MAG003 - Álgebra Linear

Prova P1 - 19 de outubro de 2022

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INSTRUÇÕES

1. Preencha o cabeçalho acima.
2. A prova deve ser feita com consulta a uma folha de papel a4 com o conteúdo livre.
3. O fonte desenvolvido deverá ser apenas na linguagem Haskell.
4. Responda cada questão no espaço correspondente (mesma folha)

DURACÃO DA PROVA: 3 horas e 30 minutos

1.(2.5 pontos) Considere as bases do R−espaço vetorial $\mathbb{R}^3$, $A = \{$(4, 2, 0),(1, -1, 1),(5, 3, 3)$\}$ e $B = \{$ (1, -2, 1),(1, 5, 2),(1, 0, 1) $\}$. Exiba as matrizes de mudança de base $M_{\text{B}\to A}$ e $M_{\text{A}\to B}$ .Escreva também os vetores abaixo nas bases indicadas:

• $M_{\text{A}\to B}$

$$\left[\begin{array}{ccc|c} 4 & 1 & 5 & 1 \\ 2 & \text{-1} & 3 & \text{-2} \\ 0 & 1 & 3 & 1 \end{array}\right] \quad l_1 \leftarrow \tfrac{1}{4} \cdot l_1 \left[\begin{array}{ccc|c} 1 & \frac{1}{4} & \frac{5}{4} & \frac{1}{4} \\ 2 & \text{-1} & 3 & \text{-2} \\ 0 & 1 & 3 & 1 \end{array}\right] \quad l_2 \leftarrow \tfrac{1}{2} \cdot l_2$$

$$\left[\begin{array}{ccc|c} 1 & \frac{1}{4} & \frac{5}{4} & \frac{1}{4} \\ 1 & \frac{-1}{2} & \frac{3}{2} & \text{-1} \\ 0 & 1 & 3 & 1 \end{array}\right] \quad l_2 \leftarrow l_2 - l_1 \left[\begin{array}{ccc|c} 1 & \frac{1}{4} & \frac{5}{4} & \frac{1}{4} \\ 0 & \text{-}\frac{3}{2} & \frac{1}{4} & \text{-}\frac{5}{4} \\ 0 & 1 & 3 & 1 \end{array}\right] \quad l_2 \leftarrow \text{-}\tfrac{4}{3} \cdot l_2$$

$$\left[\begin{array}{ccc|c} 1 & \frac{1}{4} & \frac{5}{4} & \frac{1}{4} \\ 0 & 1 & \frac{-1}{3} & \frac{5}{3} \\ 0 & 1 & 3 & 1 \end{array}\right] \quad l_3 \leftarrow l_3\text{-}l_2 \left[\begin{array}{ccc|c} 1 & \frac{1}{4} & \frac{5}{4} & \frac{1}{4} \\ 0 & 1 & \frac{-1}{3} & \frac{5}{3} \\ 0 & 1 & \frac{10}{3} & \frac{-2}{3} \end{array}\right] \quad l_3 \leftarrow \tfrac{3}{10} \cdot l_3$$

$$\left[\begin{array}{ccc|c} 1 & \frac{1}{4} & \frac{5}{4} & \frac{1}{4} \\ 0 & 1 & \frac{-1}{3} & \frac{5}{3} \\ 0 & 0 & 1 & \frac{-1}{5} \end{array}\right] \quad l_2 \leftarrow l_2 + \tfrac{1}{3} \cdot l_3 \left[\begin{array}{ccc|c} 1 & \frac{1}{4} & \frac{5}{4} & \frac{1}{4} \\ 0 & 1 & 0 & \frac{8}{5} \\ 0 & 0 & 1 & \frac{-1}{5} \end{array}\right] \quad l_1 \leftarrow l_1\text{-}\tfrac{5}{4} \cdot l_3$$

$$\left[\begin{array}{ccc|c} 1 & \frac{1}{4} & 0 & \frac{1}{2} \\ 0 & 1 & 0 & \frac{8}{5} \\ 0 & 0 & 1 & \frac{-1}{5} \end{array}\right] \quad l_1 \leftarrow l_1\text{-}\tfrac{1}{4} \cdot l_2 \left[\begin{array}{ccc|c} 1 & 0 & 0 & \frac{1}{10} \\ 0 & 1 & 0 & \frac{8}{5} \\ 0 & 0 & 1 & \frac{-1}{5} \end{array}\right] S = \{(x = \tfrac{1}{10}, y = \tfrac{8}{5}, z = \tfrac{-1}{5})\}$$

$$\left[\begin{array}{ccc|c} 4 & 1 & 5 & 1 \\ 2 & -1 & 3 & 5 \\ 0 & 1 & 3 & 2 \end{array}\right] l_1 \leftarrow \frac{1}{4} \cdot l_1 \left[\begin{array}{ccc|c} 1 & \frac{1}{4} & \frac{5}{4} & \frac{1}{4} \\ 2 & -1 & 3 & -2 \\ 0 & 1 & 3 & 2 \end{array}\right] l_2 \leftarrow \frac{1}{2} \cdot l_2$$

$$\left[\begin{array}{ccc|c} 1 & \frac{1}{4} & \frac{5}{4} & \frac{1}{4} \\ 2 & -\frac{1}{2} & \frac{3}{2} & \frac{5}{2} \\ 0 & 1 & 3 & 2 \end{array}\right] l_2 \leftarrow l_2\text{-}1\cdot \mathrm{l}_1 \left[\begin{array}{ccc|c} 1 & \frac{1}{4} & \frac{5}{4} & \frac{1}{4} \\ 1 & -\frac{3}{4} & \frac{1}{4} & \frac{9}{4} \\ 0 & 1 & 3 & 2 \end{array}\right] l_2 \leftarrow -\frac{4}{3} \cdot l_2$$

$$\left[\begin{array}{ccc|c} 1 & \frac{1}{4} & \frac{5}{4} & \frac{1}{4} \\ 0 & 1 & -\frac{1}{3} & -3 \\ 0 & 1 & 3 & 2 \end{array}\right] l_3 \leftarrow l_3\text{-}1\cdot l_2 \left[\begin{array}{ccc|c} 1 & \frac{1}{4} & \frac{5}{4} & \frac{1}{4} \\ 0 & 1 & -\frac{1}{3} & -3 \\ 0 & 1 & \frac{10}{3} & 5 \end{array}\right] l_3 \leftarrow \frac{3}{10} \cdot l_3$$

$$\left[\begin{array}{ccc|c} 1 & \frac{1}{4} & \frac{5}{4} & \frac{1}{4} \\ 0 & 1 & -\frac{1}{3} & -3 \\ 0 & 1 & 1 & \frac{3}{2} \end{array}\right] l_2 \leftarrow l_2 + \frac{1}{3} \cdot l_3 \left[\begin{array}{ccc|c} 1 & \frac{1}{4} & \frac{5}{4} & \frac{1}{4} \\ 0 & 1 & 0 & -\frac{5}{2} \\ 0 & 0 & 1 & \frac{3}{2} \end{array}\right] l_1 \leftarrow l_1\text{-}\frac{5}{4} \cdot l_3$$

$$\left[\begin{array}{ccc|c} 1 & \frac{1}{4} & 0 & -\frac{13}{8} \\ 0 & 1 & 0 & -\frac{5}{2} \\ 0 & 0 & 1 & \frac{3}{2} \end{array}\right] l_1 \leftarrow l_1\text{-}\frac{1}{4} \cdot l_2 \left[\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -\frac{5}{2} \\ 0 & 0 & 1 & \frac{3}{2} \end{array}\right] \mathbf{S} = \{(x = -1, y = -\frac{5}{2}, z = \frac{3}{2})\}$$

$$\left[\begin{array}{ccc|c} 4 & 1 & 5 & 1 \\ 2 & -1 & 3 & 0 \\ 0 & 1 & 3 & 1 \end{array}\right] l_1 \leftarrow \frac{1}{4} \cdot l_1 \left[\begin{array}{ccc|c} 1 & \frac{1}{4} & \frac{5}{4} & \frac{1}{4} \\ 2 & -1 & 3 & 0 \\ 0 & 1 & 3 & 1 \end{array}\right] l_2 \leftarrow \frac{1}{2} \cdot l_2$$

$$\left[\begin{array}{ccc|c} 1 & \frac{1}{4} & \frac{5}{4} & \frac{1}{4} \\ 2 & \frac{-1}{2} & \frac{3}{2} & 0 \\ 0 & 1 & 3 & 1 \end{array}\right] l_2 \leftarrow l_2 \ \text{-}1 \cdot l_1 \left[\begin{array}{ccc|c} 1 & \frac{1}{4} & \frac{5}{4} & \frac{1}{4} \\ 2 & \frac{-3}{4} & \frac{1}{4} & -\frac{1}{4} \\ 0 & 1 & 3 & 1 \end{array}\right] l_2 \leftarrow \frac{-4}{3} \cdot l_2$$

$$\left[\begin{array}{ccc|c} 1 & \frac{1}{4} & \frac{5}{4} & \frac{1}{4} \\ 0 & 1 & -\frac{1}{3} & \frac{1}{3} \\ 0 & 1 & 3 & 1 \end{array}\right] l_3 \leftarrow l_3\text{-}1\cdot l_2 \left[\begin{array}{ccc|c} 1 & \frac{1}{4} & \frac{5}{4} & \frac{1}{4} \\ 0 & 1 & -\frac{1}{3} & \frac{1}{3} \\ 0 & 1 & \frac{10}{3} & \frac{2}{3} \end{array}\right] l_3 \leftarrow \frac{3}{10} \cdot l_3$$

$$\left[\begin{array}{ccc|c} 1 & \frac{1}{4} & \frac{5}{4} & \frac{1}{4} \\ 0 & 1 & -\frac{1}{3} & \frac{1}{3} \\ 0 & 0 & 1 & \frac{1}{5} \end{array}\right] l_2 \leftarrow l_2 + \frac{1}{3} \cdot l_3 \left[\begin{array}{ccc|c} 1 & \frac{1}{4} & \frac{5}{4} & \frac{1}{4} \\ 0 & 1 & 0 & \frac{2}{5} \\ 0 & 0 & 1 & \frac{1}{5} \end{array}\right] l_1 \leftarrow l_1\text{-}\frac{5}{4} \cdot l_3$$

$$\left[\begin{array}{ccc|c} 1 & \frac{1}{4} & 0 & 0 \\ 0 & 1 & 0 & \frac{2}{5} \\ 1 & 0 & 1 & \frac{1}{5} \end{array}\right] l_1 \leftarrow l_1\text{-}\frac{1}{4} \cdot l_2 \left[\begin{array}{ccc|c} 1 & 0 & 0 & -\frac{1}{10} \\ 0 & 1 & 0 & \frac{2}{5} \\ 0 & 0 & 1 & \frac{1}{5} \end{array}\right] \mathbf{S} = \{(x = -\frac{1}{10}, y = \frac{2}{5}, z = \frac{1}{5})\}$$

$\bullet M_{\mathrm{B}\to\, A}$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 4 \\ 1 & 5 & 0 & 2 \\ 1 & 2 & 1 & 0 \end{array}\right] \quad l_2 \leftarrow l_2 \cdot \text{-}\tfrac{1}{2} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 4 \\ 1 & \text{-}\frac{5}{2} & 0 & \text{-}1 \\ 1 & 2 & 1 & 0 \end{array}\right] \quad l_2 \leftarrow l_2 - l_1$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 4 \\ 0 & \text{-}\frac{7}{2} & -1 & \text{-}5 \\ 1 & 2 & 1 & 0 \end{array}\right] \quad l_3 \leftarrow l_3 - l_1 \left[\begin{array}{ccc|c} 1 & 1 & 1 & 4 \\ 0 & \text{-}\frac{7}{2} & -1 & \text{-}5 \\ 0 & 1 & 0 & \text{-}4 \end{array}\right] \quad l_2 \leftarrow \text{-}\tfrac{2}{7} \cdot l2$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 4 \\ 0 & 1 & \frac{2}{7} & \frac{10}{7} \\ 0 & 1 & 0 & \text{-}4 \end{array}\right] \quad l_3 \leftarrow \text{-}\tfrac{7}{2} \cdot l_3 \left[\begin{array}{ccc|c} 1 & 1 & 1 & 4 \\ 0 & 1 & \frac{2}{7} & \frac{10}{7} \\ 0 & 0 & 1 & 19 \end{array}\right] \quad l_2 \leftarrow l_2 \text{ -}\tfrac{2}{7} \cdot l_3$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 4 \\ 0 & 1 & 0 & \text{-}4 \\ 0 & 0 & 1 & 19 \end{array}\right] \quad l_1 \leftarrow l_1 - l_3 \left[\begin{array}{ccc|c} 1 & 1 & 0 & \text{-}15 \\ 0 & 1 & 0 & \text{-}4 \\ 0 & 0 & 1 & 19 \end{array}\right] l_1 \leftarrow l_1\text{-}l_2$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 11 \\ 0 & 1 & 0 & \text{-}4 \\ 0 & 0 & 1 & 19 \end{array}\right] \mathbf{S} = \{(x = 11, y = \text{-}4, z = 19)\}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 1 & 5 & 0 & \text{-}1 \\ 1 & 2 & 1 & 1 \end{array}\right] l_2 \leftarrow l_2\text{-}1 \cdot l_1 \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 4 & -1 & \text{-}2 \\ 1 & 2 & 1 & 1 \end{array}\right] l_3 \leftarrow l_3\text{-}1 \cdot l_1$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 4 & 1 & \text{-}2 \\ 0 & 1 & 0 & 0 \end{array}\right] l_2 \leftarrow \tfrac{1}{4} \cdot l_2 \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1 & \text{-}\frac{1}{4} & \text{-}\frac{1}{2} \\ 0 & 1 & 0 & 0 \end{array}\right] l_3 \leftarrow l_3\text{-}1 \cdot l_2$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1 & \text{-}\frac{1}{4} & \text{-}\frac{1}{2} \\ 0 & 0 & \frac{1}{4} & \frac{1}{2} \end{array}\right] l_3 \leftarrow 4 \cdot l_3 \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1 & \text{-}\frac{1}{4} & \text{-}\frac{1}{2} \\ 0 & 0 & 1 & 2 \end{array}\right] l_2 \leftarrow l_2 + \tfrac{1}{4} \cdot l_3$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 \end{array}\right] l_1 \leftarrow l_1\text{-}1 \cdot l_3 \left[\begin{array}{ccc|c} 1 & 0 & 0 & \text{-}1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 \end{array}\right] \mathbf{S} = \{(x = \text{-}1, y = 0, z = 2)\}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 5 \\ 1 & 5 & 0 & 3 \\ 1 & 2 & 1 & 3 \end{array}\right] l_2 \leftarrow l_2\text{-}1 \cdot l_1 \left[\begin{array}{ccc|c} 1 & 1 & 1 & 5 \\ 0 & 4 & \text{-}1 & \text{-}2 \\ 1 & 2 & 1 & 3 \end{array}\right] l_3 \leftarrow l_3\text{-}1 \cdot l_1$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 5 \\ 0 & 4 & \text{-}1 & \text{-}2 \\ 0 & 1 & 0 & \text{-}2 \end{array}\right] l_2 \leftarrow \tfrac{1}{4} \cdot l_2 \left[\begin{array}{ccc|c} 1 & 1 & 1 & 5 \\ 0 & 1 & \text{-}\frac{1}{4} & \text{-}\frac{1}{2} \\ 0 & 1 & 0 & \text{-}2 \end{array}\right] l_3 \leftarrow l_3\text{-}1 \cdot l_2$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 5 \\ 0 & 1 & \text{-}\frac{1}{4} & \text{-}\frac{1}{2} \\ 0 & 0 & \frac{1}{4} & \text{-}\frac{3}{2} \end{array}\right] l_3 \leftarrow 4 \cdot l_3 \left[\begin{array}{ccc|c} 1 & 1 & 1 & 5 \\ 0 & 1 & \text{-}\frac{1}{4} & \text{-}\frac{1}{2} \\ 0 & 0 & 1 & \text{-}6 \end{array}\right] l_2 \leftarrow l_2 + \tfrac{1}{4} \cdot l_3$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 5 \\ 0 & 1 & 0 & \text{-}2 \\ 0 & 0 & 1 & \text{-}6 \end{array}\right] l_1 \leftarrow l_1\text{-}1 \cdot l_3 \left[\begin{array}{ccc|c} 1 & 1 & 0 & 11 \\ 0 & 1 & 0 & \text{-}2 \\ 0 & 0 & 1 & \text{-}6 \end{array}\right] l_1 \leftarrow l_1\text{-}1 \cdot l_2$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 13 \\ 0 & 1 & 0 & \text{-}2 \\ 0 & 0 & 1 & \text{-}6 \end{array}\right] \mathbf{S} = \{(x = 11, y = \text{-}2, z = \text{-}6)\}$$

- $\mathbf{v} = (0, 1, 2)_A$ *em* B
- $\mathbf{v} = (1, 3, -1)_B$ *em* A

2. (2.5 pontos) Considere os conjuntos $S = \{$(1, 1, 1, 1, 1),(2, 0, -1, 1, 3),(3, 1, 0, 2, 4),(2, 2, 5, 8, -1),(0, 1, 0, 2, 3)$\}$.

(a) S é li ou ld?

Resposta: É ld, pois não há um "C"(constante) que multiplicada a um vetor resulte na combinação linear de outros vetores.

(b) S forma uma base do $\mathbb{R}$-espaço vetorial $\mathbb{R}^5$?

Resposta: Um conjunto deve ser linermente independente e o conjunto S desrespeita esse axioma das bases de um espaço vetorial.

3. (2.5 pontos)

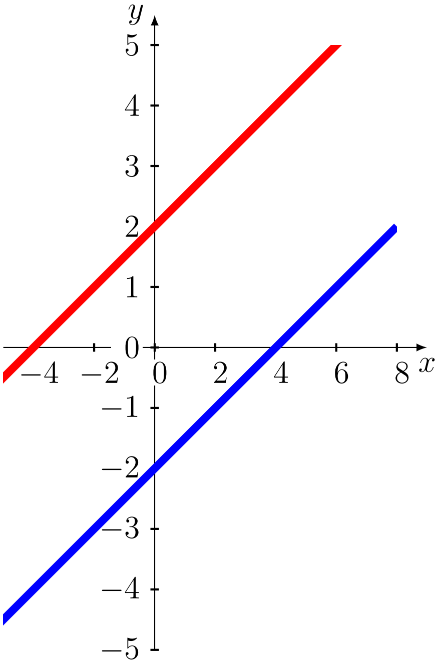
•Considere o conjunto $\mathbb{W} = \{$(x,y,z,w,t,u) | $x, y, z, w, t, u \in \mathbb{R} \wedge x+y+w+z+t+u$ $= 0 \wedge$ w + t − x = 0$\} \subseteq \mathbb{R}^6$. Mostre que o conjunto $\mathbb{W}$ é um subespaço do $\mathbb{R}$-espaço vetorial $\mathbb{R}^6$.

•O conjunto $\mathbb{W} = \{$(x, y, z)|x, y $\in$ R x − z = 1 y + x = 0$\}$ é um espaço vetorial de $\mathbb{R}^3$? Esboce graficamente W.

Resposta: $0 \in \mathbb{W}$

"(0,0,0)", mas 0−0 (x−z) $\neq$ 1, ao contrário da condição do conjunto. Logo, não é subespaço de $\mathbb{R}^3$.

Graficamente:



Invente seu subespaço vetorial qualquer $\mathbb{R}^n$ com n $\geq$ 2 . Mostre que o conjunto apresentado é um subespaço vetorial. Não vale usar nenhum exemplo da aula ou da prova.

Resposta:

$A = (a, b, c)|a + b + c = 0)$

•Axioma do elemento neutro:

(0,0,0) = (0+0+0) = 0

•Associatividade v + w $\in A$

v = (a1+b1+c1) w = (a2+b2+c2)

=(a1+a2,b1+b2,c1+c2)

a1+a2+b1+b2+c1+c2=0

$Logo$,v+w$\in A$

•Multiplicação por escalar:

k $\cdot$ v

= k $\cdot$ (a1,b1,c1)

=(k$\cdot$a1,k$\cdot$b1,k$\cdot$c1)

= $k$$\cdot$a1+k$\cdot$b1+k$\cdot$c1 =$k\cdot$ (a1+b1+c1) = 0

k $\cdot$ 0 = 0

Logo, A é subespaço vetorial de $\mathbb{R}^3$

4. Mostre que o conjunto $\{(1, 1, 1, 1, 0, 1, 1), (1, 0, 1, 1, 1, 1, 0), (2, 2, 1, 1, 1, 1, 1), (1, 0, 0, 1, 2, 1, 1),(2, 0, 2, 0, 2, 0, 2), (1, 1, 1, 1, 1, 1, 1), (3, 0, 2, 0, 2, 1, 2)\}$ forma uma base para o $\mathbb{R}$-espaço vetorial $\mathbb{R}^7$. Escreva o vetor $(0, 1, 1, 1, 1, 0, 1)$ nesta base.