PROCEEDINGS OF THE ROYAL SOCIETY A | MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

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*Proc. R. Soc. Lond. A* 1955 **231**, 505-514
doi: 10.1098/rspa.1955.0191

# The influence of solid boundaries upon aerodynamic sound

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(*Communicated by M. J. Lighthill, F.R.S.—Received* 13 *April* 1955)

An extension is made to Lighthill's general theory of aerodynamic sound, so as to incorporate the influence of solid boundaries upon the sound field. This influence is twofold, namely (i) reflexion and diffraction of the sound waves at the solid boundaries, and (ii) a resultant dipole field at the solid boundaries which are the limits of Lighthill's quadrupole distribution. It is shown that these effects are exactly equivalent to a distribution of dipoles, each representing the force with which unit area of solid boundary acts upon the fluid.

A dimensional analysis shows that the intensity of the sound generated by the dipoles should at large distances $x$ be of the general form $I \propto \rho_0 U_0^6 a_0^{-3} L^2 x^{-2}$, where $U_0$ is a typical velocity of the flow, $L$ is a typical length of the body, $a_0$ is the velocity of sound in fluid at rest and $\rho_0$ is the density of the fluid at rest. Accordingly, these dipoles should be more efficient generators of sound than the quadrupoles of Lighthill's theory if the Mach number is small enough. It is shown that the fundamental frequency of the dipole sound is one half of the frequency of the quadrupole sound.

## 1. Introduction

The subject of aerodynamic sound is one which has claimed a great deal of attention during the past two or three years. Previously, much work had been done upon various matters concerning frequencies, such as showing that any frequencies in a fluid flow are equal to those of the sound generated, but the problem of estimating the intensity of the sound generated by a given flow was not seriously investigated.

This omission has been rectified by Lighthill (1952) in his paper 'On sound generated aerodynamically'. In this paper he considers the exact equations of motion of a fluid, and compares these with the equations of sound propagation in a medium at rest. He shows that the exact equations may be written in the form

$$\frac{\partial^2 \rho}{\partial t^2} - a_0^2 \nabla^2 \rho = \frac{\partial^2}{\partial x_i \partial x_j}(T_{ij}), \tag{1·1}$$

where $$T_{ij} = \rho v_i v_j + p_{ij} - a_0^2 \rho \delta_{ij}.$$

Here $\rho$ = density, $p_{ij}$ = compressive stress tensor, $a_0$ = velocity of sound in fluid at rest, $v_i$ = component of velocity in direction $x_i$ $(i = 1, 2, 3)$. Physically, this means that sound is generated by a fluid flow exactly as in a uniform medium at rest which is acted upon by externally applied fluctuating stresses. The equation (1·1) has been solved by Lighthill in the form

$$\rho - \rho_0 = \frac{1}{4\pi a_0^2} \frac{\partial^2}{\partial x_i \partial x_j} \int_v \frac{T_{ij}\left(\mathbf{y}, t - \dfrac{|\mathbf{x}-\mathbf{y}|}{a_0}\right)}{|\mathbf{x}-\mathbf{y}|} \, d\mathbf{y}. \tag{1·2}$$

It follows that if we regard the flow as known it is possible by means of equation (1·2) to evaluate the sound generated.

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This process has been carried through in a number of fundamental cases which adequately illustrate the value of the theory. In a second paper Lighthill (1954) uses his general theory to consider the sound generated by turbulence, and in particular makes a detailed theoretical examination of the sound-field of a turbulent jet. The predictions of the theory are well borne out by experiment both for subsonic and supersonic jets. In that paper Lighthill makes use of the work of Proudman (1952), who has estimated the sound generated by unit volume of isotropic turbulence having no mean flow. Use has also been made of the theory in considering jets at low subsonic speeds, and this has proved valuable in the study of the related phenomenon of edge-tones (Curle 1953).

All the work described above, however, refers to applications of the theory to flows in which there are no solid boundaries near, or in which it is possible to neglect the effects of such boundaries. Now it has already been pointed out by Lighthill (1954) that solid boundaries might well play an important role in sound production in certain cases, even when they remain rigid, as for example by the fluctuating lift on a rigid circular cylinder in a uniform stream. The purpose of this paper is to consider in detail the part played by the solid boundaries which are present in a given flow. One can see physically that solid boundaries could make their presence felt in two ways:

(i) The sound generated by the quadrupoles of Lighthill's theory will be reflected and diffracted by the solid boundaries.

(ii) The quadrupoles will no longer be distributed over the whole of space, but only throughout the region external to the solid boundaries, and it seems that there might be a resultant distribution of dipoles (or even sources) at the boundaries. Dipoles are especially likely, since in acoustics they correspond to externally applied forces, and such forces are present between the fluid and the solid boundary. This expectation will be confirmed in what follows.

The equation (1·2) no longer represents the solution of the inhomogeneous wave equation (1·1). In § 2 below the most general solution of equation (1·1) is considered, thus taking into account all such effects as those foreshadowed above. It is found that this solution consists of the sum of the doubly-differentiated volume integral solution of Lighthill's theory and a surface integral over all the solid boundaries. This surface integral includes forces (corresponding to (i) and (ii) above) (*a*) due to the impact of sound waves from the quadrupoles on the solid surface, and (*b*) due to the hydrodynamic flow itself, including turbulence, etc. Part (*a*) is exactly the diffracted wave. This surface integral therefore represents the modifications to Lighthill's theory necessitated by the presence of solid boundaries, and a detailed examination shows it to be equivalent exactly to the sound generated by a distribution of dipoles representing the fluctuating forces with which the solid boundaries act on the fluid. This result is shown to hold provided either the solid boundaries are rigid or each vibrates in its own plane.

The sound field may thus be regarded as being derived from two distinct origins, namely (i) the quadrupole field which represents, as in Lighthill's theory, the fluctuating applied stresses, and (ii) a dipole field which represents the fluctuating force with which the solid boundaries act on the fluid. A dimensional analysis is used to obtain a general idea of the relative magnitudes of the intensities of sound generated by quadrupoles and dipoles, and it is shown that the dipoles become increasingly important as the Mach number is decreased. It is also shown that the fundamental frequency of the sound generated by the dipoles is one half of that generated by the quadrupoles. This is because the quadrupole strength per unit volume, $\rho_0 v_i v_j$, being essentially proportional to (velocity)$^2$, will have double the frequency of the fluctuating velocity. On the other hand, the fluctuating force exerted on the fluid at the solid boundaries will have the same frequency as the velocity fluctuations. Physically this means that in an antisymmetrical oscillation (of a wake or jet, for example) the quadrupole strength per unit volume is the same when a vortex is formed at the upper or lower side of the jet or wake, but the fluctuating force exerted at the solid boundaries will be of opposite sign in these two cases.

Phillips (1955) has considered a general application of the theory to turbulent flows, relating the motion of a fluid about a body to the stress-distribution over the surface, and applying this in several cases to derive properties of the surface noise radiation field.

## 2. General theory of aerodynamic sound

In his well-known paper 'On sound generated aerodynamically', Lighthill (1952) considers the exact equations of motion of a fluid under no external forces. He points out that these may be written in the very convenient form

$$\left.\begin{aligned}\frac{\partial \rho}{\partial t}+\frac{\partial}{\partial x_i}(\rho v_i) &= 0,\\ \frac{\partial}{\partial t}(\rho v_i)+\frac{\partial}{\partial x_j}(\rho v_i v_j+p_{ij}) &= 0.\end{aligned}\right\} \tag{2·1}$$

The first of these equations is the usual Cartesian form of the equation of continuity. The second is the Reynolds form of the momentum equation, which may be derived from the better known form by addition of a multiple of the continuity equation. More fundamentally it may be derived by considering the flow of momentum across surfaces fixed in the fluctuating fluid flow. By elimination of $(\rho v_i)$ between the two equations (2·1) it is found that

$$\frac{\partial^2 \rho}{\partial t^2} = \frac{\partial^2}{\partial x_i \partial x_j}(\rho v_i v_j + p_{ij}),$$

and hence

$$\left.\begin{aligned}\frac{\partial^2 \rho}{\partial t^2} - a_0^2 \nabla^2 \rho &= \frac{\partial^2}{\partial x_i \partial x_j}(T_{ij}),\\ T_{ij} &= \rho v_i v_j + p_{ij} - a_0^2 \rho \delta_{ij}.\end{aligned}\right\} \tag{2·2}$$

where

Lighthill shows that for flows at low Mach numbers it is sufficient to put

$$T_{ij} \approx \rho_0 v_i v_j. \tag{2·3}$$

Equation (2·2) is the inhomogeneous wave equation, and the most general solution of this is well known (Stratton 1941). The solution is

$$\rho - \rho_0 = \frac{1}{4\pi a_0^2}\int_V \frac{\partial^2 T_{ij}}{\partial y_i \partial y_j}\frac{\mathrm{d}\mathbf{y}}{|\mathbf{x}-\mathbf{y}|} + \frac{1}{4\pi}\int_S \left\{\frac{1}{r}\frac{\partial \rho}{\partial n} + \frac{1}{r^2}\frac{\partial r}{\partial n}\rho + \frac{1}{a_0 r}\frac{\partial r}{\partial n}\frac{\partial \rho}{\partial t}\right\}\mathrm{d}S(\mathbf{y}). \tag{2·4}$$

In this equation all the quantities $\frac{\partial^2 T_{ij}}{\partial y_i \partial y_j}$, $\frac{\partial \rho}{\partial n}$, $\rho$, $\frac{\partial \rho}{\partial t}$, are taken at retarded times $t - r/a_0$, where $r = |\mathbf{x}-\mathbf{y}|$, and $\mathbf{n}$ is the outward normal from the fluid. The first integral is taken over the total volume $V$ external to the solid boundaries, and the second integral is taken over the surface $S$ of the solid boundaries.

In the absence of solid boundaries the surface integral will not appear and only the volume integral remains, i.e. the retarded potential solution

$$\rho - \rho_0 = \frac{1}{4\pi a_0^2} \int_V \frac{\partial^2 T_{ij}}{\partial y_i \partial y_j} \frac{\mathrm{d}\mathbf{y}}{|\mathbf{x}-\mathbf{y}|}. \tag{2·5}$$

It is seen that the sound is radiated as if by a distribution of quadrupoles of strength $T_{ij}$ per unit volume in a medium at rest. This is not quite the form in which Lighthill expressed his result, but his form is easily obtained by considering the quadrupole field as four separate source fields which come infinitely close together. This immediately leads to the alternative form of (2·5),

$$\rho - \rho_0 = \frac{1}{4\pi a_0^2} \frac{\partial^2}{\partial x_i \partial x_j} \int_V \frac{T_{ij}\left(\mathbf{y}, t - \dfrac{|\mathbf{x}-\mathbf{y}|}{a_0}\right)}{|\mathbf{x}-\mathbf{y}|} \mathrm{d}\mathbf{y}. \tag{2·6}$$

This integral can be simplified in certain cases. If $|\mathbf{x}|$ is sufficiently large, so that $\mathbf{x}$ lies in the radiation field of each quadrupole; that is $|\mathbf{x}|$ is at least a few acoustic wavelengths, then (2·6) may be simplified to

$$\rho - \rho_0 = \frac{1}{4\pi a_0^2} \int_V \frac{(x_i - y_i)(x_j - y_j)}{|\mathbf{x}-\mathbf{y}|^3} \frac{1}{a_0^2} \frac{\partial^2}{\partial t^2} T_{ij}\left(y, t - \frac{|\mathbf{x}-\mathbf{y}|}{a_0}\right) \mathrm{d}\mathbf{y}. \tag{2·7}$$

Furthermore, if $|\mathbf{x}|$ is large compared with even the dimensions of the flow then $|\mathbf{x}| \gg |\mathbf{y}|$ and (2·7) reduces to

$$\rho - \rho_0 = \frac{1}{4\pi a_0^4} \frac{x_i x_j}{x^3} \frac{\partial^2}{\partial t^2} \int_V T_{ij}\left(\mathbf{y}, t - \frac{|\mathbf{x}-\mathbf{y}|}{a_0}\right) \mathrm{d}\mathbf{y}. \tag{2·8}$$

When there are solid boundaries this position is somewhat modified. Equation (2·4), the required solution, differs from Lighthill's solution in two respects. First, there is the extra surface integral,

$$\frac{1}{4\pi} \int_S \left\{ \frac{1}{r} \frac{\partial \rho}{\partial n} + \frac{1}{r^2} \frac{\partial r}{\partial n} \rho + \frac{1}{a_0 r} \frac{\partial r}{\partial n} \frac{\partial \rho}{\partial t} \right\} \mathrm{d}S(\mathbf{y}),$$

representing the effect upon the hydrodynamic flow itself of the solid boundaries. Secondly, the volume integral takes a form different from that of Lighthill, (2·6), the difference representing the effect of the impact (on the solid surface) of sound waves from the quadrupoles. It must be noted that the presence of fixed solid boundaries invalidates the idea of regarding the quadrupole distribution as the limiting case of four source distributions. However, this physical idea is exactly equivalent to the mathematical process of twice applying the divergence theorem, and carrying out this process is still permissible. We have

$$\int_V \frac{\partial^2 T_{ij}}{\partial y_i \partial y_j}\frac{\mathrm{d}\mathbf{y}}{r} - \frac{\partial}{\partial x_i}\int_V \frac{\partial T_{ij}}{\partial y_j}\frac{\mathrm{d}\mathbf{y}}{r} = \int_V \frac{\partial}{\partial y_i}\left[\frac{\partial T_{ij}}{\partial y_j}\Big/ r\right]\mathrm{d}\mathbf{y}$$
$$= \int_S l_i \frac{\partial T_{ij}}{\partial y_j}\frac{\mathrm{d}S(\mathbf{y})}{r}, \qquad (2\cdot 9)$$

where $l_i$ are the direction cosines of the outward normal from the fluid, i.e.,

$$(l_1, l_2, l_3) = \mathbf{n}. \qquad (2\cdot 10)$$

By repeating this a second time we find

$$\int_V \frac{\partial T_{ij}}{\partial y_j}\frac{\mathrm{d}\mathbf{y}}{r} - \frac{\partial}{\partial x_j}\int_V T_{ij}\frac{\mathrm{d}\mathbf{y}}{r} = \int_S l_j T_{ij}\frac{\mathrm{d}S(\mathbf{y})}{r}. \qquad (2\cdot 11)$$

From (2·9) and (2·11) it follows that

$$\int_V \frac{\partial^2 T_{ij}}{\partial y_i \partial y_j}\frac{\mathrm{d}\mathbf{y}}{r} = \frac{\partial^2}{\partial x_i \partial x_j}\int_V \frac{T_{ij}\left(\mathbf{y}, t-\frac{r}{a_0}\right)}{r}\mathrm{d}\mathbf{y} + \frac{\partial}{\partial x_i}\int_S l_j T_{ij}\left(\mathbf{y}, t-\frac{r}{a_0}\right)\frac{\mathrm{d}S(\mathbf{y})}{r} + \int_S l_i \frac{\partial T_{ij}}{\partial y_j}\frac{\mathrm{d}S(\mathbf{y})}{r}. \qquad (2\cdot 12)$$

The surface integral which appears explicitly in (2·4) may also be transformed into a more convenient form. We have

$$\int_S \left\{\frac{1}{r}\frac{\partial \rho}{\partial n} + \frac{1}{r^2}\frac{\partial r}{\partial n}\rho + \frac{1}{a_0 r}\frac{\partial r}{\partial n}\frac{\partial \rho}{\partial t}\right\}\mathrm{d}S(\mathbf{y})$$
$$= \int_S l_i \left\{\frac{1}{r}\frac{\partial \rho}{\partial y_i} + \frac{1}{r^2}\frac{\partial r}{\partial y_i}\rho + \frac{1}{a_0 r}\frac{\partial r}{\partial y_i}\frac{\partial \rho}{\partial t}\right\}\mathrm{d}S(\mathbf{y})$$
$$= \int_S l_i \frac{1}{r}\frac{\partial}{\partial y_j}(\rho\delta_{ij})\,\mathrm{d}S(\mathbf{y}) - \int_S l_i \left\{\frac{1}{r^2}\frac{\partial r}{\partial x_i}\rho + \frac{1}{a_0 r}\frac{\partial r}{\partial x_i}\frac{\partial \rho}{\partial t}\right\}\mathrm{d}S(\mathbf{y})$$
$$= \int_S l_i \frac{1}{r}\frac{\partial}{\partial y_j}(\rho\delta_{ij})\,\mathrm{d}S(\mathbf{y}) + \int_S l_j \frac{\partial}{\partial x_i}\left(\frac{1}{r}\rho\delta_{ij}\right)\mathrm{d}S(\mathbf{y}). \qquad (2\cdot 13)$$

The form of the last integral is due to the fact that retarded times apply in all the foregoing work, and

$$\frac{\partial}{\partial x_i}\left\{\frac{1}{r}f\left(t-\frac{r}{a_0}\right)\right\} = -\left\{\frac{1}{r^2}f + \frac{1}{a_0 r}f'\right\}\frac{\partial r}{\partial x_i}. \qquad (2\cdot 14)$$

Substituting from (2·12) and (2·13) into (2·4) then leads to

$$\rho - \rho_0 = \frac{1}{4\pi a_0^2}\frac{\partial^2}{\partial x_i \partial x_j}\int_V \frac{T_{ij}\left(\mathbf{y}, t-\frac{r}{a_0}\right)}{r}\mathrm{d}\mathbf{y}$$
$$+ \frac{1}{4\pi a_0^2}\int_S l_i \frac{1}{r}\frac{\partial}{\partial y_j}(T_{ij} + a_0^2\rho\,\delta_{ij})\,\mathrm{d}S(\mathbf{y})$$
$$+ \frac{1}{4\pi a_0^2}\frac{\partial}{\partial x_i}\int_S l_j \frac{1}{r}(T_{ij} + a_0^2\rho\delta_{ij})\,\mathrm{d}S(\mathbf{y}),$$

and by substituting for $T_{ij} = \rho v_i v_j + p_{ij} - a_0^2 \rho \delta_{ij}$ this becomes

$$\rho - \rho_0 = \frac{1}{4\pi a_0^2} \frac{\partial^2}{\partial x_i \partial x_j} \int_V \frac{T_{ij}\left(\mathbf{y}, t - \frac{r}{a_0}\right)}{r} \mathrm{d}\mathbf{y} + \frac{1}{4\pi a_0^2} \int_S l_i \frac{1}{r} \frac{\partial}{\partial y_j} (\rho v_i v_j + p_{ij}) \, \mathrm{d}S(\mathbf{y}) + \frac{1}{4\pi a_0^2} \frac{\partial}{\partial x_i} \int_S l_j \frac{1}{r} (\rho v_i v_j + p_{ij}) \, \mathrm{d}S(\mathbf{y}). \tag{2·15}$$

Now

$$l_i \frac{\partial}{\partial y_j} (\rho v_i v_j + p_{ij}) = -l_i \frac{\partial}{\partial t} (\rho v_i), \tag{2·16}$$

and if there is zero normal velocity at the solid boundaries, that is, if each surface is fixed or is vibrating in its own plane, then

$$l_i v_i \equiv 0. \tag{2·17}$$

Hence (2·15) reduces simply to

$$\begin{aligned} \rho - \rho_0 &= \frac{1}{4\pi a_0^2} \frac{\partial^2}{\partial x_i \partial x_j} \int_V \frac{T_{ij}\left(\mathbf{y}, t - \frac{r}{a_0}\right)}{r} \mathrm{d}\mathbf{y} + \frac{1}{4\pi a_0^2} \frac{\partial}{\partial x_i} \int_S \frac{1}{r} l_j p_{ij} \, \mathrm{d}S(\mathbf{y}) \\ &= \frac{1}{4\pi a_0^2} \frac{\partial^2}{\partial x_i \partial x_j} \int_V \frac{T_{ij}\left(\mathbf{y}, t - \frac{r}{a_0}\right)}{r} \mathrm{d}\mathbf{y} - \frac{1}{4\pi a_0^2} \frac{\partial}{\partial x_i} \int_S \frac{P_i\left(\mathbf{y}, t - \frac{r}{a_0}\right)}{r} \mathrm{d}S(\mathbf{y}), \end{aligned} \tag{2·18}$$

where

$$P_i = -l_j p_{ij}. \tag{2·19}$$

Equation (2·18) is the fundamental result of this paper. In it, the surface integral, representing the modification to Lighthill's theory, is exactly equivalent to the sound generated in a medium at rest by a distribution of dipoles of strength $P_i$ per unit area, and, by (2·19), $P_i$ is exactly the force per unit area exerted on the fluid by the solid boundaries in the $x_i$ direction. Physically, therefore, one can look upon the sound field as the sum of that generated by a volume distribution of quadrupoles and by a surface distribution of dipoles.

It is important to notice that, just as in Lighthill's theory, the analysis is exact and no simplifying assumptions have been made regarding the relationship between stresses and rates of strain. Also in the Lighthill theory the external stress system, $T_{ij}$, incorporates not only the generation of sound but also its convection with the flow, its propagation with variable speed, and its dissipation by conduction and viscosity. Similarly, in the present work all such effects as reflexion and diffraction at the solid boundaries are completely accounted for by incorporation into the applied dipole field $P_i$. The sound field is therefore exactly that which would be generated in a hypothetical unbounded uniform medium at rest, acted upon by (i) a volume distribution of quadrupoles $T_{ij}$ throughout the region external to the solid bodies which in fact are present, and (ii) a surface distribution of dipoles $P_i$ acting at the surfaces of the solid bodies.

## 3. Dimensional analysis of sound generated

In much the same way as Lighthill simplified the volume integral in the cases when $|\mathbf{x}|$ is large enough, so the surface integral in (2·18) may be simplified. When $|\mathbf{x}| \gg \lambda$, where $\lambda$ is a typical wavelength of the sound generated, so that $\mathbf{x}$ lies in the radiation field of each dipole, then $\dfrac{1}{4\pi a_0^2}\dfrac{\partial}{\partial x_i}\displaystyle\int_S \dfrac{P_i\left(\mathbf{y}, t-\frac{a_0}{r}\right)}{r}\,\mathrm{d}S(\mathbf{y})$ takes the simpler form

$$\frac{1}{4\pi a_0^3}\frac{\partial}{\partial t}\int_S \frac{x_i-y_i}{|\mathbf{x}-\mathbf{y}|^2} P_i\left(\mathbf{y}, t-\frac{r}{a_0}\right)\mathrm{d}S(\mathbf{y}). \tag{3·1}$$

Again, if $|\mathbf{x}| \gg L$, where $L$ is a typical dimension of the solid bodies, then (3·1) reduces still further to

$$\frac{1}{4\pi a_0^3}\frac{x_i}{x^2}\frac{\partial}{\partial t}\int_S P_i\left(\mathbf{y}, t-\frac{r}{a_0}\right)\mathrm{d}S(\mathbf{y}). \tag{3·2}$$

Also, if $L \ll a_0/\omega$, where $\omega$ is a typical sound frequency, then in (3·2) the $r/a_0$ of the retarded time may be neglected, and the integral becomes simply

$$\frac{1}{4\pi a_0^3}\frac{x_i}{x^2}\frac{\partial}{\partial t}F_i(t), \tag{3·3}$$

where $F_i(t) = \int_S P_i(\mathbf{y}, t)\,\mathrm{d}S(\mathbf{y})$, the total resultant force exerted upon the fluid by the solid boundaries. Since $\omega$ will in general be of order $U_0/L$, where $U_0$ is a typical velocity in the flow, it follows that this last simplification should be possible if

$$L \ll \frac{a_0}{\omega} \approx \frac{a_0 L}{U_0}, \quad \text{i.e. if} \quad \frac{U_0}{a_0} \ll 1.$$

Hence for flows at low Mach number the 'surface noise' generated is that generated by a single dipole* representing the resultant fluctuating force exerted on the fluid by the solid boundaries.

This simple result probably contains within it the basis of an explanation of the Aeolian tones emitted by wires in a wind. Too little is known concerning the fluctuating lift on such wires, as a function of Reynolds number, to make quantitative inferences in the matter. However, Gerrard in unpublished work has verified that the sound field in this flow is a dipole field with the direction of the dipole at right angles to the direction of wind.

Now Lighthill has shown that for similar flows the intensity $I_Q$ of sound generated by the quadrupoles should vary very nearly as

$$I_Q \sim \rho_0 U_0^8 a_0^{-5} L^2 x^{-2} f(R) \tag{3·4}$$

* This result is proved here for finite bodies. For a two-dimensional flow the sound field will be that due to a line of dipoles. The proof assumes that frequencies will be of order $U_0/L$, but Phillips (1955) shows that for turbulent flows the result will hold for all frequencies.

at a distance $x$ from the centre of the flow field. By similar reasoning one can estimate the sound generated by the dipoles. $F_i(t) = \int P_i(t)\,\mathrm{d}S$ is of order $\rho_0 U_0^2 L^2 g(R)$, and so

$$\begin{aligned}\rho - \rho_0 &\sim a_0^{-3}\frac{1}{x}\frac{U_0}{L}\rho_0 U_0^2 L^2 g(R) \\ &\sim \rho_0 U_0^3 a_0^{-3} L x^{-1} g(R).\end{aligned} \tag{3·5}$$

Hence the sound intensity $I_D$ generated by the dipoles, being given by $a_0^3\rho_0^{-1}(\rho-\rho_0)^2$, is of order

$$I_D \sim \rho_0 U_0^6 a_0^{-3} L^2 x^{-2} g(R). \tag{3·6}$$

From (3·4) and (3·6) it follows that

$$\frac{I_Q}{I_D} \sim \left(\frac{U_0}{a_0}\right)^2 \text{ times a function of } R. \tag{3·7}$$

It follows that at sufficiently low Mach numbers the contribution to the sound field from the dipoles should be greater than that for the quadrupoles. Exactly how small the Mach number must be before this is so will depend upon the flow in question. (Mathematically speaking, it will depend upon what the function of $R$ is in (3·7).)

From (3·6) it follows that the total acoustic power output will be roughly proportional to

$$\rho_0 U_0^6 a_0^{-3} L^2. \tag{3·8}$$

In a steadily maintained flow the total rate of supply of energy will be proportional to $(\rho_0 U_0^2)(U_0 L^2)$, and the acoustic efficiency, described as the ratio of acoustic power output to energy supplied, will then vary roughly as

$$\eta \propto \left(\frac{U_0}{a_0}\right)^3. \tag{3·9}$$

This is to be compared with the acoustic efficiency $\eta \propto M^5$ found by Lighthill for the quadrupole field.

## 4. Some properties of the sound generated

An important difference between the sound generated by the dipoles and that generated by the quadrupoles is that the frequencies are not equal. If one considers a flow in which the velocity in the $x_j$-direction is given to a first approximation as

$$v_j(\mathbf{x}) \approx A_j(\mathbf{x}) \cos(nt + \epsilon_j), \tag{4·1}$$

(where $\epsilon_j$ is also a function of $\mathbf{x}$), then the quadrupole strength per unit volume is

$$\begin{aligned}T_{jk} &\approx \rho_0 v_j v_k \approx \rho_0 A_j A_k \cos(nt+\epsilon_j)\cos(nt+\epsilon_k) \\ &= \tfrac{1}{2}\rho_0 A_j A_k \{\cos(2nt+\epsilon_j+\epsilon_k) + \cos(\epsilon_j - \epsilon_k)\}.\end{aligned} \tag{4·2}$$

It follows that the frequency of the sound generated by the quadrupoles will be double that of the velocity fluctuations. On the other hand, the frequency of the sound generated by the fluctuating forces at the solid boundaries will have the same frequency as the fluctuating velocity field.

In unpublished work the author attempted to estimate the intensity of the sound generated when a uniform flow impinging upon a circular wire leads to the formation of a Kármán vortex street behind the wire. In so doing he found an excellent example of this phenomenon. In trying to estimate the sound generated by the quadrupoles he found that the sound so generated would have double the frequency of the sound normally associated with such a flow. This shows that at the low speeds at which a Kármán vortex street is obtained the sound is not generated principally by the quadrupoles. In view of equation (3·7) this is not surprising, for at such low speeds one would expect dipole sound to dominate.

Physically, the quadrupole sound frequency is doubled because the quadrupole strength per unit volume, being essentially proportional to (velocity)$^2$, is the same when a vortex is cast off at the upper or lower edge of the cylinder; that is, one cannot distinguish between these two cases as far as the quadrupoles are concerned. For the same reason one would equally well expect the frequency of the sound generated by the fluctuating drag force to have the doubled frequency. The lift force, however, would have the same frequency as the velocity fluctuations in the wake, for clearly it would be of opposite sign in the two cases of the shedding of a vortex from the upper or lower edge of the cylinder. In fact, as is mentioned above in § 3, Gerrard has verified that the sound field is a dipole field with the direction of the dipole at right angles to the direction of flow.*

In investigating the dependence of sound intensity upon velocity Gerrard found that in the principal range of Reynolds numbers it varied essentially as $U^7$ when the cylinder diameter was kept constant. In addition, however, he carried out experiments in which he varied both the cylinder diameter and the velocity in such a way as to keep the Reynolds number constant, and found a dependence on the fourth power of the velocity. This is a surprising result, for it is the dependence upon velocity which would be associated with a distribution of sources. No sound field corresponding to a distribution of sources could be present in this problem, for, by the same reasoning as above, such sound would be of double the observed frequency. A possible explanation of this dilemma has been provided by Sir Geoffrey Taylor, F.R.S. It is easily demonstrated that if a toasting-fork is 'swished' in the plane of the prongs much more noise is heard than if it is swished in a plane at right angles to this, the reason for this presumably being that in the former case each prong is in the wake of the one preceding it. Now in Gerrard's experiments two wires were rotated about a parallel axis, and therefore each would be continually moving in the wake of the other. This might well cause a similar change in the intensity of the sound generated.

The author is greatly indebted to Professor M. J. Lighthill, F.R.S., for many helpful discussions on the subject of this paper, and on aerodynamic noise in general.

* Phillips (1955) has shown that for turbulent flow past a cylinder the fluctuations in lift per unit length are very much greater than the fluctuations in drag. A similar directional distribution of sound intensity would therefore be expected for turbulent flow as for the Kármán vortex street.

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# Relative intensities of totally symmetrical vibrations in the Raman spectrum of gaseous neopentane

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(*Communicated by R. P. Bell, F.R.S.—Received* 15 *April* 1955)

Raman spectra are photographed for neopentane in the liquid state and in the gaseous state at approximately $1\frac{1}{2}$ atm pressure. Only two of the three totally symmetrical vibrational modes appear. The ratio of their intensities is determined for the gas, and the result is used to test the validity of the Wolkenstein bond polarizability theory. It is concluded that Wolkenstein's assumption, that the polarizability of a bond is not affected by a change of bond orientation, is a quite good approximation for the particular modes considered.

In a previous paper (Long, Matterson & Woodward 1954, hereafter to be referred to as L.M.&W.) it was pointed out that determinations of relative Raman intensities of the three totally symmetrical modes of a molecule like neopentane $C(CH_3)_4$ are specially suitable for testing bond polarizability theories. Such theories apply to free molecules, and the tests should therefore be made with Raman intensities determined for gases at not too high pressures.

In L.M.& W. use was made of the measurements of Rank, Saksena & Shull (1950) for liquid neopentane, since at the time no data were available for the gas. We have now made corresponding measurements for gaseous neopentane at approximately $1\frac{1}{2}$ atm pressure.

## Experimental

As pointed out by Woodward & Long (1949) no special relation between incident and scattered directions is necessary in determining (as in this work) the relative intensities of totally symmetrical modes of spherically symmetrical molecules. A conventional Wood's tube type of Raman vessel was therefore used. Its capacity was approximately 1 l., and it was placed with its window directly in front of the slit of the spectrograph, a Hilger E 518 instrument with an $F/4$ camera. Suitable diaphragms placed inside the Raman tube near the window end ensured that no light from the walls could enter the spectrograph. The other end of the Raman tube was drawn off into the usual 'horn' and connected to a vacuum line. Only