Q1 Consider the following grammar $S \to aS | aSbS | \varepsilon$ Show that derivation for string aab is ambiguous

Ans 1 By Leftmost derivation :

1)	2)
$S \Rightarrow aSbS$ $(S \to aSbS)$	$S \Rightarrow aS$ $(S \to aS)$
$\Rightarrow aaSbS$ $(S \to aS)$	$\Rightarrow aaSbS$ $(S \to aSbS)$
$\Rightarrow aabS$ $(S \to \varepsilon)$	$\Rightarrow aabS$ $(S \to \varepsilon)$
$\Rightarrow aab$ $(S \to \varepsilon)$	$\Rightarrow aab$ $(S \to \varepsilon)$

∴ Since we can derive string "aab" in more than one way,
∴ The given grammar is ambiguous.

Q2 Simplify the given grammar : $S \to aAa | bBb | BB$, $A \to C$, $B \to A | S\varepsilon$, $C \to S | \varepsilon$

Ans 2 Step 1 : Elimination of Null Production

Productions (P)	New Production (NP)
$S \to aAa$	$S \to aa$
$S \to bBb$	$S \to bb$
$S \to BB$	$S \to B$, $S \to \varepsilon$
$A \to C$	$A \to \varepsilon$
$B \to A$	$B \to \varepsilon$
$B \to S$	$B \to \varepsilon$
$C \to S$	$C \to \varepsilon$
$C \to \varepsilon$	

P + NP : $S \to aAa | aa | BB | bBb | bb | B | \varepsilon$
$A \to C | \varepsilon$
$B \to A | \varepsilon | S$
$C \to S | \varepsilon$

Delete all null productions, we get

S → aAa | aa | bBb | bb | BB | B
A → C
B → A | S
C → S

Step 2 : Elimination of unit productions

Production (P)	New Production (NP)
S → aAa	
S → bBb	
S → aa	
S → bb	
S → BB	
S → B	
A → C	A → aAa | aa | bb | bBb
B → A	B → aAa | bb | aa | bBb | BB | B
B → S	B → aAa | bb | aa | bBb | BB | B
C → S	C → aAa | aa | bBb | bb | BB | B

P+NP S → aAa | aa | BB | bBb | bb | B
A → aAa | aa | bBb | bb | BB | B | C
B → aAa | aa | bBb | bb | BB | B | A | S
C → aAa | aa | bBb | bb | BB | B | S

Delete all unit productions, we get
S → aAa | aa | BB | bBb | bb
A → aAa | aa | bBb | bb | BB
B → aAa | aa | bBb | bb | BB
C → aAa | aa | bBb | bb | BB

Step 3 : Elimination of useless productions.
Since variable C is not reachable from state S, hence it is a useless production

∴ Simplified Grammar :

S → aAa | aa | bBb | bb | BB
A → aAa | aa | bBb | bb | BB
B → aAa | aa | bBb | bb | BB

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Q3 Generate and reduce the grammars for the following to CNF:
$L = \{a^n b^m c^n \mid n >= 1, m >= 1\}$ over $\Sigma = \{a, b, c\}$

Ans 3 Here, the logic is equal no of a's followed by no of b's dependent on m and equal no of c's. Therefore, the grammar is:

$S \to aSc \mid aXc$
$X \to bX \mid b$

$\therefore G = (V, T, P, S)$

$\therefore G = (\{S, X\}, \{a, b, c\}, \{S \to aSc \mid aXc, X \to bX \mid b\})$

Convert to CNF:

Step 1: Simplification of grammar

Null productions: As there are no null productions, the grammar stays as it is after removal of null productions, i.e

$S \to aSc \mid aXc$, $X \to bX \mid b$

Unit productions: As there are no unit productions, the grammar stays as it is after removal of unit productions, i.e

$S \to aSc \mid aXc$, $X \to bX \mid b$

Useless productions: All variables are reachable from start state S therefore no useless productions present $\therefore$ Grammar is:

$S \to aSc \mid aXc$, $X \to bX \mid b$

Step 2: Conversion to CNF

First, write all productions already in CNF

$X \to b$,

$S \to aSc$, $C_1 \to a$ and $C_2 \to c$

$\therefore S \to C_1 S C_2$

$C_3 \to 3C_2$

$\therefore S \to C_1 C_3$

$S \to aXc$	$X \to bX$
$S \to C_1 X C_2$	$C_5 \to 6b$
$C_4 \to XC_2$	
$\therefore S \to C_1 C_4$	

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∴ CNF is

$S \to C_1C_3 \mid C_1C_4$
$X \to C_5X \mid b$
$C_1 \to a$
$C_2 \to c$
$C_3 \to SC_2$
$C_4 \to XC_2$
$C_5 \to b$

Q4 Convert the following into GNF : $S \to AA \mid 0$, $A \to SS \mid 1$

Ans 4 Step 1 : Simplification of grammar

Null productions : No null productions present in grammar

∴ After elimination of null productions, $S \to AA \mid 0$
$A \to SS \mid 1$

Unit productions : No unit productions present in grammar.

∴ Grammar is : $S \to AA \mid 0$
$A \to SS \mid 1$

Useless productions : All variables are reachable from start state S.

∴ Simplified grammar is : $S \to AA \mid 0$
$A \to SS \mid 1$

Step 2 : Conversion to GNF

$A \to SS \mid 1$

By substitution we get

$A \to AAS \mid 0S \mid 1$

(A → α_1 | B_1 | B_2)

After replacement, By rule 2.

$B \to \alpha_1 \mid \alpha_1 B$

i.e., $B \to AS \mid ASB$ — (I)

And $A \to B_i \mid B_iB$

i.e $A \to 0S \mid 0SB \mid 1 \mid B1$ — (II)

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After substitution in S

$S \rightarrow AA \mid 0$

$\therefore S \rightarrow 0SA \mid 0SBA \mid 1A \mid 1BA \mid 0$

Substitute A in B

$B \rightarrow 0SS \mid 0SBS \mid 1BS \mid 1S \mid 0SBB \mid 0SBSB \mid 1BSB \mid 1S$

~~$B \rightarrow 0SS \mid 0$~~

$A \rightarrow 0S \mid 0SB \mid 1 \mid 1B$