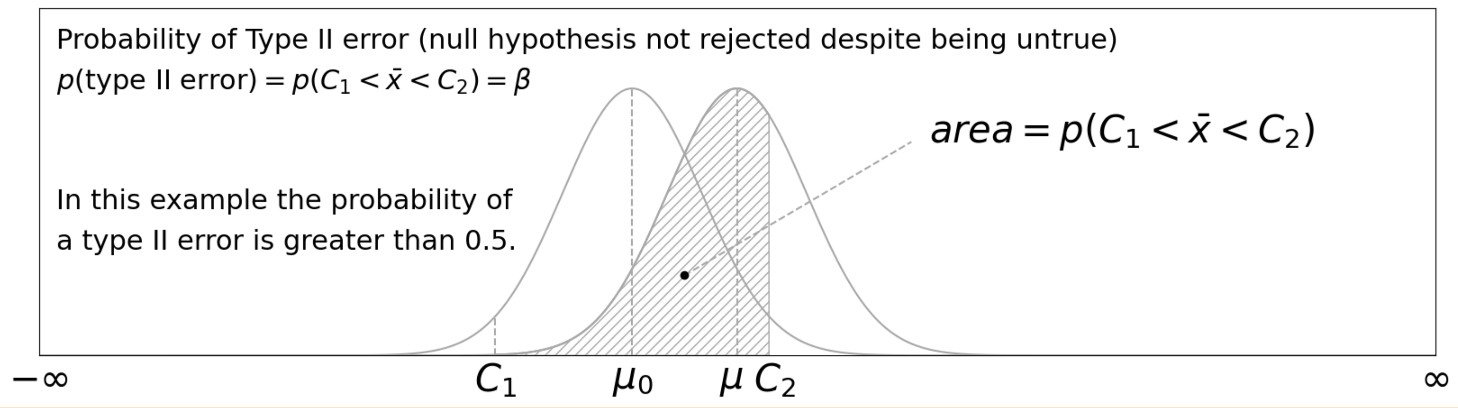


Probability of Type I error (null hypothesis rejected even though it is true)

$p(\text{type I error}) = p(\bar{x} < C_1) + p(\bar{x} > C_2) = \alpha$

$area = p(\bar{x} < C_1) = \alpha/2$

$area = p(\bar{x} > C_2) = \alpha/2$

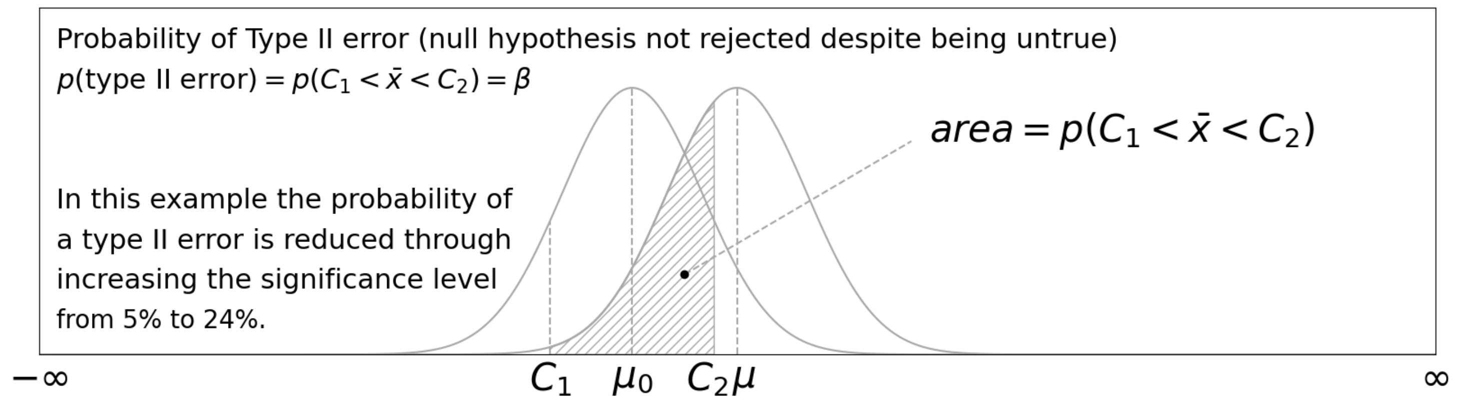


Probability of Type II error (null hypothesis not rejected despite being untrue)

$p(\text{type II error}) = p(C_1 < \bar{x} < C_2) = \beta$

$area = p(C_1 < \bar{x} < C_2)$

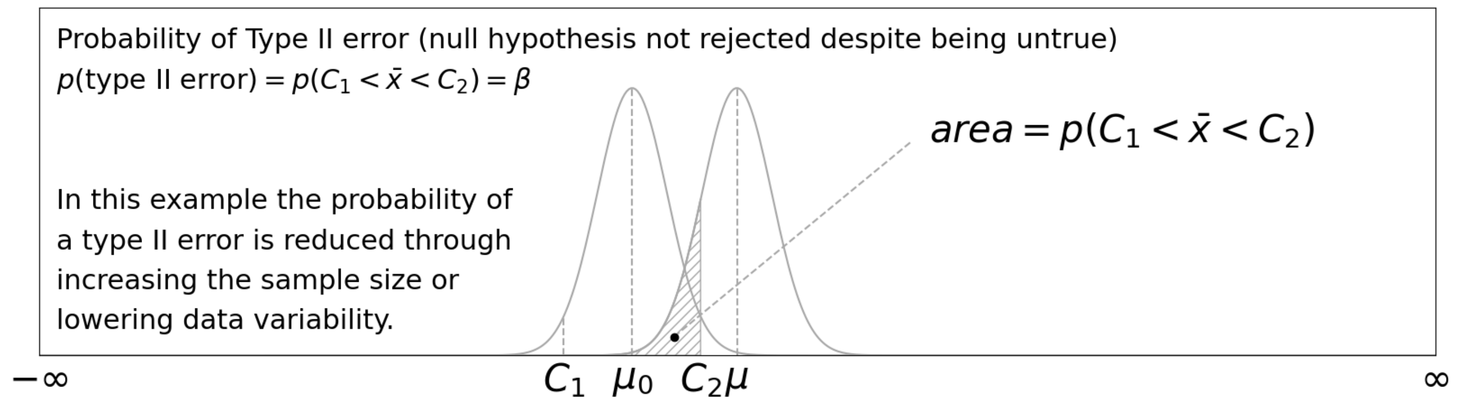
In this example the probability of a type II error is greater than 0.5.



Probability of Type II error (null hypothesis not rejected despite being untrue)

$p(\text{type II error}) = p(C_1 < \bar{x} < C_2) = \beta$

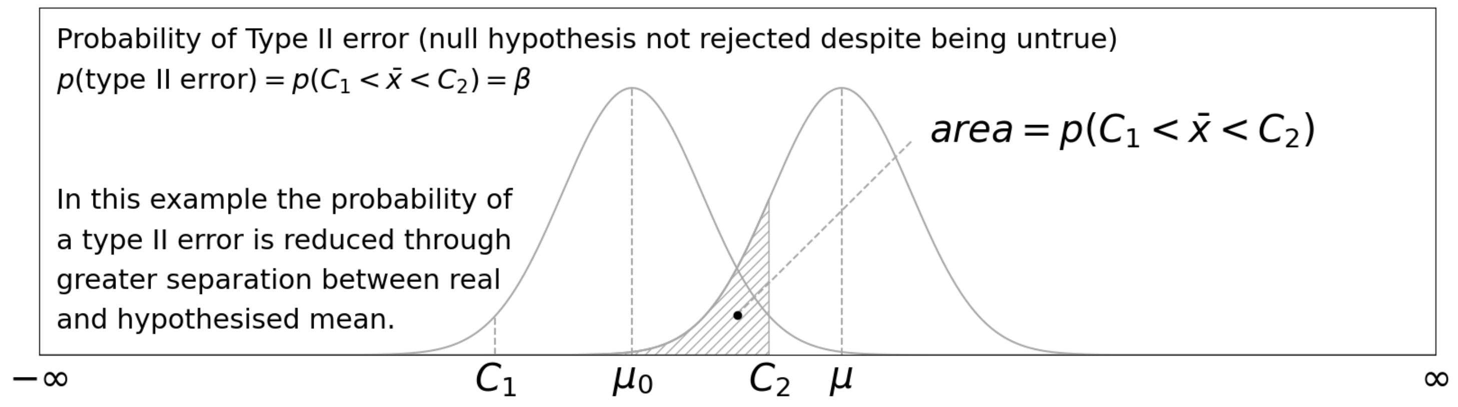
In this example the probability of a type II error is reduced through increasing the significance level from 5% to 24%.



Probability of Type II error (null hypothesis not rejected despite being untrue)

$p(\text{type II error}) = p(C_1 < \bar{x} < C_2) = \beta$

In this example the probability of a type II error is reduced through increasing the sample size or lowering data variability.



Probability of Type II error (null hypothesis not rejected despite being untrue)

$p(\text{type II error}) = p(C_1 < \bar{x} < C_2) = \beta$

In this example the probability of a type II error is reduced through greater separation between real and hypothesised mean.