

1.5em

# AA 272C: Global Positioning Systems

## Problem Set 3

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### Algorithm Sensor Fusion

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**Algorithm 1** Sensor Fusion EKF

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1: Inputs:
2:   GNSS pseudoranges
3:   Satellite positions
4:   IMU odometry
5:   barometer measurements
6: Initialize:
7:    $\mathbf{T} \leftarrow$  concatenated and sorted timesteps
8: for  $t \in \mathbf{T}$  do
9:   if odometry exists at  $t$  then
10:    Predict:
11:     $v_t \leftarrow$  imu odometry
12:     $\hat{\mu}_{t|t-1} = F\hat{\mu}_{t-1|t-1} + Bv_t$ 
13:     $P_{t|t-1} = F_t P_{t-1|t-1} F_t^T + Q$ 
14:   end if
15:   if GNSS exists at  $t$  then
16:    Update GNSS:
17:     $z_t \leftarrow$  GNSS measurements
18:     $\tilde{y}_t = z_t - h(\hat{\mu}_{t|t-1})$ 
19:     $K_t = P_{t|t-1} H_t^T (R + H_t P_{t|t-1} H_t^T)^{-1}$ 
20:     $\hat{\mu}_{t|t} = \hat{\mu}_{t|t-1} + K_t \tilde{y}_t$ 
21:     $P_{t|t} = (I - K_t H_t) P_{t|t-1} (I - K_t H_t)^T + K_t R K_t^T$ 
22:   end if
23:   if Barometer exists at  $t$  then
24:    Update Barometer:
25:     $z_t \leftarrow$  barometer measurements
26:     $\tilde{y}_t = z_t - h(\hat{\mu}_{t|t-1})$ 
27:     $K_t = P_{t|t-1} H_t^T (R + H_t P_{t|t-1} H_t^T)^{-1}$ 
28:     $\hat{\mu}_{t|t} = \hat{\mu}_{t|t-1} + K_t \tilde{y}_t$ 
29:     $P_{t|t} = (I - K_t H_t) P_{t|t-1} (I - K_t H_t)^T + K_t R K_t^T$ 
30:   end if
31: end for
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## EKF Model Update

For the state:

$$\hat{\mu} = \begin{bmatrix} x \\ y \\ z \\ \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix}$$

For the predict step, I will use:

$$\hat{\mu}_{t|t-1} = F \hat{\mu}_{t-1|t-1}$$

$$\hat{\mu}_{t|t-1} = \begin{bmatrix} 1 & 0 & 0 & \Delta t & 0 & 0 \\ 0 & 1 & 0 & 0 & \Delta t & 0 \\ 0 & 0 & 1 & 0 & 0 & \Delta t \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix}$$

For Q, I will use

$$\sigma = 0.03km/h * \frac{1000m}{1km} * \frac{1h}{3600s} * dt$$

$$Q = \begin{bmatrix} \sigma^2 & 0 & 0 \\ 0 & \sigma^2 & 0 \\ 0 & 0 & \sigma^2 \end{bmatrix}$$

For the update step, I will use

$$h = \begin{bmatrix} \sqrt{(x_1 - x)^2 + (y_1 - y)^2 + (z_1 - z)^2} \\ \sqrt{(x_2 - x)^2 + (y_2 - y)^2 + (z_2 - z)^2} \\ \sqrt{(x_3 - x)^2 + (y_3 - y)^2 + (z_3 - z)^2} \\ \sqrt{(x_4 - x)^2 + (y_4 - y)^2 + (z_4 - z)^2} \\ \sqrt{(x_5 - x)^2 + (y_5 - y)^2 + (z_5 - z)^2} \\ \sqrt{(x_6 - x)^2 + (y_6 - y)^2 + (z_6 - z)^2} \end{bmatrix}$$

$$H = \begin{bmatrix} \frac{-(x_1 - x)}{\sqrt{(x_1 - x)^2 + (y_1 - y)^2 + (z_1 - z)^2}} & \frac{-(y_1 - y)}{\sqrt{(x_1 - x)^2 + (y_1 - y)^2 + (z_1 - z)^2}} & \frac{-(z_1 - z)}{\sqrt{(x_1 - x)^2 + (y_1 - y)^2 + (z_1 - z)^2}} \\ \frac{-(x_2 - x)}{\sqrt{(x_2 - x)^2 + (y_2 - y)^2 + (z_2 - z)^2}} & \frac{-(y_2 - y)}{\sqrt{(x_2 - x)^2 + (y_2 - y)^2 + (z_2 - z)^2}} & \frac{-(z_2 - z)}{\sqrt{(x_2 - x)^2 + (y_2 - y)^2 + (z_2 - z)^2}} \\ \frac{-(x_3 - x)}{\sqrt{(x_3 - x)^2 + (y_3 - y)^2 + (z_3 - z)^2}} & \frac{-(y_3 - y)}{\sqrt{(x_3 - x)^2 + (y_3 - y)^2 + (z_3 - z)^2}} & \frac{-(z_3 - z)}{\sqrt{(x_3 - x)^2 + (y_3 - y)^2 + (z_3 - z)^2}} \\ \frac{-(x_4 - x)}{\sqrt{(x_4 - x)^2 + (y_4 - y)^2 + (z_4 - z)^2}} & \frac{-(y_4 - y)}{\sqrt{(x_4 - x)^2 + (y_4 - y)^2 + (z_4 - z)^2}} & \frac{-(z_4 - z)}{\sqrt{(x_4 - x)^2 + (y_4 - y)^2 + (z_4 - z)^2}} \\ \frac{-(x_5 - x)}{\sqrt{(x_5 - x)^2 + (y_5 - y)^2 + (z_5 - z)^2}} & \frac{-(y_5 - y)}{\sqrt{(x_5 - x)^2 + (y_5 - y)^2 + (z_5 - z)^2}} & \frac{-(z_5 - z)}{\sqrt{(x_5 - x)^2 + (y_5 - y)^2 + (z_5 - z)^2}} \\ \frac{-(x_6 - x)}{\sqrt{(x_6 - x)^2 + (y_6 - y)^2 + (z_6 - z)^2}} & \frac{-(y_6 - y)}{\sqrt{(x_6 - x)^2 + (y_6 - y)^2 + (z_6 - z)^2}} & \frac{-(z_6 - z)}{\sqrt{(x_6 - x)^2 + (y_6 - y)^2 + (z_6 - z)^2}} \end{bmatrix}$$

$$\sigma = 8mR = \begin{bmatrix} \sigma^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & \sigma^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sigma^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & \sigma^2 \end{bmatrix}$$