Implementation exercises for the course Heuristic Optimization

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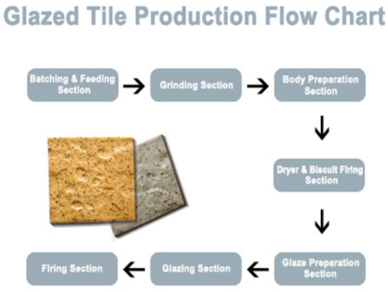
[1] Slides based on 2014 excersises by Dr. Franco Mascia.

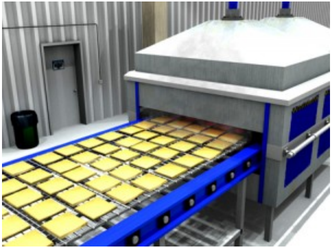
Exercise 1.1: Iterative Improvement for the PFSP

Implement perturbative local search algorithms for the PFSP

1. Permutation Flow Shop Scheduling Problem (PFSP)
2. First-improvement and Best-Improvement
3. Transpose, exchange and insert neighborhoods
4. Random initialization vs. Simplified RZ heuristic
5. Statistical Empirical Analysis

The Permutation Flow Shop Scheduling Problem (1/4)





Example in ceramic tile production

- Tiles need several processing steps with different machines
- Tiles of different type require specific processing times for each machine
- Goal: find a schedule of the jobs that minimizes an objective function (makespan or total completion time)

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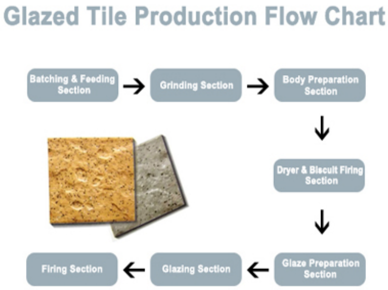

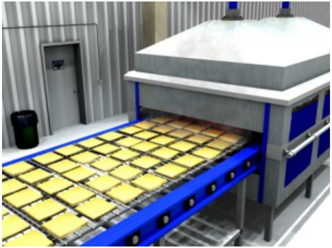

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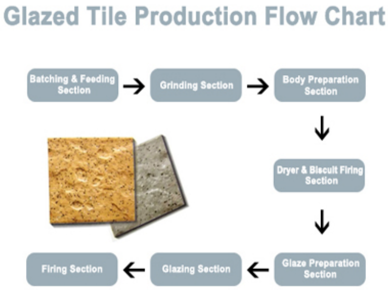

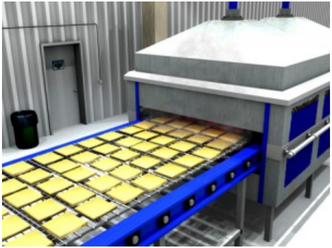

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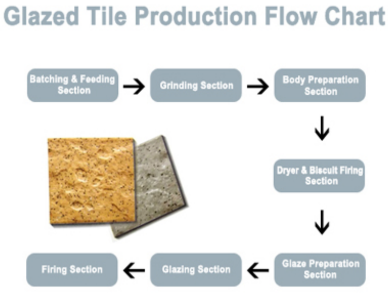

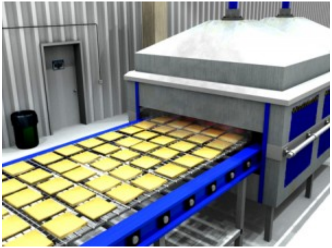

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Flow Shop Scheduling

- Several scheduling problems have been proposed with different formulations and constraints.
- In permutation flow shop problems:
  - jobs composed by operations to be executed on several machines
  - all jobs pass through the machines in the same order
  - all jobs available at time zero
  - pre-emption not allowed
  - each operation has to be performed on a specific machine
  - each job at most on one machine at a time
  - each machine at most one job at a time

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- Jobs pass trough all machines in the same order (FCFS queues)
- No constraints: infinite buffers between machines, no blocking, no no-wait requirements (steel production)

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Given

A set of n jobs $J_1, \ldots, J_n$ jobs, where each job J_i consists of m operations $o_{i1}, \ldots, o_{im}$ performed on $M_1, \ldots, M_m$ machines in that order, with processing time p_{ij} for operation o_{ij}.

Objective

Find a permutation π that minimizes the sum of the completion times $\sum_{i=1}^{n} w_i \cdot C_i$.

The Permutation Flow Shop Scheduling Problem (3/4)

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Makespan = 21

Sum of Completion times = 73

Weighted sum of Completion Times = 186

The Permutation Flow Shop Scheduling Problem (4/4)

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Makespan = 21

Sum of Completion times = 73

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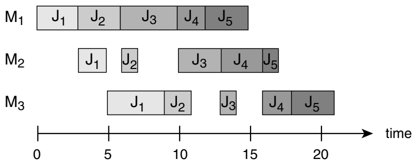

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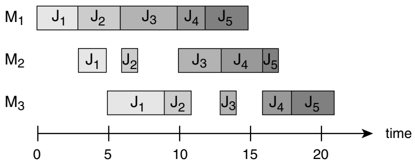

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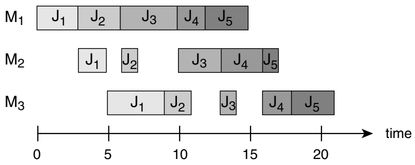

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The Permutation Flow Shop Scheduling Problem (4/4)

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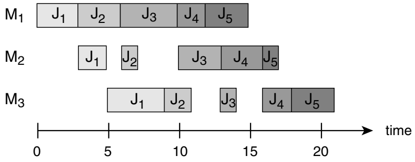

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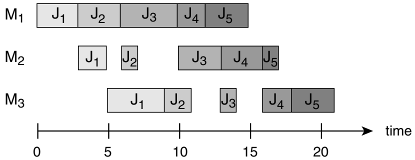

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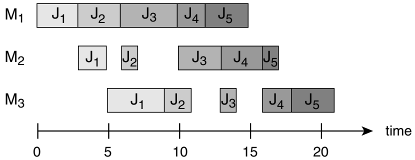

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Implement 12 iterative improvements algorithms for the PFSP

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  2. best-improvement
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  2. Exchange
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- Initial solution:
  1. Random permutation
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2 pivoting rules $\times$ 3 neighborhoods $\times$ 2 initialization methods $=$
12 combinations

Exercise 1.1: Iterative Improvement for the PFSP

Implement 12 iterative improvements algorithms for the PFSP

Don't implement 12 programs!
Reuse code and use command-line parameters

```
pfsp-ii --first --transpose --srz
pfsp-ii --best --exchange --random-init
...
```

Exercise 1.1: Iterative Improvement for the PFSP

Iterative Improvement

$\pi := \texttt{GenerateInitialSolution()}$
while π is not a local optimum **do**
  choose a neighbour $\pi' \in \mathcal{N}(\pi)$ such that $F(\pi') < F(\pi)$
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  - ✘ Requires evaluation of all neighbours in each step
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Simplified RZ heuristic

Start by ordering the jobs in ascending order of their weighted sum of processing times.
Construct the solution by inserting **one job at a time** in the position that minimize the WCT.

The weighted sum of processing times of job J_i is computed as $\frac{1}{w_i} \cdot \sum_1^m p_{ij}$
Note: the solution is constructed incrementally, and at each iteration C_i corresponds to the makespan of the partial solution.

Simplified RZ heuristic: an example

$$C_{\pi(1)j} = \sum_{h=1}^{j} p_{\pi(1)h} \qquad j = 1, \ldots m$$
$$C_{\pi(k)1} = \sum_{h=1}^{k} p_{\pi(h)1} \qquad k = 1, \ldots n$$
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$$T_i = \frac{1}{w_i} \cdot \sum_1^m p_{ij}$$

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Starting sequence $= \{J_3\ J_5\ J_2\ J_4\ J_1\}$

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Step 1 $\pi = \{\}$

$\{J_3\ J_5\}$ $WCT = 65$

$\{J_5\ J_3\}$ $WCT = 65$

Step 2 $\pi = \{J_3\ J_5\}$

$\{J_2\ J_3\ J_5\}$ $WCT = 98$

$\{J_3\ J_2\ J_5\}$ $WCT = 94$

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Step 3 $\pi = \{J_3\ J_5\ J_2\}$

$\{J_4\ J_3\ J_5\ J_2\}$ $WCT = 123$

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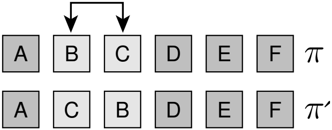
Iterative Improvement

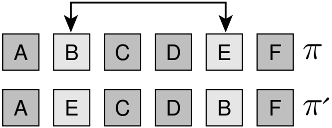
$\pi :=$ `GenerateInitialSolution()`
while π is not a local optimum **do**
  choose a neighbour $\pi' \in \mathcal{N}(\pi)$ such that $F(\pi') < F(\pi)$
  $\pi := \pi'$

Which neighborhood $\mathcal{N}(\pi)$?

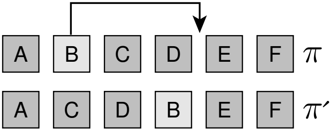
- Transpose
- Exchange
- Insertion



transpose neighbourhood

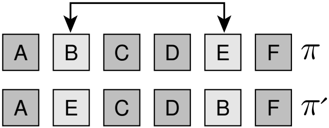


exchange neighbourhood



insert neighbourhood

Exercise 1.1: Iterative Improvement for the PFSP



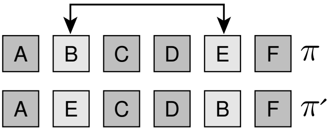
exchange neighbourhood

Example: Exchange π_i and π_j ($i < j$), $\pi' = \texttt{Exchange}(\pi, i, j)$

Only jobs after i are affected!

Do not recompute the evaluation function from scratch!

Equivalent speed-ups with Transpose and Insertion



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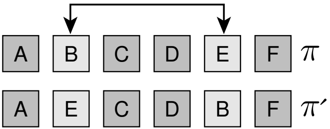

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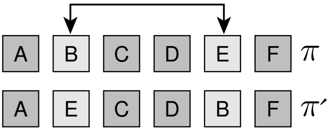

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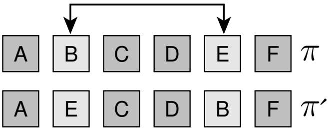

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Instances

- PFSP instances with 50 and 100 jobs, and 20 machines.
- More info: http://iridia.ulb.ac.be/~stuetzle/Teaching/HO/

Experiments

Apply each algorithm k once to each instance i and compute:

1. Relative percentage deviation $\Delta_{ki} = 100 \cdot \dfrac{\text{cost}_{ki} - \text{best-known}_i}{\text{best-known}_i}$
2. Computation time (t_{ki})

Report for each algorithm k

- Average relative percentage deviation
- Sum of computation time

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Exercise 1.1: Iterative Improvement for the PFSP

Is there a statistically significant difference between the solution quality generated by the different algorithms?

Statistical test

- Paired t-test
- Wilcoxon signed-rank test

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Background: Statistical hypothesis tests (1)

- *Statistical hypothesis tests* are used to assess the validity of statements about properties of or relations between sets of statistical data.
- The statement to be tested (or its negation) is called the *null hypothesis* (H_0) of the test.
  Example: For the Wilcoxon signed-rank test, the null hypothesis is that 'the median of the differences is zero'.
- The *significance level* (α) determines the maximum allowable probability of incorrectly rejecting the null hypothesis.
  Typical values of α are 0.05 or 0.01.

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- Most common statistical hypothesis tests are already implemented in statistical software such as the *R software environment* (http://www.r-project.org/).

Is there a statistically significant difference between the solution quality generated by the different algorithms?

Example in R

```
best.known <- read.csv ("bestSolutions.txt")
a.cost <- read.table("ii-best-ex-rand.dat")$V1
a.cost <- 100 * (a.cost - best.known) / best.known$BS
b.cost <- read.table("ii-best-ins-rand.dat")$V1
b.cost <- 100 * (b.cost - best.known) / best.known$BS
t.test (a.cost, b.cost, paired=T)$p.value
[1] 0.8819112
wilcox.test (a.cost, b.cost, paired=T)$p.value
[1] 0.0019212
```

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Exercise 1.1: Iterative Improvement for the PFSP

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Exercise 1.2 VND algorithms for the PFSP

Implement 4 VND algorithms for the PFSP

- Pivoting rule: first-improvement
- Neighborhood order:
  1. transpose $\rightarrow$ exchange $\rightarrow$ insert
  2. transpose $\rightarrow$ insert $\rightarrow$ exchange
- Initial solution:
  1. Random permutation
  2. Simplified RZ heuristic

Variable Neighbourhood Descent (VND)

k neighborhoods $\mathcal{N}_1, \ldots, \mathcal{N}_k$
$\pi := \texttt{GenerateInitialSolution()}$
$i := 1$
repeat
  choose the first improving neighbor $\pi' \in \mathcal{N}_i(\pi)$
  if $\nexists \pi'$ **then**
    $i := i + 1$
  else
    $\pi := \pi'$
    $i := 1$
until $i > k$

Implement 4 VND algorithms for the PFSP

- Instances: Same as 1.1
- Experiments: one run of each algorithm per instance
- Report: Same as 1.1
- Statistical tests: Same as 1.1

- Instances and barebone code will be soon available at:
  http://iridia.ulb.ac.be/~stuetzle/Teaching/HO/
- Deadline is April 4 (23:59)
- Questions in the meantime?
  `federico.pagnozzi@ulb.ac.be`