Atividade de Matemática

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1) Deduza a determinante 4x4 usando a formula

$$\det(A) = \sum_{\sigma \in S_4} \left(\prod_{j=1}^{4} (-1)^{sgn(\sigma)} \cdot a_{i,\sigma(i)} \right)$$

Resolução:

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix}$$

$$\begin{array}{ll}
+(\text{sgn}(1,2,3,4)) \cdot a_{1,1} \cdot a_{2,2} \cdot a_{3,3} \cdot a_{4,4} & (+\text{sgn}(1,2,4,3)) \cdot a_{1,1} \cdot a_{2,2} \cdot a_{3,4} \cdot a_{4,3} \\
+(\text{sgn}(1,3,2,4)) \cdot a_{1,1} \cdot a_{2,3} \cdot a_{3,2} \cdot a_{4,4} & (+\text{sgn}(1,3,4,2))) \cdot a_{1,1} \cdot a_{2,3} \cdot a_{3,4} \cdot a_{4,2} \\
+(\text{sgn}(1,4,2,3)) \cdot a_{1,1} \cdot a_{2,4} \cdot a_{3,2} \cdot a_{4,3} & (+\text{sgn}(1,4,3,2)) \cdot a_{1,1} \cdot a_{2,4} \cdot a_{3,3} \cdot a_{4,2} \\
+(\text{sgn}(2,1,3,4)) \cdot a_{1,2} \cdot a_{2,1} \cdot a_{3,3} \cdot a_{4,4} & (+\text{sgn}(2,1,4,3)) \cdot a_{1,2} \cdot a_{2,1} \cdot a_{3,4} \cdot a_{4,3} \\
+(\text{sgn}(2,3,1,4)) \cdot a_{1,2} \cdot a_{2,3} \cdot a_{3,1} \cdot a_{4,4} & (+\text{sgn}(2,3,4,1)) \cdot a_{1,2} \cdot a_{2,3} \cdot a_{3,4} \cdot a_{4,1} \\
+(\text{sgn}(2,4,1,3)) \cdot a_{1,2} \cdot a_{2,4} \cdot a_{3,1} \cdot a_{4,3} & (+\text{sgn}(2,4,3,1)) \cdot a_{1,2} \cdot a_{2,4} \cdot a_{3,3} \cdot a_{4,1} \\
+(\text{sgn}(3,1,2,4)) \cdot a_{1,3} \cdot a_{2,1} \cdot a_{3,2} \cdot a_{4,4} & (+\text{sgn}(3,1,4,2)) \cdot a_{1,3} \cdot a_{2,1} \cdot a_{3,4} \cdot a_{4,2} \\
+(\text{sgn}(3,2,1,4)) \cdot a_{1,3} \cdot a_{2,2} \cdot a_{3,1} \cdot a_{4,4} & (+\text{sgn}(3,2,4,1)) \cdot a_{1,3} \cdot a_{2,2} \cdot a_{3,4} \cdot a_{4,1} \\
+(\text{sgn}(3,4,1,2)) \cdot a_{1,4} \cdot a_{2,4} \cdot a_{3,1} \cdot a_{4,2} & (+\text{sgn}(3,4,2,1)) \cdot a_{1,3} \cdot a_{2,4} \cdot a_{3,2} \cdot a_{4,1} \\
+(\text{sgn}(4,1,2,3)) \cdot a_{1,4} \cdot a_{2,1} \cdot a_{3,2} \cdot a_{4,3} & (+\text{sgn}(4,1,3,2)) \cdot a_{1,4} \cdot a_{2,1} \cdot a_{3,3} \cdot a_{4,2} \\
+(\text{sgn}(4,2,1,3)) \cdot a_{1,4} \cdot a_{2,2} \cdot a_{3,1} \cdot a_{4,3} & (+\text{sgn}(4,2,3,1)) \cdot a_{1,4} \cdot a_{2,2} \cdot a_{3,3} \cdot a_{4,1} \\
+(\text{sgn}(4,3,1,2)) \cdot a_{1,4} \cdot a_{2,3} \cdot a_{3,1} \cdot a_{4,2} & (+\text{sgn}(4,3,2,1)) \cdot a_{1,4} \cdot a_{2,3} \cdot a_{3,2} \cdot a_{4,1}
\end{array}$$

$$\begin{aligned}
&= a_{1,1}a_{2,2}a_{3,3}a_{4,4} - a_{1,1}a_{2,2}a_{3,4}a_{4,3} - a_{1,1}a_{2,3}a_{2,3}a_{3,4}a_{4,2} \\
&\quad + a_{1,1}a_{2,4}a_{3,2}a_{4,3} - a_{1,1}a_{2,4}a_{3,3}a_{4,2} - a_{1,2}a_{2,1}a_{3,3}a_{4,4} \\
&\quad + a_{1,2}a_{2,1}a_{3,4}a_{4,3} + a_{1,2}a_{2,3}a_{3,1}a_{4,4} - a_{1,2}a_{2,3}a_{3,4}a_{4,1} \\
&\quad - a_{1,2}a_{2,4}a_{3,1}a_{4,3} + a_{1,2}a_{2,4}a_{3,3}a_{4,1} + a_{1,3}a_{2,1}a_{3,2}a_{4,4} \\
&\quad - a_{1,3}a_{2,1}a_{3,4}a_{4,2} - a_{1,3}a_{2,2}a_{3,1}a_{4,4} + a_{1,3}a_{2,2}a_{3,4}a_{4,1} \\
&\quad + a_{1,3}a_{2,4}a_{3,1}a_{4,2} - a_{1,3}a_{2,4}a_{3,2}a_{4,1} - a_{1,4}a_{2,1}a_{3,2}a_{4,3} \\
&\quad + a_{1,4}a_{2,1}a_{3,3}a_{4,2} + a_{1,4}a_{2,2}a_{3,1}a_{4,3} - a_{1,4}a_{2,2}a_{3,3}a_{4,1} \\
&\quad - a_{1,4}a_{2,3}a_{3,1}a_{4,2} + a_{1,4}a_{2,3}a_{3,2}a_{4,1}
\end{aligned}$$

2) Calcule o determinante, usando o que foi deduzido, de duas matrizes definidas pelo autor.

$$det(a) = 0$$
$$det(a) \neq 0$$

$\det = 0$

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ -1 & -2 & -3 & -4 \\ 0 & 1 & 2 & 3 \end{bmatrix}$$

Solução: Para calcular o determinante dessa matriz, podemos usar a eliminação gaussiana para transformá-la em uma matriz triangular superior e depois multiplicar os elementos da diagonal principal. No entanto, podemos ver que a segunda coluna é igual a primeira coluna multiplicada por 2, e a quarta coluna é igual a terceira coluna adicionada de duas vezes a segunda coluna. Portanto, **a matriz é linearmente dependente e seu determinante é igual a 0**.

$$\det(a) \neq 0$$

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \\ -1 & 0 & 2 & -1 \\ 2 & -1 & 3 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \\ -1 & 0 & 2 & -1 \\ 2 & -1 & 3 & 2 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 7 & 8 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 0 & 13 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Podemos calcular o determinante da matriz triangular superior multiplicando os elementos da diagonal principal, obtendo:

$$\det \begin{pmatrix} 1 & 2 & 0 & 13 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 1 \end{pmatrix} = 1 \times 1 \times 1 \times 1 = 1$$

Portanto, o determinante da matriz original é igual a 1.

3) Programe em Python.

Resolução da questão 2:

$$det(a) = 0$$

main.py

```python
import numpy as np

A = np.array([[1, 2, 3, 4], [2, 4, 6, 8], [-1, -2, -3, -4], [0, 1, 2, 3]])

det_A = np.linalg.det(A)

print(det_A)
```

>_ Console

```
0.0
```

$$det(a) \neq 0$$

main.py

```python
import numpy as np

matrix = np.array([[1, 2, 3, 4], [0, 1, 2, 3], [-1, 0, 2, -1], [2, -1, 3, 2]])

det = np.linalg.det(matrix)

print("O determinante da matriz é:", det)
```

>_ Console Shell

```
O determinante da matriz é: 29.999999999999999
```