

Formulae

Confidence intervals

$$\bar{x} \pm z_{\alpha/2} \times \frac{S}{\sqrt{n}} \quad \bar{x} \pm t_{\alpha/2} \times \frac{S}{\sqrt{n}}$$

T-statistic for the mean and for the difference of two means

$$T = \frac{\bar{x} - \mu_0}{\frac{S}{\sqrt{n}}} \quad T = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\left(\frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2} \right) \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}}$$

Standard deviation (population and sample)

$$\sigma = \sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}} \quad S = \sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n - 1}}$$

Kendall's Tau

$$\tau_A = \frac{n_c - n_d}{n(n - 1)/2} \quad \tau_B = \frac{n_c - n_d}{\sqrt{(n_c + n_d + t_x)(n_c + n_d + t_y)}}$$

F-statistic

$$F = \frac{\left(\sum_{i=1}^k n_i (\bar{x}_i - \bar{\bar{x}})^2 \right) / (k - 1)}{\left(\sum_{i=1}^k (n_i - 1) S_i^2 \right) / (N - k)}$$

Chi-squared

$$\chi^2 = \sum_{r=1, c=1}^{n_R, n_C} \frac{(O_{rc} - E_{rc})^2}{E_{rc}}$$

Tables

Upper-tail percentage points of the standard normal distribution

The table gives the values of z for which $P(Z > z) = p$, where the distribution of Z is $N(0, 1)$.

p	z	p	z	p	z	p	z	p	z
.50	0.000	.15	1.036	.025	1.960	.010	2.326	.0 ³ 4	3.353
.45	0.126	.14	1.080	.024	1.977	.009	2.366	.0 ³ 3	3.432
.40	0.253	.13	1.126	.023	1.995	.008	2.409	.0 ³ 2	3.540
.35	0.385	.12	1.175	.022	2.014	.007	2.457	.0 ³ 1	3.719
.30	0.524	.11	1.227	.021	2.034	.006	2.512	.0 ⁴ 5	3.891
.25	0.674	.10	1.282	.020	2.054	.005	2.576	.0 ⁴ 1	4.265
.24	0.706	.09	1.341	.019	2.075	.004	2.652	.0 ⁵ 5	4.417
.23	0.739	.08	1.405	.018	2.097	.003	2.748	.0 ⁵ 1	4.753
.22	0.772	.07	1.476	.017	2.120	.002	2.878	.0 ⁶ 5	4.892
.21	0.806	.06	1.555	.016	2.144	.001	3.090	.0 ⁶ 1	5.199
.20	0.842	.050	1.645	.015	2.170	.0 ³ 9	3.121	.0 ⁷ 5	5.327
.19	0.878	.045	1.695	.014	2.197	.0 ³ 8	3.156	.0 ⁷ 1	5.612
.18	0.915	.040	1.751	.013	2.226	.0 ³ 7	3.195	.0 ⁸ 5	5.731
.17	0.954	.035	1.812	.012	2.257	.0 ³ 6	3.239	.0 ⁸ 1	5.998
.16	0.994	.030	1.881	.011	2.290	.0 ³ 5	3.291	.0 ⁹ 5	6.109

Critical values for Kendall's τ (two-tailed)

n	5%	1%	n	5%	1%	n	5%	1%	n	5%	1%
4	*	*	11	.491	.600	18	.346	.451	25	.287	.367
5	1.000	*	12	.455	.576	19	.333	.439	26	.280	.360
6	.867	1.000	13	.436	.564	20	.326	.421	27	.271	.356
7	.714	.905	14	.407	.516	21	.314	.410	28	.265	.344
8	.643	.786	15	.390	.505	22	.307	.394	29	.261	.340
9	.556	.722	16	.383	.483	23	.296	.391	30	.255	.333
10	.511	.644	17	.368	.471	24	.290	.377	40	.218	.285

Critical values for the t-distribution (two-tailed)

α (1 tail)	0.05	0.025	0.01	0.005	0.0025	0.001	0.0005
α (2 tail)	0.1	0.05	0.02	0.01	0.005	0.002	0.001
df							
1	6.3138	12.7065	31.8193	63.6551	127.3447	318.4930	636.0450
2	2.9200	4.3026	6.9646	9.9247	14.0887	22.3276	31.5989
3	2.3534	3.1824	4.5407	5.8408	7.4534	10.2145	12.9242
4	2.1319	2.7764	3.7470	4.6041	5.5976	7.1732	8.6103
5	2.0150	2.5706	3.3650	4.0322	4.7734	5.8934	6.8688
6	1.9432	2.4469	3.1426	3.7074	4.3168	5.2076	5.9589
7	1.8946	2.3646	2.9980	3.4995	4.0294	4.7852	5.4079
8	1.8595	2.3060	2.8965	3.3554	3.8325	4.5008	5.0414
9	1.8331	2.2621	2.8214	3.2498	3.6896	4.2969	4.7809
10	1.8124	2.2282	2.7638	3.1693	3.5814	4.1437	4.5869
11	1.7959	2.2010	2.7181	3.1058	3.4966	4.0247	4.4369
12	1.7823	2.1788	2.6810	3.0545	3.4284	3.9296	4.3178
13	1.7709	2.1604	2.6503	3.0123	3.3725	3.8520	4.2208
14	1.7613	2.1448	2.6245	2.9768	3.3257	3.7874	4.1404
15	1.7530	2.1314	2.6025	2.9467	3.2860	3.7328	4.0728
16	1.7459	2.1199	2.5835	2.9208	3.2520	3.6861	4.0150
17	1.7396	2.1098	2.5669	2.8983	3.2224	3.6458	3.9651
18	1.7341	2.1009	2.5524	2.8784	3.1966	3.6105	3.9216
19	1.7291	2.0930	2.5395	2.8609	3.1737	3.5794	3.8834
20	1.7247	2.0860	2.5280	2.8454	3.1534	3.5518	3.8495

Critical values for Chi-squared (χ^2)

df	0.995	0.99	0.975	0.95	0.90	0.10	0.05	0.025	0.01	0.005
1	---	---	0.001	0.004	0.016	2.706	3.841	5.024	6.635	7.879
2	0.010	0.020	0.051	0.103	0.211	4.605	5.991	7.378	9.210	10.597
3	0.072	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.345	12.838
4	0.207	0.297	0.484	0.711	1.064	7.779	9.488	11.143	13.277	14.860
5	0.412	0.554	0.831	1.145	1.610	9.236	11.070	12.833	15.086	16.750
6	0.676	0.872	1.237	1.635	2.204	10.645	12.592	14.449	16.812	18.548
7	0.989	1.239	1.690	2.167	2.833	12.017	14.067	16.013	18.475	20.278
8	1.344	1.646	2.180	2.733	3.490	13.362	15.507	17.535	20.090	21.955
9	1.735	2.088	2.700	3.325	4.168	14.684	16.919	19.023	21.666	23.589
10	2.156	2.558	3.247	3.940	4.865	15.987	18.307	20.483	23.209	25.188
11	2.603	3.053	3.816	4.575	5.578	17.275	19.675	21.920	24.725	26.757
12	3.074	3.571	4.404	5.226	6.304	18.549	21.026	23.337	26.217	28.300
13	3.565	4.107	5.009	5.892	7.042	19.812	22.362	24.736	27.688	29.819
14	4.075	4.660	5.629	6.571	7.790	21.064	23.685	26.119	29.141	31.319
15	4.601	5.229	6.262	7.261	8.547	22.307	24.996	27.488	30.578	32.801

Critical values for the F-statistic

		Degrees of freedom in the numerator								
<i>p</i>		1	2	3	4	5	6	7	8	9
Degrees of freedom in the denominator	1	.100	39.86	49.50	53.59	55.83	57.24	58.20	58.91	59.86
		.050	161.45	199.50	215.71	224.58	230.16	233.99	236.77	240.54
		.025	647.79	799.50	864.16	899.58	921.85	937.11	948.22	956.66
		.010	4052.2	4999.5	5403.4	5624.6	5763.6	5859.0	5928.4	5981.1
		.001	405284	500000	540379	562500	576405	585937	592873	598144
	2	.100	8.53	9.00	9.16	9.24	9.29	9.33	9.35	9.37
		.050	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37
		.025	38.51	39.00	39.17	39.25	39.30	39.33	39.36	39.37
		.010	98.50	99.00	99.17	99.25	99.30	99.33	99.36	99.37
		.001	998.50	999.00	999.17	999.25	999.30	999.33	999.36	999.37
	3	.100	5.54	5.46	5.39	5.34	5.31	5.28	5.27	5.24
		.050	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.81
		.025	17.44	16.04	15.44	15.10	14.88	14.73	14.62	14.54
		.010	34.12	30.82	29.46	28.71	28.24	27.91	27.67	27.35
		.001	167.03	148.50	141.11	137.10	134.58	132.85	131.58	129.86
	4	.100	4.54	4.32	4.19	4.11	4.05	4.01	3.98	3.94
		.050	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.00
		.025	12.22	10.65	9.98	9.60	9.36	9.20	9.07	8.90
		.010	21.20	18.00	16.69	15.98	15.52	15.21	14.98	14.66
		.001	74.14	61.25	56.18	53.44	51.71	50.53	49.66	48.47
	5	.100	4.06	3.78	3.62	3.52	3.45	3.40	3.37	3.32
		.050	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.77
		.025	10.01	8.43	7.76	7.39	7.15	6.98	6.85	6.68
		.010	16.26	13.27	12.06	11.39	10.97	10.67	10.46	10.16
		.001	47.18	37.12	33.20	31.09	29.75	28.83	28.16	27.24
	6	.100	3.78	3.46	3.29	3.18	3.11	3.05	3.01	2.96
		.050	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.10
		.025	8.81	7.26	6.60	6.23	5.99	5.82	5.70	5.52
		.010	13.75	10.92	9.78	9.15	8.75	8.47	8.26	7.98
		.001	35.51	27.00	23.70	21.92	20.80	20.03	19.46	18.69
	7	.100	3.59	3.26	3.07	2.96	2.88	2.83	2.78	2.72
		.050	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.68
		.025	8.07	6.54	5.89	5.52	5.29	5.12	4.99	4.82
		.010	12.25	9.55	8.45	7.85	7.46	7.19	6.99	6.72
		.001	29.25	21.69	18.77	17.20	16.21	15.52	15.02	14.33

Critical values for the F-statistic (continued)

		Degrees of freedom in the numerator								
<i>p</i>		1	2	3	4	5	6	7	8	9
Degrees of freedom in the denominator	8	.100	3.46	3.11	2.92	2.81	2.73	2.67	2.62	2.59
		.050	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44
		.025	7.57	6.06	5.42	5.05	4.82	4.65	4.53	4.43
		.010	11.26	8.65	7.59	7.01	6.63	6.37	6.18	6.03
		.001	25.41	18.49	15.83	14.39	13.48	12.86	12.40	12.05
	9	.100	3.36	3.01	2.81	2.69	2.61	2.55	2.51	2.47
		.050	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23
		.025	7.21	5.71	5.08	4.72	4.48	4.32	4.20	4.10
		.010	10.56	8.02	6.99	6.42	6.06	5.80	5.61	5.47
		.001	22.86	16.39	13.90	12.56	11.71	11.13	10.70	10.37
	10	.100	3.29	2.92	2.73	2.61	2.52	2.46	2.41	2.38
		.050	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07
		.025	6.94	5.46	4.83	4.47	4.24	4.07	3.95	3.85
		.010	10.04	7.56	6.55	5.99	5.64	5.39	5.20	5.06
		.001	21.04	14.91	12.55	11.28	10.48	9.93	9.52	9.20
	11	.100	3.23	2.86	2.66	2.54	2.45	2.39	2.34	2.30
		.050	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95
		.025	6.72	5.26	4.63	4.28	4.04	3.88	3.76	3.66
		.010	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74
		.001	19.69	13.81	11.56	10.35	9.58	9.05	8.66	8.35
	12	.100	3.18	2.81	2.61	2.48	2.39	2.33	2.28	2.24
		.050	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85
		.025	6.55	5.10	4.47	4.12	3.89	3.73	3.61	3.51
		.010	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50
		.001	18.64	12.97	10.80	9.63	8.89	8.38	8.00	7.71
	13	.100	3.14	2.76	2.56	2.43	2.35	2.28	2.23	2.20
		.050	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77
		.025	6.41	4.97	4.35	4.00	3.77	3.60	3.48	3.39
		.010	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30
		.001	17.82	12.31	10.21	9.07	8.35	7.86	7.49	7.21
	14	.100	3.10	2.73	2.52	2.39	2.31	2.24	2.19	2.15
		.050	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70
		.025	6.30	4.86	4.24	3.89	3.66	3.50	3.38	3.29
		.010	8.86	6.51	5.56	5.04	4.69	4.46	4.28	4.14
		.001	17.14	11.78	9.73	8.62	7.92	7.44	7.08	6.80
	15	.100	3.07	2.70	2.49	2.36	2.27	2.21	2.16	2.12
		.050	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64
		.025	6.20	4.77	4.15	3.80	3.58	3.41	3.29	3.20
		.010	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00
		.001	16.59	11.34	9.34	8.25	7.57	7.09	6.74	6.47
	16	.100	3.05	2.67	2.46	2.33	2.24	2.18	2.13	2.09
		.050	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59
		.025	6.12	4.69	4.08	3.73	3.50	3.34	3.22	3.12
		.010	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89
		.001	16.12	10.97	9.01	7.94	7.27	6.80	6.46	6.19
	17	.100	3.03	2.64	2.44	2.31	2.22	2.15	2.10	2.06
		.050	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55
		.025	6.04	4.62	4.01	3.66	3.44	3.28	3.16	3.06
		.010	8.40	6.11	5.19	4.67	4.34	4.10	3.93	3.79
		.001	15.72	10.66	8.73	7.68	7.02	6.56	6.22	5.96