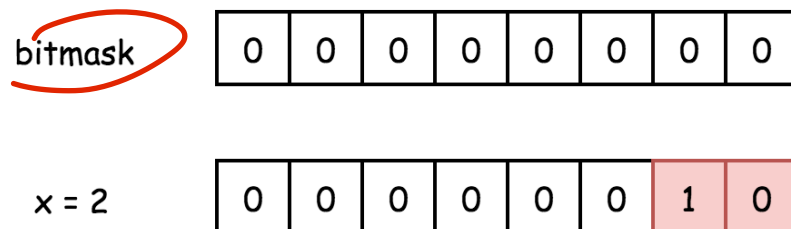


Approach 2: Two bitmasks

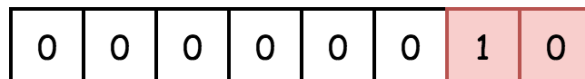
Prerequisites

This article will use two bitwise tricks, discussed in details last week :

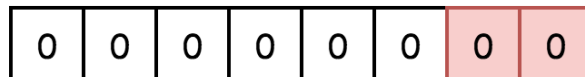
- If one builds an array bitmask with the help of XOR operator, following `bitmask ^= x` strategy, the bitmask would keep only the bits which appear odd number of times. That was discussed in details in the article Single Number II (<https://leetcode.com/articles/single-number-ii/>).



$\text{bitmask} \oplus x$,
the first appearance of x



$\text{bitmask} \oplus x \oplus x$,
the second appearance of x



- $x \& (-x)$ is a way to isolate the rightmost 1-bit, i.e. to keep the rightmost 1-bit and to set all the others bits to zero. Please refer to the article Power of Two (<https://leetcode.com/articles/power-of-two/>) for the detailed explanation.

$x \& (-x)$
保留最后一位

$x \& (-x)$
keeps the rightmost 1-bit
and sets all the other bits to 0

$x \& (\sim x) + x \& 1$
↓
1

$x = 7$	0	0	0	0	0	1	1	1
$-x = \sim x + 1$	1	1	1	1	1	0	0	1
$x \& (-x)$	0	0	0	0	0	0	1	1

$x = 6$	0	0	0	0	0	1	1	0
$-x = \sim x + 1$	1	1	1	1	1	0	1	0
$x \& (-x)$	0	0	0	0	0	0	1	0

Intuition

An interview tip. Imagine, you have a problem to identify an array element (or elements), which appears exactly given number of times. Probably, the key is to build first an array bitmask using XOR operator. Examples: In-Place Swap (leetcode.com/articles/single-number-ii/356460/Single-Number-II/324042), Single Number (<https://leetcode.com/articles/single-number/>), Single Number II (leetcode.com/articles/single-number-ii/356460/Single-Number-II/324042).

So let's create an array bitmask: $\text{bitmask} \oplus x$. This bitmask will *not* keep any number which appears twice because XOR of two equal bits results in a zero bit $a \oplus a = 0$.

Instead, the bitmask would keep only the difference between two numbers (let's call them x and y) which appear just once. The difference here it's the bits which are different for x and y .

$\text{bitmask} \oplus x \oplus y \oplus a \oplus a$
 $=$
 difference
 between x and y

bitmask

0	0	0	0	0	0	0	0
---	---	---	---	---	---	---	---

x = 1

0	0	0	0	0	0	0	1
---	---	---	---	---	---	---	---

y = 2

0	0	0	0	0	0	1	0
---	---	---	---	---	---	---	---

a = 3

0	0	0	0	0	0	1	1
---	---	---	---	---	---	---	---

a = 3

0	0	0	0	0	0	1	1
---	---	---	---	---	---	---	---

 $\text{bitmask} \oplus x \oplus y \oplus a \oplus a$

0	0	0	0	0	0	1	1
---	---	---	---	---	---	---	---

Could we extract x and y directly from this bitmask? No. Though we could use this bitmask as a marker to separate x and y.

Let's do $\text{bitmask} \& (-\text{bitmask})$ to isolate the rightmost 1-bit, which is different between x and y.
 Let's say this is 1-bit for x, and 0-bit for y.

isolate rightmost 1-bit
which is different for
x and y

bitmask

0	0	0	0	0	0	0	0
---	---	---	---	---	---	---	---

x = 1

0	0	0	0	0	0	0	1
---	---	---	---	---	---	---	---

y = 2

0	0	0	0	0	0	1	0
---	---	---	---	---	---	---	---

a = 3

0	0	0	0	0	0	1	1
---	---	---	---	---	---	---	---

a = 3

0	0	0	0	0	0	1	1
---	---	---	---	---	---	---	---

 $\text{bitmask} =$
 $\text{bitmask} \oplus x \oplus y \oplus a \oplus a$

0	0	0	0	0	0	1	1
---	---	---	---	---	---	---	---

 $\text{diff} =$
 $\text{bitmask} \& (-\text{bitmask})$

0	0	0	0	0	0	0	1
---	---	---	---	---	---	---	---

$x \oplus y = 1$
 $1 \oplus 1 \oplus 0 \oplus 1 \oplus 0$

Now let's use XOR as before, but for the new bitmask $x_bitmask$, which will contain only the numbers which have 1-bit in the position of $bitmask \& (-bitmask)$. This way, this new bitmask will contain only number x $x_bitmask = x$, because of two reasons:

- y has 0-bit in the position $bitmask \& (-bitmask)$ and hence will not enter this new bitmask.
- All numbers but x will not be visible in this new bitmask because they appear two times.

总共有 1

$x_bitmask$	0	0	0	0	0	0	0	1
$x = 1$	0	0	0	0	1	0	0	1
$y = 2$	0	0	0	0	0	0	1	0
$a = 3$	0	0	0	0	0	0	1	1
$a = 3$	0	0	0	0	0	0	1	1

$$x_bitmask = x_bitmask \oplus x \oplus a \oplus a$$

$$diff = bitmask \& (-bitmask)$$

$x_bitmask$	0	0	0	0	1	0	0	1
$diff$	0	0	0	0	0	0	0	1

Voila, x is identified. Now to identify y is simple: $y = bitmask \oplus x$.

Implementation

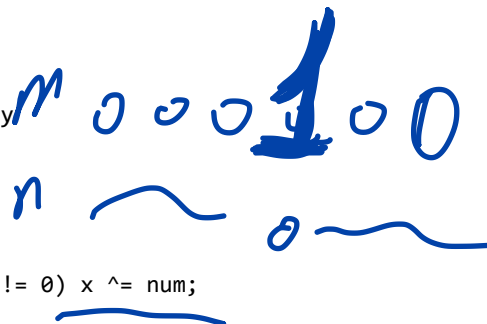
Java Python

Copy

```

1 class Solution {
2     public int[] singleNumber(int[] nums) {
3         // difference between two numbers (x and y) which were seen only once
4         int bitmask = 0;
5         for (int num : nums) bitmask ^= num;
6
7         // rightmost 1-bit diff between x and y
8         int diff = bitmask & (-bitmask);
9
10        int x = 0;
11        // bitmask which will contain only x
12        for (int num : nums) if ((num & diff) != 0) x ^= num;
13
14        return new int[]{x, bitmask^x};
15    }
16 }

```



Complexity Analysis

- Time complexity : $\mathcal{O}(N)$ to iterate over the input array.
- Space complexity : $\mathcal{O}(1)$, it's a constant space solution.

Analysis written by @liaison (<https://leetcode.com/liaison/>) and @andvary (<https://leetcode.com/andvary/>)

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