Discussion Handout 3

Problem One: Graphs

In an undirected graph with no self-loops (that is, an undirected graph where no node has an edge to itself), the *degree* of a node is the number of edges connected to it.

An undirected graph $G = (V, E)$ is called *connected* iff for any $u, v \in V$, there is a path from u to v (that is, $u \leftrightarrow v$). Let $G = (V, E)$ be an undirected graph with no self-loops. Prove that if the degree of every node in G is $|V| / 2$ or greater, then G is connected.

Problem Two: The Pigeonhole Principle

Prove that if you pick any five points in the unit square, there must be some pair of points chosen whose distance from one another is at most $\sqrt{2}/2$.

Problem Three: Binary Relations

Consider the following relation R defined over $\mathbb{N}^2$:

$$(a, b)R(c, d) \quad \text{iff} \quad a + d = b + c$$

   i.   Prove that R is an equivalence relation.

   ii. What are the equivalence classes of R?