

## Practical 4 (weeks 7 - 8)

### Theory Questions

1. Symbolize the following proposition and discuss the truth.

1. Everyone has black hair.
2. Some people boarded the moon.
3. No one has boarded Jupiter
4. Students studying in the US are not necessarily Asians.

*your answer here...*

1. Since the individual domain is not proposed in this question, the total individual domain is used.
  - assume  $M(x)$  : x is a people.
  - assume  $F(x)$  : x with black hair.
  - Propositional symbolization as  $x(M(x) \Rightarrow F(x))$  .
  - assume a is a girl with blonde hair, then  $M(a)$  is true,  $F(a)$  is false.
  - so  $M(a) \Rightarrow F(a)$  is false,
  - Therefore Propositional is false.
2. assume that,  $G(x)$ : x boarded the moon.
  - Propositional symbolization as  $x(M(x) \wedge G(x))$ .
  - assume a is an American who completed the Apollo program on the moon in 1969.
  - so  $M(a) \wedge G(a)$  is true.
  - Therefore Propositional is true.
3. assume that,  $J(x)$ : x boarded the Jupiter.
  - Propositional symbolization as  $\neg x(M(x) \wedge J(x))$ .
  - assume anyone is a, then  $M(a) \wedge J(a)$  of all is false.
  - Therefore  $x(M(x) \wedge J(x))$  is false.
  - so Propositional is true.
4. assume  $U(x)$  : x is a student studying in the US.
  - $A(x)$  : x is Asian.
  - Propositional symbolization as  $\neg x(U(x) \Rightarrow A(x))$ .
  - Therefore Propositional is true.

2. Judge the following formula, which is tautology? What is the contradiction?

1.  $\forall x F(x) \Rightarrow (\exists x \exists y G(x, y)) \Rightarrow \forall x F(x)$
2.  $\neg (\forall x F(x) \Rightarrow \exists y G(y)) \wedge \exists y G(y)$
3.  $\forall x (F(x) \Rightarrow G(y))$

*your answer here...*

1. tautology
2. contradiction
3. neither tautology nor contradiction

3. Which of the following are correct?

1.  $\text{False} \models \text{True}$ .
2.  $(A \wedge B) \models (A \Leftrightarrow B)$ .
3.  $(A \wedge B) \Rightarrow C \models (A \Rightarrow C) \vee (B \Rightarrow C)$ .
4.  $(A \vee B) \wedge (\neg C \vee \neg D \vee E) \models (A \vee B)$ .
5.  $(A \vee B) \wedge (\neg C \vee \neg D \vee E) \models (A \vee B) \wedge (\neg D \vee E)$ .

*your answer here...*

1. True.
2. True.
3. True.
4. True.
5. False.

4. Conjunctive normal form. link: <https://baike.baidu.com/item/%E5%90%88%E5%8F%96%E8%8C%83%E5%BC%8F/2459360>

1. Obtaining conjunctive paradigm:  $P \wedge (Q \Rightarrow R) \Rightarrow S$

**Basic steps to find a conjunctive normal form.**

2. Cut redundant connectives, Reserved  $\{\vee, \wedge, \neg\}$
3. Move or remove the negation  $\sim$
4. distribution rates

*your answer here...*

- $P \wedge (Q \Rightarrow R) \Rightarrow S$
- $= \neg(P \wedge (\neg Q \vee R)) \vee S$
- $= \neg P \vee \neg(\neg Q \vee R) \vee S$
- $= \neg P \vee (\neg \neg Q \wedge \neg R) \vee S$
- $= \neg P \vee (Q \wedge \neg R) \vee S$
- $= \neg P \vee S \vee (Q \wedge \neg R)$
- $= (\neg P \vee S \vee Q) \wedge (\neg P \vee S \vee \neg R)$

5. Arithmetic assertions can be written in first-order logic with the predicate symbol  $<$ , the function symbols  $+$  and  $\times$ , and the constant symbols 0 and 1. Additional predicates can also be defined with biconditionals. (Chapter 8.20)

1. Represent the property "x is an even number."
2. Represent the property "x is prime."
3. Goldbach's conjecture is the conjecture (unproven as yet) that every even number is equal to the sum of two primes. Represent this conjecture as a logical sentence.

*your answer here...*

1.  $\forall x \text{ Even}(x) \Leftrightarrow \exists y \ x=y+y.$
2.  $\forall x \text{ Prime}(x) \Leftrightarrow \forall y, z \ x=y \times z \Rightarrow y=1 \vee z=1.$
3.  $\forall x \text{ Even}(x) \Rightarrow \exists y, z \text{ Prime}(y) \wedge \text{Prime}(z) \wedge x=y+z.$