ICON4Py and IBM documentation

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Abstract

Work in progress documentation of the immersed boundary method implementation in ICON.

1 Equations

1.1 Normal wind

Advection

wind and kinetic energy

$$v_{t\,e\,k}^{\,n} = \sum_{e2c2e} \overset{\text{rbf}}{\chi}\, v_{n\,e\,k}^{\,n} \tag{1}$$

$$K_{h\,e\,k}^{\,n} = \frac{1}{2}\left(v_{n\,e\,k}^{\,n\ 2} + v_{t\,e\,k}^{\,n\ 2}\right) \tag{2}$$

$$K_{h\,c\,k}^{\,n} = \sum_{c2e} \overset{\text{hor}}{\chi}\, K_{h\,e\,k}^{\,n} \tag{3}$$

$$v_{n\,e\,k-1/2}^{\,n} = \overset{\text{lev}}{\chi}\, v_{n\,e\,k}^{\,n} + (1-\overset{\text{lev}}{\chi})\, v_{n\,e\,k-1}^{\,n} \tag{4}$$

$$\tag{5}$$

vorticity

$$\zeta_{v\,k}^{\,n} = \sum_{v2e} \overset{\text{rot}}{\chi}\, v_{n\,e\,k}^{\,n} \tag{6}$$

contravariant-corrected vertical wind

$$w_{\text{cc}\,e\,k}^{\,n} = v_{n\,e\,k}^{\,n}\frac{\partial z}{\partial n} + v_{t\,e\,k}^{\,n}\frac{\partial z}{\partial t} \tag{7}$$

$$w_{\text{cc}\,c\,k}^{\,n} = \sum_{c2e} \overset{\text{hor}}{\chi}\, w_{\text{cc}\,e\,k}^{\,n} \tag{8}$$

$$w_{\text{cc}\,c\,k-1/2}^{\,n} = \overset{\text{lev}}{\chi}\, w_{\text{cc}\,c\,k}^{\,n} + (1-\overset{\text{lev}}{\chi})\, w_{\text{cc}\,c\,k-1}^{\,n} \tag{9}$$

$$(w_{c\,k-1/2}^{\,n} - w_{\text{cc}\,c\,k-1/2}^{\,n}) = \begin{cases} w_{c\,k-1/2}^{\,n}, & k \in [0, \texttt{flat\_lev}+1) \\ w_{c\,k-1/2}^{\,n} - w_{\text{cc}\,c\,k-1/2}^{\,n}, & k \in [\texttt{flat\_lev}+1, \texttt{num\_lev}) \\ 0, & k = \texttt{num\_lev} \end{cases} \tag{10}$$

$$(w_{c\,k}^{\,n} - w_{\text{cc}\,c\,k}^{\,n}) = \frac{1}{2}\left[(w_{c\,k-1/2}^{\,n} - w_{\text{cc}\,c\,k-1/2}^{\,n}) + (w_{c\,k+1/2}^{\,n} - w_{\text{cc}\,c\,k+1/2}^{\,n})\right] \tag{11}$$

$$\tag{12}$$

sum all contributions

$$\begin{aligned}
\text{adv}(v_n)_{e\,k}^{\,n} &= \frac{\partial K_h}{\partial n} + v_t(\zeta + f) + \frac{\partial v_n}{\partial z}(w - w_{\text{cc}}) \\
&= \Delta_{e2c}\overset{\text{grad}}{\chi}\, K_{h\,c\,k}^{\,n} + K_{h\,e\,k}^{\,n}\Delta_{e2c}\overset{\text{grad}}{\chi} \\
&\quad + v_{t\,e\,k}^{\,n}(f_e + 1/2\sum_{e2v}\zeta_{v\,k}^{\,n}) \\
&\quad + \frac{v_{n\,e\,k-1/2}^{\,n} - v_{n\,e\,k+1/2}^{\,n}}{\Delta z_k}\sum_{e2c}\overset{\text{hor}}{\chi}(w_{c\,k}^{\,n} - w_{\text{cc}\,c\,k}^{\,n})
\end{aligned} \tag{13}$$

Exner

$$\pi'^{\bar{n}}_{c\,k} = (1+\gamma)\pi'^{\,n}_{c\,k} - \gamma\pi'^{\,n-1}_{c\,k} \tag{14}$$

$$\pi'^{\bar{n}}_{c\,k-1/2} = \overset{\text{lev}}{\chi}\,\pi'^{\bar{n}}_{c\,k} + (1-\overset{\text{lev}}{\chi})\,\pi'^{\bar{n}}_{c\,k-1}, \quad k \in [\max(1,\texttt{flat\_lev}), \texttt{num\_lev}) \tag{15}$$

$$\pi'^{\bar{n}}_{c\,\texttt{num\_lev}-1/2} = \sum_{k=\texttt{num\_lev}-1}^{\texttt{num\_lev}-3} \overset{\text{lev}}{\chi}_k\,\pi'^{\bar{n}}_{c\,k} \tag{16}$$

horizontal gradient (at constant height) of π' on flat and non-flat levels

$$\frac{\partial \pi'^{\bar{n}}}{\partial n}\bigg|_{e\,k} = \overset{\text{grad}}{\chi}\Delta_{e2c}\pi'^{\bar{n}}_{c\,k}, \quad k \in [0, \texttt{flat\_lev}) \tag{17}$$

$$\begin{aligned}
\frac{\partial \pi'^{\bar{n}}}{\partial n}\bigg|_{e\,k} &= \frac{\partial \pi'}{\partial n}\bigg|_s - \frac{\partial h}{\partial n}\bigg|_s\frac{\partial \pi'}{\partial z}, \quad k \in [\texttt{flat\_lev}, \texttt{flat\_gradp\_lev}] \\
&= \frac{1}{d_{12}}\Delta_{e2c}\pi'^{\bar{n}}_{c\,k} - \frac{\partial h}{\partial n}\sum_{e2c}\overset{\text{hor}}{\chi}\frac{\partial \pi'^{\bar{n}}}{\partial z}\bigg|_{c\,k}
\end{aligned} \tag{18}$$

$$\begin{aligned}
\frac{\partial \pi'^{\bar{n}}}{\partial n}\bigg|_{e\,k} &= \frac{1}{d_{12}}(\pi'^{*}_{c_1} - \pi'^{*}_{c_0}), \quad k \in [\texttt{flat\_gradp\_lev}+1, \texttt{num\_lev}) \\
&= \frac{1}{d_{12}}\Delta_{e2c}\left[\pi'^{\bar{n}}_{c\,k^*} + (h^* - h_k)\left(\frac{\partial \pi'^{\bar{n}}}{\partial z}\bigg|_{c\,k^*} + (h^* - h_k)\frac{1}{2}\frac{\partial^2 \pi'^{\bar{n}}}{\partial z^2}\bigg|_{c\,k^*}\right)\right]
\end{aligned} \tag{19}$$

Hydrostatic correction term to $\frac{\partial \pi'}{\partial n}$.

$$\theta_{v_{c_i\,k}} = \theta_{v_{c_i\,k^*}} + (h^* - h_k)\frac{\theta_{v_{c_i\,k^*-1/2}} - \theta_{v_{c_i\,k^*+1/2}}}{\Delta z_{k^*}} \tag{20}$$

$$\frac{g}{c_{pd}\theta_v{}^2}\frac{\partial \theta_v}{\partial n}\Big|_e = \frac{g}{c_{pd}}\frac{1}{d_{12}}4\frac{\theta_{v_{c_1\,k}} - \theta_{v_{c_0\,k}}}{(\theta_{v_{c_1\,k}} + \theta_{v_{c_0\,k}})^2} \tag{21}$$

$$\frac{\partial \pi'^{\tilde{n}}}{\partial n}\Big|_{e\,k} = \frac{\partial \pi'^{\tilde{n}}}{\partial n}\Big|_{e\,k} + \frac{g}{c_{pd}\theta_v{}^2}\frac{\partial \theta_v}{\partial n}\Big|_e (h_k - h_{k^*}), \quad e \in \overset{\partial \pi/\partial n}{\text{IDX}} \tag{22}$$

Miura scheme

$\theta_{v_{e\,k}^{\,n}}$ is computed with the Miura scheme §1.6

Final update

$$v_{n_{e\,k}^{\,n+1^*}} = v_{n_{e\,k}^{\,n}} - \Delta t\left(\text{adv}(v_n)_{e\,k}^{\,n} + c_{pd}\theta_{v_{e\,k}^{\,n}}\frac{\partial \pi'^{\tilde{n}}}{\partial n}\Big|_{e\,k}\right) \tag{23}$$

1.2 Vertical wind

Exner

$$\frac{\partial \pi'^{\bar{n}}}{\partial z}\bigg|_{c\,k} \approx \frac{\pi'^{\bar{n}}_{c\,k-1/2} - \pi'^{\bar{n}}_{c\,k+1/2}}{\Delta z_k} \tag{24}$$

1.3 Exner

Coefficients

$$\alpha_{k??} = \eta \rho^n_{c\,k-1/2} \theta'^{\,n}_{v\,c\,k-1/2} \tag{25}$$

$$\beta_{k??} = \Delta t \frac{R_d}{c_{vd}} \frac{\Gamma^n_k}{\Delta z} = \Delta t \frac{R_d}{c_{vd}} \frac{\pi'^{\,n}_{c\,k}}{\rho^n_{c\,k} \theta'^{\,n}_{v\,c\,k}} \frac{1}{\Delta z} \tag{26}$$

Average normal wind
"needed to obtain a nearly 2nd order accurate divergence (Zangl 2015)"

$$\texttt{z\_vn\_avg}_{ek} = \sum_{e2c2eo} \overset{\text{hor}}{\chi} v_{n\,e\,k}^{n+1^*} \tag{27}$$

Miura scheme
$\rho^n_{e\,k}$ is computed with the Miura scheme §1.6

Explicit term

$$\texttt{z\_theta\_v\_fl\_e}_{ek} = \texttt{mass\_fl\_e}_{ek}\texttt{z\_theta\_v\_e}_{ek} = \rho^n_{e\,k}\texttt{z\_vn\_avg}_{ek}/\Delta z\texttt{z\_theta\_v\_e}_{ek} \tag{28}$$

$$\texttt{z\_flxdiv\_theta}_{ck} = \sum_{c2e} \overset{\text{div}}{\chi}\texttt{z\_theta\_v\_fl\_e}_{ek} \tag{29}$$

$$\begin{aligned}
Z^{\pi\,\text{expl}} = \pi'^{\,n}_{c\,k} - \beta_{k??} &\Big(\texttt{z\_flxdiv\_theta}_{ck} + \theta'^{\,n}_{v\,c\,k-1/2}\texttt{z\_contr\_w\_fl\_l}_{k-1/2} \\
&-\theta'^{\,n}_{v\,c\,k+1/2}\texttt{z\_contr\_w\_fl\_l}_{k+1/2} \Big) + \Delta t \texttt{ddt\_exner\_phy}
\end{aligned} \tag{30}$$

Final update

$$\pi'^{\,n+1^*}_{c\,k} = Z^{\pi\,\text{expl}} + \pi_{0\,c\,k} - \beta \left(\alpha_{k-1/2} w^{n+1^*}_{c\,k-1/2} - \alpha_{k+1/2} w^{n+1^*}_{c\,k+1/2} \right) \tag{31}$$

1.4 Density

Average normal wind

"needed to obtain a nearly 2nd order accurate divergence (Zangl 2015)"

$$\texttt{z\_vn\_avg}_{ek} = \sum_{e2c2eo} \overset{\text{hor}}{\chi} \, v_{n_{ek}}^{n+1*} \tag{32}$$

Miura scheme

ρ_{ek}^{n} is computed with the Miura scheme §1.6

Explicit term

$$\texttt{mass\_fl\_e}_{ek} = \rho_{ek}^{n} \texttt{z\_vn\_avg}_{ek} / \Delta z \tag{33}$$

$$\texttt{z\_flxdiv\_mass}_{ck} = \sum_{c2e} \overset{\text{div}}{\chi} \texttt{mass\_fl\_e}_{ek} \tag{34}$$

$$Z^{\rho \, \text{expl}} = \rho_{ck}^{n} - \frac{\Delta t}{\Delta z} \left(\texttt{z\_flxdiv\_mass}_{ck} + \texttt{z\_contr\_w\_fl\_l}_{ck-1/2} - \texttt{z\_contr\_w\_fl\_l}_{ck+1/2} \right) \tag{35}$$

Final update

$$\rho_{ck}^{n+1*} = Z^{\rho \, \text{expl}} - \eta \frac{\Delta t}{\Delta z} \left(\rho_{ck-1/2}^{n} w_{ck-1/2}^{n+1*} - \rho_{ck+1/2}^{n} w_{ck+1/2}^{n+1*} \right) \tag{36}$$

1.5 Virtual potential temperature

$$\theta'^{n+1^*}_{v_{c\,k}} = \frac{\rho^n_{c\,k}\,\theta'^n_{v_{c\,k}}}{\rho^{n+1^*}_{c\,k}} \left(\left(\frac{\pi'^{n+1^*}_{c\,k}}{\pi'^n_{c\,k}} - 1 \right) \frac{c_{vd}}{R_d} + 1 \right) \tag{37}$$

1.6 Miura scheme

No artificial diffusion term is needed for the scalar quantities because the advection of ρ and θ_v is discretized with an upwind-biased second-order accurate scheme following [Miura, 2007].

$$\rho_{e\,k}^n = \rho_{e\,k} + \rho'^n_{c0/1\,k} + \Delta^{\hat{x}}_{c-\text{btrj}} \frac{\partial \rho'^n}{\partial x}\bigg|_{c\,k} + \Delta^{\hat{y}}_{c-\text{btrj}} \frac{\partial \rho'^n}{\partial y}\bigg|_{c\,k} \tag{38}$$

$$\theta_{v\,e\,k}^n = \theta_{v0\,e\,k} + \theta'^n_{v\,c0/1\,k} + \Delta^{\hat{x}}_{c-\text{btrj}} \frac{\partial \theta_v'^n}{\partial x}\bigg|_{c\,k} + \Delta^{\hat{y}}_{c-\text{btrj}} \frac{\partial \theta_v'^n}{\partial y}\bigg|_{c\,k} \tag{39}$$

$$\frac{\partial \rho'^n}{\partial x}\bigg|_{c\,k} = \sum_{c2e2co} \texttt{geofac\_grg\_x}\ \rho'^n_{c\,k} \tag{40}$$

$$\frac{\partial \rho'^n}{\partial y}\bigg|_{c\,k} = \sum_{c2e2co} \texttt{geofac\_grg\_y}\ \rho'^n_{c\,k} \tag{41}$$

$$\frac{\partial \theta_v'^n}{\partial x}\bigg|_{c\,k} = \sum_{c2e2co} \texttt{geofac\_grg\_x}\ \theta'^n_{v\,c\,k} \tag{42}$$

$$\frac{\partial \theta_v'^n}{\partial y}\bigg|_{c\,k} = \sum_{c2e2co} \texttt{geofac\_grg\_y}\ \theta'^n_{v\,c\,k} \tag{43}$$

The immersed boundary method (IBM) handles these operations by zeroing the gradient computation in cells adjacent to masked cells (i.e., those indicated with a grid pattern)
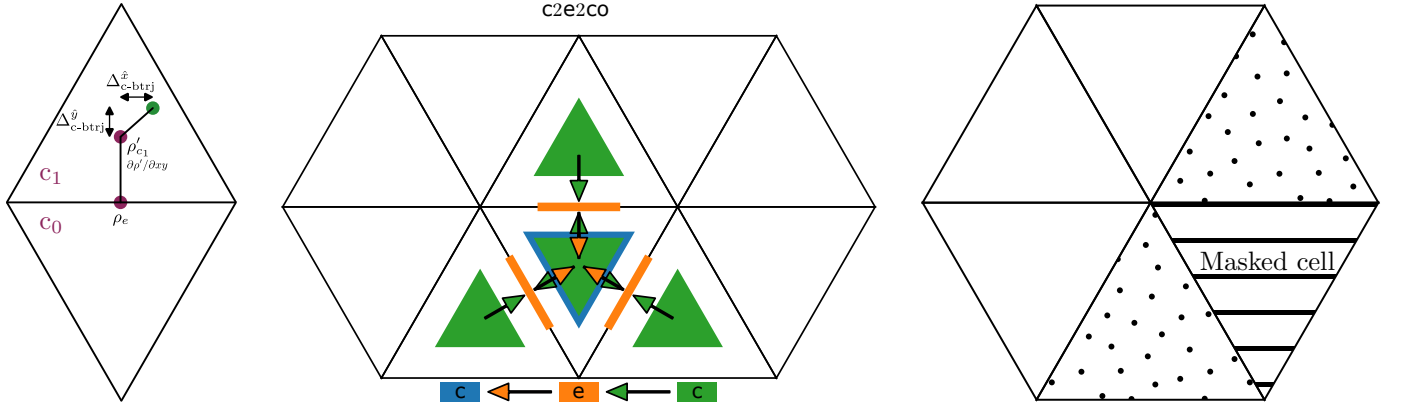

Figure 1: Miura scheme definition and offset provider used for the horizontal derivative computations

1.7 Diffusion

e2v interpolation

$$u_{\mathrm{v}} = \sum_{v2e} \overset{\mathrm{rbf}}{\chi}_1 v_{n_{\mathrm{e}}} \tag{44}$$

$$v_{\mathrm{v}} = \sum_{v2e} \overset{\mathrm{rbf}}{\chi}_2 v_{n_{\mathrm{e}}} \tag{45}$$

Smagorinsky coefficient

$$
\begin{aligned}
K_{h}^{\mathrm{smag^n}}{}_{\mathrm{e\,k}} &= \min\left(\texttt{smag\_limit}, \max\left(0, \texttt{diff\_multfac\_smag}\sqrt{*} - \texttt{smag\_offset}\right)\right) \\
* \quad &= \left(\frac{-\left(u_{\mathrm{v2}}\,\hat{n}_{x\mathrm{v2}} + v_{\mathrm{v2}}\,\hat{n}_{y\mathrm{v2}}\right) + \left(u_{\mathrm{v3}}\,\hat{n}_{x\mathrm{v3}} + v_{\mathrm{v3}}\,\hat{n}_{y\mathrm{v3}}\right)}{\ell_{\mathrm{vv}}} - \frac{\left(-\left(u_{\mathrm{v0}}\,\hat{t}_{x\mathrm{v0}} + v_{\mathrm{v0}}\,\hat{t}_{y\mathrm{v0}}\right) + \left(u_{\mathrm{v1}}\,\hat{t}_{x\mathrm{v1}} + v_{\mathrm{v1}}\,\hat{t}_{y\mathrm{v1}}\right)\right)\hat{t}_{\mathrm{e}}}{\ell_{\mathrm{e}}}\right)^2 \\
&\quad + \left(\frac{\left(-\left(u_{\mathrm{v0}}\,\hat{n}_{x\mathrm{v0}} + v_{\mathrm{v0}}\,\hat{n}_{y\mathrm{v0}}\right) + \left(u_{\mathrm{v1}}\,\hat{n}_{x\mathrm{v1}} + v_{\mathrm{v1}}\,\hat{n}_{y\mathrm{v1}}\right)\right)\hat{t}_{\mathrm{e}}}{\ell_{\mathrm{e}}} + \frac{-\left(u_{\mathrm{v2}}\,\hat{t}_{x\mathrm{v2}} + v_{\mathrm{v2}}\,\hat{t}_{y\mathrm{v2}}\right) + \left(u_{\mathrm{v3}}\,\hat{t}_{x\mathrm{v3}} + v_{\mathrm{v3}}\,\hat{t}_{y\mathrm{v3}}\right)}{\ell_{\mathrm{vv}}}\right)^2
\end{aligned}
\tag{46}
$$

Nabla 2

$$\nabla^2(v_n{}^{\mathrm{n}})_{\mathrm{e\,k}} = 4\left[\frac{u_{\mathrm{v0}}\,\hat{n}_{x\mathrm{v0}} + v_{\mathrm{v0}}\,\hat{n}_{y\mathrm{v0}} + u_{\mathrm{v1}}\,\hat{n}_{x\mathrm{v1}} + v_{\mathrm{v1}}\,\hat{n}_{y\mathrm{v1}} - 2v_{n_{\mathrm{e}}}}{\ell_{\mathrm{e}}^2} + \frac{u_{\mathrm{v2}}\,\hat{n}_{x\mathrm{v2}} + v_{\mathrm{v2}}\,\hat{n}_{y\mathrm{v2}} + u_{\mathrm{v3}}\,\hat{n}_{x\mathrm{v3}} + v_{\mathrm{v3}}\,\hat{n}_{y\mathrm{v3}} - 2v_{n_{\mathrm{e}}}}{\ell_{\mathrm{vv}}^2}\right] \tag{47}$$

$$\nabla^2(w)_{\mathrm{c\,k-1/2}} = \sum_{c2e2co} \overset{\mathrm{hor}}{\chi}\, w_{\mathrm{c\,k-1/2}} \tag{48}$$

$$\nabla^2(\theta_v'{}^{\mathrm{n}})_{\mathrm{c\,k}} = \sum_{c2e}\left(\overset{\mathrm{hor}}{\chi} K_{h}^{\mathrm{smag}}{}_{\mathrm{e\,k}} \overset{\mathrm{grad}}{\chi} \Delta_{e2c}\theta_{v\,\mathrm{c\,k}}'{}^{\mathrm{n}}\right) \tag{49}$$

e2v interpolation

$$\nabla^2(v_n)_{\mathrm{v}^x} = \sum_{v2e} \overset{\mathrm{rbf}}{\chi}_1 \nabla^2(v_n)_{\mathrm{e}} \tag{50}$$

$$\nabla^2(v_n)_{\mathrm{v}^y} = \sum_{v2e} \overset{\mathrm{rbf}}{\chi}_2 \nabla^2(v_n)_{\mathrm{e}} \tag{51}$$

Nabla 4

$$\texttt{nabv\_tang} = \nabla^2(v_n)_{\mathrm{v}^{x_0}}\,\hat{n}_{x\mathrm{v0}} + \nabla^2(v_n)_{\mathrm{v}^{y_0}}\,\hat{n}_{y\mathrm{v0}} + \nabla^2(v_n)_{\mathrm{v}^{x_1}}\,\hat{n}_{x\mathrm{v1}} + \nabla^2(v_n)_{\mathrm{v}^{y_1}}\,\hat{n}_{y\mathrm{v1}} \tag{52}$$

$$\texttt{nabv\_norm} = \nabla^2(v_n)_{\mathrm{v}^{x_2}}\,\hat{n}_{x\mathrm{v2}} + \nabla^2(v_n)_{\mathrm{v}^{y_2}}\,\hat{n}_{y\mathrm{v2}} + \nabla^2(v_n)_{\mathrm{v}^{x_3}}\,\hat{n}_{x\mathrm{v3}} + \nabla^2(v_n)_{\mathrm{v}^{y_3}}\,\hat{n}_{y\mathrm{v3}} \tag{53}$$

$$\nabla^4(v_n{}^{\mathrm{n}})_{\mathrm{e\,k}} = 4\left[\frac{\texttt{nabv\_norm} - 2\nabla^2(v_n{}^{\mathrm{n}})_{\mathrm{e\,k}}}{\ell_{\mathrm{vv}}^2} + \frac{\texttt{nabv\_tang} - 2\nabla^2(v_n{}^{\mathrm{n}})_{\mathrm{e\,k}}}{\ell_{\mathrm{e}}^2}\right] \tag{54}$$

Final updates

$$v_{n\,\mathrm{e\,k}}^{d} = v_{n\,\mathrm{e\,k}}^{\mathrm{n}} + a_e\left(K_{h}^{\mathrm{smag}}{}_{\mathrm{e\,k}} \nabla^2(v_n{}^{\mathrm{n}})_{\mathrm{e\,k}} - \texttt{diff\_multfac\_vn} a_e \nabla^4(v_n{}^{\mathrm{n}})_{\mathrm{e\,k}}\right) \tag{55}$$

$$w_{\mathrm{c\,k-1/2}}^{d} = w_{\mathrm{c\,k-1/2}}^{\mathrm{n}} - \texttt{diff\_multfac\_wa}\,a_c^2 \sum_{c2e2co} \overset{\mathrm{hor}}{\chi}\,\nabla^2(w)_{\mathrm{c\,k-1/2}} \tag{56}$$

$$\theta_{v\,\mathrm{c\,k}}^{d} = \theta_{v\,\mathrm{c\,k}}^{\mathrm{n}} + a_c \nabla^2(\theta_v'{}^{\mathrm{n}})_{\mathrm{c\,k}} \tag{57}$$

$$\pi_{\mathrm{c\,k}}^{d} = \pi_{\mathrm{c\,k}}^{\mathrm{n}}\left(1 + \frac{R_d}{c_{vd}}\left(\frac{\theta_{v\,\mathrm{c\,k}}^{d}}{\theta_{v\,\mathrm{c\,k}}^{\mathrm{n}}} - 1\right)\right) \tag{58}$$

1.8 Don't remember what these are for

$$\frac{1}{2}\frac{\partial^2 \pi'^{\bar{n}}}{\partial z^2}\bigg|_{c\,k} = -\frac{1}{2}\left((\theta'^{n}_{v_{c\,k-1/2}} - \theta'^{n}_{v_{c\,k+1/2}})\frac{1}{\Delta z\theta_{v0}}\frac{d\pi_0}{dz}\bigg|_{c\,k} + \theta'^{n}_{v_{c\,k}}\frac{d}{dz}\left(\frac{1}{\theta_{v0}}\frac{d\pi_0}{dz}\right)_{c\,k}\right), \quad k \in [\texttt{flat\_gradp\_lev}, \texttt{num\_lev}) \tag{59}$$

$$\frac{d\pi_0}{dz} = -\frac{g}{c_{pd}\theta_{v0}} \tag{60}$$

References

[Miura, 2007] Miura, H. (2007). An Upwind-Biased Conservative Advection Scheme for Spherical Hexagonal–Pentagonal Grids. *Monthly Weather Review*, 135(12):4038–4044.