# Statistical Inference - Confidence Intervals and Hypothesis Tests

### ANSWERS

#### Exercise 1 - Confidence interval

In order to define the confidence interval, we need to find the values at the lower and upper boundaries of a range, symmetrical with respect to the measured mean, in the sampling distribution. The range must include a percentage of values such that the percentage is equal to the confidence level, which in our case is 95%. The upper and lower boundaries are, hence, values on the x-axis of the distribution that 'fence out' the remaining 5% of possible measured means (2.5%) on each tail of the distribution. The boundary values can be read out of the percentage point table for a suitable distribution. In this case, the sample is large  $(n \ge 30)$ , which means that we can use the standard normal distribution (the z-distribution). This table is given in the appendix in the lab sheet. If the sample were small (n < 30), we would use a t-distribution table for degrees of freedom df = n - 1.

# Upper-tail percentage points of the standard normal distribution

The table gives the values of z for which P(Z > z) = p, where the distribution of Z is N(0, 1).

p	Z	p	Z	p	Z	p	Z	p	Z
.50	0.000	.15	1.036	.025	1.960	.010	2.326	$.0^{3}4$	3.353
.45	0.126	.14	1.080	.024	1.977	.009	2.366	$.0^{3}$ 3	3.432
.40	0.253	.13	1.126	.023	1.995	.008	2.409	$.0^{3}2$	3.540
.35	0.385	.12	1.175	.022	2.014	.007	2.457	$0^{3}1$	3.719
.30	0.524	.11	1.227	.021	2.034	.006	2.512	$0^45$	3.891

The z-value (an x-axis value in the standardised normal distribution) that is lower than 2.5% of all possible values is 1.96 (see the picture). As the standardised normal distribution is symmetrical around the mean of 0, the value that is higher than 2.5% of all possible values is -1.96.

The other calculation that we must do is to find the *standard error*, which is the standard deviation of the sampling distribution. It is equal to  $\frac{s}{\sqrt{n}}$ , where s is the standard deviation.

Finally, we can say, with a confidence of 95%, that the mean amount of shampoo in the bottles is in the range:

$$200ml \pm \frac{2ml}{10} \times 1.96$$

or

 $200ml \pm 0.39ml$ 

#### Exercise 2 - Hypothesis test

In this hypothesis test the null hypothesis is that the mean value of the amount of shampoo in the bottles is 200ml:

 $H_0: \mu = 200ml$ 

 $H_a: \mu \neq 200ml$ 

 $H_a$  is the alternative hypothesis, that the mean is different from 200ml.

For the hypothesis test we need to identify the lower and upper boundaries of the *sampling distribution* range symmetrical around the measured mean that 'fences out' the percentage of values equal to the significance level, which in our case is 1%. Similarly to the look-up in the previous problem, for the upper boundary we need to find the z-value that corresponds to a p-value of 0.005~(0.5%), since the lower boundary will 'fence out' the other 0.5% of the 1% in the negative (left) end tail of the distribution. If we were dealing with a small sample (n < 30), we would use the t-distribution table instead of a z-distribution table.

### Upper-tail percentage points of the standard normal distribution

The table gives the values of z for which P(Z > z) = p, where the distribution of Z is N(0, 1).

p	Z	p	Z	p	Z	p	Z	p	Z
50	0.000	.15	1.036	.025	1.960	.010	2.326	$.0^{3}4$	3.353
45	0.126	.14	1.080	.024	1.977	.009	2.366	$.0^{3}3$	3.432
40	0.253	.13	1.126	.023	1.995	.008	2.409	$.0^{3}2$	3.540
35	0.385	.12	1.175	.022	2.014	.007	2.457	$.0^{3}1$	3.719
30	0.524	.11	1.227	.021	2.034	.006	2.512	$.0^{4}5$	3.891
25	0.674	10	1 202	020	2.054	005	2 576	041	4.265
25	0.6/4	.10	1.282	.020	2.054				
24	0.706	.09	1.341	.019	2.075	.004	2.652		4.417
23	0.739	.08	1.405	.018	2.097	.003	2.748	$.0^{5}1$	4.753
22	0.772	.07	1.476	.017	2.120	.002	2.878	$.0^{6}5$	4.892
21	0.806	.06	1.555	.016	2.144	.001	3.090	$.0^{6}1$	5.199
				William Bridge					
20	0.842	.050	1.645	.015	2.170	$0^{39}$	3.121	$ .0^{7}5 $	5.327
19	0.878	.045	1.695	.014	2.197	$.0^{3}8$	3.156	$.0^{7}1$	5.612
18	0.915	.040	1.751	.013	2.226	$.0^{3}7$	3.195	0.085	5.731
17	0.954	.035	1.812	.012	2.257	$.0^{3}6$	3.239	0.081	5.998
16	0.994	.030	1.881	.011	2.290	$.0^{3}5$	3.291	.095	6.109
	50 45 40 35 330 25 224 223 222 19 18 17	50 0.000 45 0.126 40 0.253 35 0.385 30 0.524 25 0.674 24 0.706 23 0.739 22 0.772 21 0.806 20 0.842 19 0.878 18 0.915 17 0.954	50 0.000 .15 45 0.126 .14 40 0.253 .13 35 0.385 .12 30 0.524 .11 25 0.674 .10 24 0.706 .09 23 0.739 .08 22 0.772 .07 21 0.806 .06 20 0.842 .050 19 0.878 .045 18 0.915 .040 17 0.954 .035	50 0.000 .15 1.036   45 0.126 .14 1.080   40 0.253 .13 1.126   35 0.385 .12 1.175   30 0.524 .11 1.227   25 0.674 .10 1.282   24 0.706 .09 1.341   23 0.739 .08 1.405   22 0.772 .07 1.476   21 0.806 .06 1.555   20 0.842 .050 1.645   19 0.878 .045 1.695   18 0.915 .040 1.751   17 0.954 .035 1.812	50 0.000 .15 1.036 .025   45 0.126 .14 1.080 .024   40 0.253 .13 1.126 .023   35 0.385 .12 1.175 .022   30 0.524 .11 1.227 .021   25 0.674 .10 1.282 .020   24 0.706 .09 1.341 .019   23 0.739 .08 1.405 .018   22 0.772 .07 1.476 .017   21 0.806 .06 1.555 .016   20 0.842 .050 1.645 .015   19 0.878 .045 1.695 .014   18 0.915 .040 1.751 .013   17 0.954 .035 1.812 .012	50 0.000 .15 1.036 .025 1.960   45 0.126 .14 1.080 .024 1.977   40 0.253 .13 1.126 .023 1.995   35 0.385 .12 1.175 .022 2.014   30 0.524 .11 1.227 .021 2.034   25 0.674 .10 1.282 .020 2.054   24 0.706 .09 1.341 .019 2.075   23 0.739 .08 1.405 .018 2.097   22 0.772 .07 1.476 .017 2.120   21 0.806 .06 1.555 .016 2.144   20 0.842 .050 1.645 .015 2.170   19 0.878 .045 1.695 .014 2.197   18 0.915 .040 1.751 .013 2.226   17 0.954 .035	50 0.000 .15 1.036 .025 1.960 .010   45 0.126 .14 1.080 .024 1.977 .009   40 0.253 .13 1.126 .023 1.995 .008   35 0.385 .12 1.175 .022 2.014 .007   30 0.524 .11 1.227 .021 2.034 .006   25 0.674 .10 1.282 .020 2.054 .006   24 0.706 .09 1.341 .019 2.075 .004   23 0.739 .08 1.405 .018 2.097 .003   22 0.772 .07 1.476 .017 2.120 .002   21 0.806 .06 1.555 .016 2.144 .001   20 0.842 .050 1.645 .015 2.170 .0³8   18 0.915 .040 1.751 .013 2.226 <th>50 0.000 .15 1.036 .025 1.960 .010 2.326   45 0.126 .14 1.080 .024 1.977 .009 2.366   40 0.253 .13 1.126 .023 1.995 .008 2.409   35 0.385 .12 1.175 .022 2.014 .007 2.457   30 0.524 .11 1.227 .021 2.034 .006 2.512   25 0.674 .10 1.282 .020 2.054 .006 2.576   24 0.706 .09 1.341 .019 2.075 .004 2.652   23 0.739 .08 1.405 .018 2.097 .003 2.748   22 0.772 .07 1.476 .017 2.120 .002 2.878   21 0.806 .06 1.555 .016 2.144 .001 3.090   20 0.842 .045 <t< th=""><th>50 0.000 .15 1.036 .025 1.960 .010 2.326 .0³4   45 0.126 .14 1.080 .024 1.977 .009 2.366 .0³3   40 0.253 .13 1.126 .023 1.995 .008 2.409 .0³2   35 0.385 .12 1.175 .022 2.014 .007 2.457 .0³1   30 0.524 .11 1.227 .021 2.034 .006 2.512 .0⁴5   25 0.674 .10 1.282 .020 2.054 .005 2.576 .0⁴5   24 0.706 .09 1.341 .019 2.075 .004 2.652 .0⁵5   23 0.739 .08 1.405 .018 2.097 .003 2.748 .0⁵1   22 0.772 .07 1.476 .017 2.120 .002 2.878 .0⁶5   21 0.806 .06 &lt;</th></t<></th>	50 0.000 .15 1.036 .025 1.960 .010 2.326   45 0.126 .14 1.080 .024 1.977 .009 2.366   40 0.253 .13 1.126 .023 1.995 .008 2.409   35 0.385 .12 1.175 .022 2.014 .007 2.457   30 0.524 .11 1.227 .021 2.034 .006 2.512   25 0.674 .10 1.282 .020 2.054 .006 2.576   24 0.706 .09 1.341 .019 2.075 .004 2.652   23 0.739 .08 1.405 .018 2.097 .003 2.748   22 0.772 .07 1.476 .017 2.120 .002 2.878   21 0.806 .06 1.555 .016 2.144 .001 3.090   20 0.842 .045 <t< th=""><th>50 0.000 .15 1.036 .025 1.960 .010 2.326 .0³4   45 0.126 .14 1.080 .024 1.977 .009 2.366 .0³3   40 0.253 .13 1.126 .023 1.995 .008 2.409 .0³2   35 0.385 .12 1.175 .022 2.014 .007 2.457 .0³1   30 0.524 .11 1.227 .021 2.034 .006 2.512 .0⁴5   25 0.674 .10 1.282 .020 2.054 .005 2.576 .0⁴5   24 0.706 .09 1.341 .019 2.075 .004 2.652 .0⁵5   23 0.739 .08 1.405 .018 2.097 .003 2.748 .0⁵1   22 0.772 .07 1.476 .017 2.120 .002 2.878 .0⁶5   21 0.806 .06 &lt;</th></t<>	50 0.000 .15 1.036 .025 1.960 .010 2.326 .0³4   45 0.126 .14 1.080 .024 1.977 .009 2.366 .0³3   40 0.253 .13 1.126 .023 1.995 .008 2.409 .0³2   35 0.385 .12 1.175 .022 2.014 .007 2.457 .0³1   30 0.524 .11 1.227 .021 2.034 .006 2.512 .0⁴5   25 0.674 .10 1.282 .020 2.054 .005 2.576 .0⁴5   24 0.706 .09 1.341 .019 2.075 .004 2.652 .0⁵5   23 0.739 .08 1.405 .018 2.097 .003 2.748 .0⁵1   22 0.772 .07 1.476 .017 2.120 .002 2.878 .0⁶5   21 0.806 .06 <

The z-value we are looking for is 2.576. This is the upper boundary of the 'acceptable' range.

Now we calculate the z-value corresponding to the measured mean, in order to see where it falls with respect to the acceptable range. This we call the test statistic and calculate as:

$$T = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}}$$

where  $\bar{x}$  is the mean calculated from the sample (197ml),  $\mu_0$  is the hypothesized population mean (200ml) and the fraction acting as nominator is the *standard error* i.e. the standard deviation of the sampling distribution.

We calculate the statistic:

$$T = \frac{197ml - 200ml}{\frac{2ml}{\sqrt{100}}} = 15$$

Note that the units (ml) have been cancelled out and that we have a dimensionless number, i.e. a z-value, which has a standardised normal distribution.

The last thing we need to do is to see whether our calculated test statistic falls within the 'acceptable' range:

$$-2.576 \le T \le 2.576$$
 ?

$$-2.576 \le 15 \le 2.576$$
 **FALSE!**

It does not fall in the range, so we reject the null hypothesis.