1) Number of n-digits numbers divisible by 3 and containing "13"

dp[i][j] = # of i-digits numbers whose digits sum up to j (mod 3) not containing "13"

Base cases: dp[1][0] = 4 = {0,3,6,9} dp[2][0] = 30

dp[1][1] = 3 = {1,4,7} dp[2][1] = 29 (since "13" is excluded")

dp[1][2] = 3 = {2,5,8} dp[2][2] = 30

for i = 3, ..., n :

dp[i][0] = dp[i-1][0]·4 + dp[i-1][1]·3 + dp[i-1][2]·3 - dp[n-2][2]

since 13 is (1+3) mod 3 = 1 when you add these two digits to an i-2 number adding up to 2 the resulting i number will add up to 1+2 = 0 as wanted.

dp[i][1] = dp[i-1][0]·3 + dp[i-1][1]·4 + dp[i-1][2]·3 - dp[n-2][0]

dp[i][2] = dp[i-1][0]·3 + dp[i-1][1]·3 + dp[i-1][2]·4 - dp[n-2][1]

Return dp[n][0] + ... + dp[1][0] Running time O(3n).