# EGB210
# Individual Gearbox Conceptual Design

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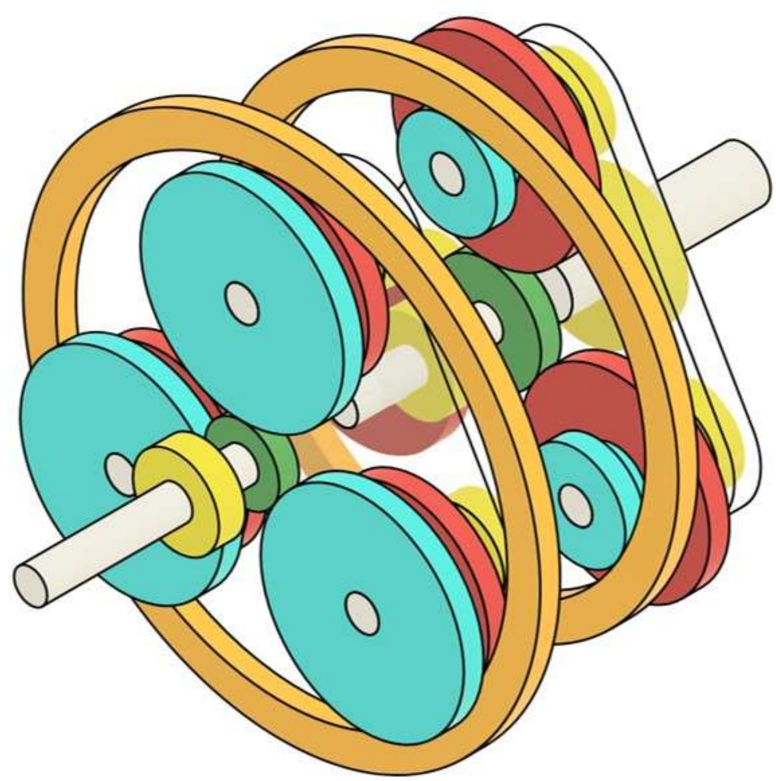

## List of Symbols

| Symbol | Unit | Description |
|---|---|---|
| $b$ | $mm$ | Face width |
| $d$ | $m$ | Diameter |
| $F_R$ | $N$ | Radial force on gear |
| $F_t$ | $N$ | Tangential force on gear |
| $i$ | $-$ | Gear ratio, $x$ indicating stage |
| $L_{ext}$ | $-$ | Losses of external spur gear mesh |
| $L_{ext}$ | $-$ | Losses of internal spur gear mesh |
| $L_{int}$ | $-$ | Losses of internal spur gear mesh |
| $m$ | $mm$ | Module |
| $N$ | $RPM$ | Revolutions Per Minute |
| $n_p$ | $-$ | Number of planet gears |
| $r$ | $m$ | Radius |
| $T$ | $Nm$ | Torque on gear |
| $v$ | $m/s$ | Pitch line velocity of gear |
| $v_{p,c}$ | $m/s$ | Planet pitch line velocity component from carrier rotation |
| $v_p'$ | $m/s$ | Planet pitch line velocity component from planets rotation |
| $\dot{W}$ | $W$ | Power |
| $Y$ | $-$ | Lewis form factor |
| $z$ | $-$ | Number of teeth |
| $\eta_0$ | $-$ | Initial efficiency estimate |
| $\eta_{actual}$ | $-$ | Actual efficiency |
| $S$ | $MPa$ | Normal Stress |
| $\tau$ | $MPa$ | Shear Stress |
| $\omega$ | $rad/s$ | Angular Velocity |

## Subscripts

| Subscript | Description |
|---|---|
| $p_1$ | Planet gear meshing with sun |
| $p_2$ | Planet gear meshing with ring |
| $c$ | Carrier |
| $r$ | Ring gear |
| $s$ | Sun gear |

# Design Overview

The design is a two-stage compound planetary gearbox. The first stage has a reduction of 8.10 and the second stage a reduction of 7.93, for a total gearbox speed reduction of 64.3. The output torque is 20 $N \cdot m$, requiring an input torque of 0.643 $N \cdot m$. The power transmission has an efficiency of 48.4%. The overall diameter of the design is 137 mm (based on the first stage ring gear and not a housing). The gears and shafts are designed to have a minimum factor of safety of 2.

The stages are aligned on the same central axis, as seen in Figure 1. The front of the gearbox is located at the side with the axis in the model, with the leftmost shaft (white) being the gearboxes input, and the output being the rightmost shaft. The sun gear is green, first planet gear blue, the second planet gear red, ring gear orange and bearings yellow. Each stage has 3 planet gears.

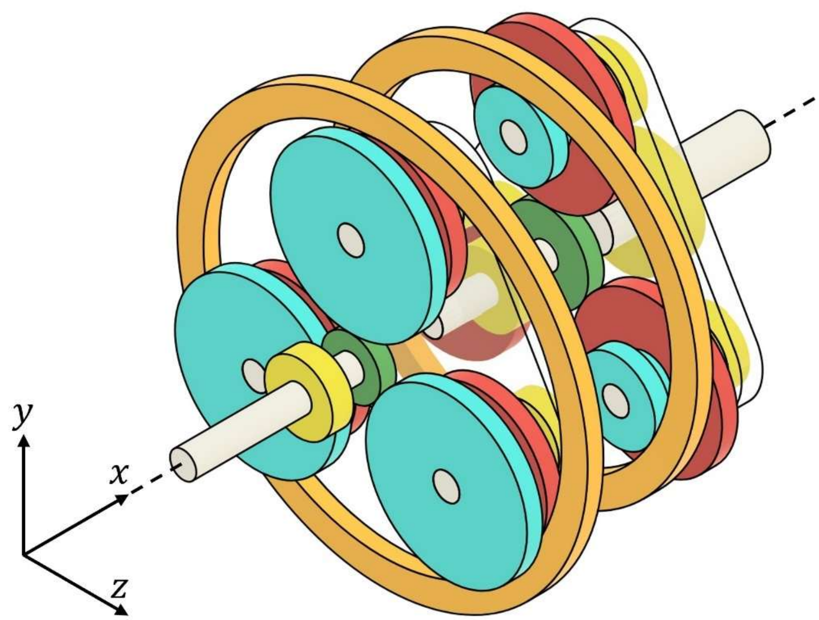


*Figure 1: 3D model of gearbox design.*

## How it Functions

A planetary gearbox has three main elements, being the sun gear, ring gear and carrier, with one inputting torque, one outputting torque, and the other held stationary to react against the other two. While any of these three elements can fulfil one of these roles, the common type (and the one used in this design) has the sun gear acting as input, carrier as output, and ring gear held stationary affixed to the gearbox housing [1, p. 331].

The carrier has planet gears attached to it that simultaneously mate with the sun gear and ring gear. The rotation of the sun gear causes the planet gears to move around the sun and thus cause the carrier to rotate. This can be understood by thinking of the planet gears motion in two parts. The first is the rotation given by the sun gear, proportional to the ratio of the two gears size, the same as any other mating spur gears. The second part of the motion is the planet gear rolling around the ring gear. A way to think about it is that if the planet gear has 15 teeth and the ring gear gas 60, the planet gear must rotate 4 times to do one revolution of the ring gear.

If the goal is to maximise gear reduction, then ideally the planet gear would be much larger than the sun gear, but also much smaller than the ring gear, a conflicting requirement. A compound planetary gearbox simply replaces each planet gear with two that are coupled together, one to mesh with the sun gear and one with the ring gear. In this way the planet gear meshing with the sun can be larger, and the planet gear meshing with the ring can be small.

The sun gear will be attached to a shaft to allow for torque to be input, and the carrier will have a shaft to allow for torque output. By using the output of one stage as the input shaft on the next stage, multiple stages can be placed in series to get a higher speed reduction and torque increase.

## Strengths and Weaknesses

The main advantage of a planetary gearbox is that a greater reduction can be achieved for a smaller overall size. This smaller size means smaller gears and a smaller housing, using less material and having a lower weight. This smaller gear size also means that the required gear reduction can be achieved with two stages given the maximum gear diameter of 300 mm (although this could be done with a single stage if allowed), meaning higher efficiency.

The disadvantage is that it is more complex to manufacture and assemble. There are more parts in a planetary gearbox compared to other designs, and these parts must be more accurately manufactured to ensure even load distribution between the planet gears. It is more complicated to assemble due to having more parts, and the compound planet gears having a specific orientation when being mated to sun and ring gears.

This design was done with both ring gears attached to the central housing. This means that with the requirements for the planet gears being connected, the first stages carrier must fit through the ring gear, which adds constraints and increases the overall size of the gearbox. The first stage ring gear and its planet gear have a factor of safety of 43 and 23 respectively, due to having a large module because to the additional constraints.

# Gear Tables

*Table 1: Gear Table for stage 1.*

| Stage 1 | | | | | |
|---|---|---|---|---|---|
| **Gear** | **Sun** | **Planet 1** | **Ring** | **Planet 2** | **Carrier** |
| $m$ | 1.5 | 1.5 | 3.5 | 3.5 | |
| Teeth | 15 | 41 | 39 | 15 | |
| $d\ (mm)$ | 22.5 | 61.5 | 136.5 | 52.5 | 84 |
| $b\ (mm)$ | 6 | 6 | 6 | 6 | |
| $N\ (RPM)$ | 1500 | -296.06 | 0 | -296.06 | 185.03 |
| Contact Angle (°) | 20 | 20 | 20 | 20 | |
| $\eta_{mesh}$ | 0.75 | | | | |
| $T\ (N \cdot m)$ | 0.643 | 0.415 | 2.29 | 0.415 | 3.69 |
| $\eta_{actual}$ | 0.708 | | | | |
| $\dot{W}\ (W)$ | 101 | 12.9 | 0 | 12.9 | 71.5 |
| $v\ (m/s)$ | 1.767 | 0.9534 | 0 | 0.8138 | 0.8138 |
| $F_t\ (N)$ | 19.1 | 13.5 | 11.2 | 15.8 | 29.3 |
| $F_R\ (N)$ | 6.96 | 4.91 | 4.08 | 5.75 | |
| $Y$ | 0.285 | 0.285 | 0.385 | 0.285 | |
| $S\ (MPa)$ | 7.45 | 3.90 | 1.39 | 2.64 | |
| $S_y\ (MPa)$ | 60 | 60 | 60 | 60 | |
| $FOS$ | 8.05 | 15.4 | 43.2 | 22.7 | |

*Table 2: Gear table for stage 2.*

| Stage 2 | | | | | |
|---|---|---|---|---|---|
| **Gear** | **Sun** | **Planet 1** | **Ring** | **Planet 2** | **Carrier** |
| $m$ | 2.1951 | 2.1951 | 2 | 2 | |
| Teeth | 15 | 26 | 60 | 15 | |
| $d\ (mm)$ | 32.93 | 57.07 | 126 | 30 | 90 |
| $b\ (mm)$ | 6 | 6 | 6 | 6 | |
| $N\ (RPM)$ | 185.03 | -69.99 | 0 | -69.99 | 23.32 |
| Contact Angle (°) | 20 | 20 | 20 | 20 | |
| $\eta_{mesh}$ | 0.75 | | | | |
| $T\ (N \cdot m)$ | 3.69 | 1.46 | 8.95 | 1.46 | 20 |
| $\eta_{actual}$ | 0.683 | | | | |
| $\dot{W}\ (W)$ | 71.5 | 10.7 | 0 | 10.7 | 48.4 |
| $v\ (m/s)$ | 0.3190 | 0.2091 | 0 | 0.1099 | 0.1099 |
| $F_t\ (N)$ | 74.7 | 51.1 | 66.3 | 97.1 | 148.1 |
| $F_R\ (N)$ | 27.2 | 18.6 | 24.1 | 35.3 | |
| $Y$ | 0.285 | 0.345 | 0.415 | 0.285 | |
| $S\ (MPa)$ | 19.9 | 11.2 | 13.3 | 28.4 | |
| $S_y\ (MPA)$ | 60 | 60 | 60 | 60 | |
| $FOS$ | 3.02 | 5.36 | 4.51 | 2.11 | |

## Stage 2 Gear Analysis

A diagram of the second stage is found in Figure 2. The shafts are white, sun gear green, first planet gear red, second planet gear blue, ring gear orange and the bearings are yellow. The carrier is clear with its triangular outline seen behind the gears. The second stage has its input coming from the output of the first stage. The equations used in this analysis can be found in the Gear Analysis and Equations appendix.

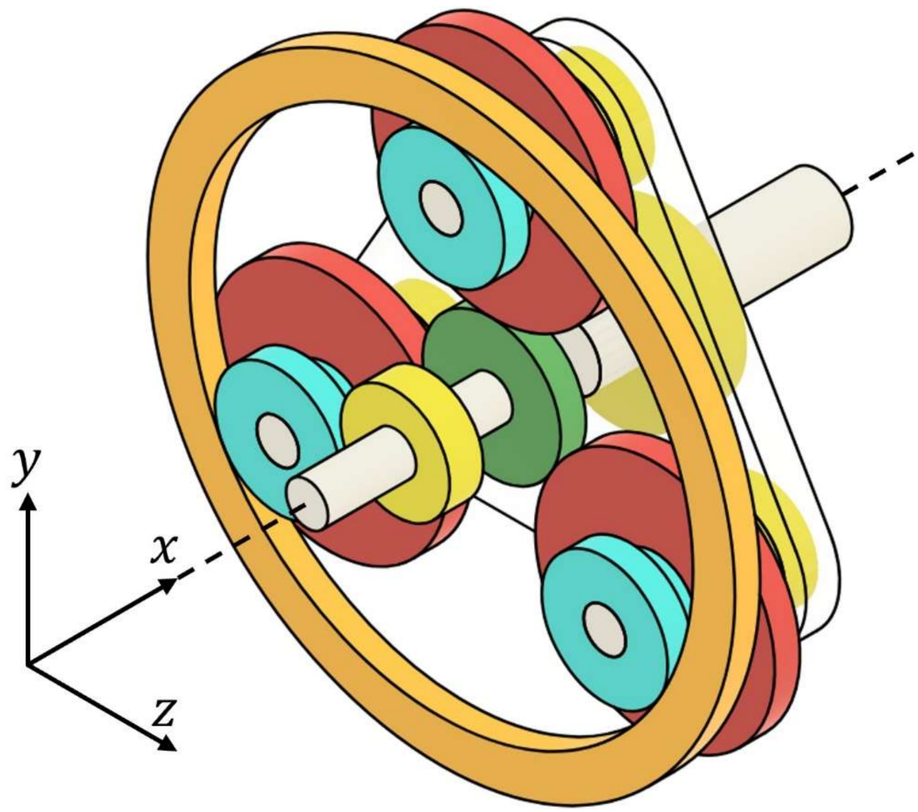


*Figure 2: Diagram of the second stage.*

$N_s = 185.03\ RPM$ $\qquad$ $T_c = 20\ N \cdot m$ $\qquad$ $n_p = 3$

### Dimensions

$d = m \cdot z$ $\qquad$ $r_c = r_s + r_{p_1}$ Eq. 8

*Table 3: Stage 2 dimension.*

| Gear | Sun | Planet 1 | Ring | Planet 2 | Carrier |
|---|---|---|---|---|---|
| $m\ (mm)$ | 2.1951 | 2.1951 | 2 | 2 | |
| $z$ | 15 | 26 | 60 | 15 | |
| $d\ (mm)$ | 32.93 | 57.07 | 126 | 30 | 90 |
| $r\ (mm)$ | 16.46 | 28.54 | 60 | 15 | 45 |
| $b\ (mm)$ | 6 | 6 | 6 | 6 | |
| $Y$ | 0.285 | 0.345 | 0.415 | 0.285 | |

## Assembly Condition

$k$ must be a whole number to ensure the planet gears can all mate at the same time.

$$k = \frac{z_s \cdot z_{p_2} - z_r \cdot z_{p_1}}{n_p \cdot \gcd(z_{p_1}, z_{p_2})} = \frac{15 \cdot 15 - 60 \cdot 26}{3 \cdot 1} = -445 \quad \text{Eq. 10}$$

## Gear Ratio

$$i = 1 + \frac{z_{p_1} \cdot z_r}{z_{p_2} \cdot z_s} = 1 + \frac{26 \cdot 60}{15 \cdot 15} = 7.933 \quad \text{Eq. 7}$$

## Output Torque and Efficiency

$$\eta_0 = (1 - L_{ext})\left(1 - \frac{z_r/z_s - 1}{z_r/z_s + 1} \cdot L_{ext}\right) \quad \text{Eq. 13}$$

$$\eta_0 = (1 - 0.25)\left(1 - \frac{60/15 - 1}{60/15 + 1} \cdot 0.25\right) = 0.6375 = 63.75\%$$

$$N_c = \frac{N_s}{i} = \frac{185.03}{7.933} = 23.32\ RPM$$

$$T_s = \frac{T_c \cdot N_c}{\eta_0 \cdot (N_s - N_c) + N_c} = \frac{20 \cdot 23.32}{0.6375 \cdot (185.03 - 23.32) + 23.32} \quad \text{Eq. 12}$$

$$T_s = 3.69\ N \cdot m$$

$$\eta_{actual} = \frac{T_c \cdot N_c}{T_s \cdot N_s} = \frac{20 \cdot 23.32}{3.69 \cdot 185.03} = 0.683 = 68.3\% \quad \text{Eq. 14}$$

## Gear Forces and Stresses

### *Sun*

$$F_{t,s} = \frac{T_s}{n_p \cdot r_s} = \frac{3.69}{3 \cdot 0.01646} = 74.7\ N \quad \text{Eq. 1}$$

$$F_{R,s} = F_{t,s} \cdot \tan(20°) = 74.7 \cdot 0.364 = 27.2\ N \quad \text{Eq. 2}$$

$$S = \frac{F_{t,s}}{m_s \cdot b_s \cdot Y_s} = \frac{74.7}{2.1951 \cdot 6 \cdot 0.285} = 19.9\ MPa \quad \text{Eq. 11}$$

### *Carrier*

$$F_{t,c} = \frac{T_c}{n_p \cdot r_c} = \frac{20}{3 \cdot 0.045} = 148.1\ N \quad \text{Eq. 3}$$

### *Planet 2*

$$F_{t,p_2} = \frac{F_{t,c}}{\left(1 + \frac{r_{p_2}}{r_{p_1}}\right)} = \frac{148.1}{\left(1 + \frac{0.015}{0.02854}\right)} = 97.1\ N \quad \text{Eq. 6}$$

$$F_{R,p_2} = F_{t,p_2} \cdot \tan(20°) = 97.1 \cdot 0.364 = 35.3\ N$$ Eq. 2

$$T_{p_2} = F_{t,p_2} \cdot r_{p_2} = 97.1 \cdot 0.015 = 1.46\ N \cdot m$$

$$S = \frac{F_{t,p_2}}{m_r \cdot b_{p_2} \cdot Y_{p_2}} = \frac{97.1}{2 \cdot 6 \cdot 0.285} = 28.4\ MPa$$ Eq. 11

*Planet 1*

$$F_{t,p_1} = F_{t,p_2} \cdot \frac{r_{p_2}}{r_{p_1}} = 97.1 \cdot \frac{0.015}{0.02854} = 51.0\ N$$ Eq. 5

$$F_{R,p_1} = F_{t,p_1} \cdot \tan(20°) = 51.0 \cdot 0.364 = 18.6\ N$$ Eq. 2

$$T_{p_1} = F_{t,p_1} \cdot r_{p_1} = 51.0 \cdot 0.02854 = 1.46\ N \cdot m$$

$$S = \frac{F_{t,p_1}}{m_s \cdot b_{p_1} \cdot Y_{p_1}} = \frac{51.0}{2.1951 \cdot 6 \cdot 0.345} = 11.2\ MPa$$ Eq. 11

*Ring*

$$F_{t,r} = F_{t,p_2} \cdot \eta_{actual} = 97.1 \cdot 0.683 = 66.3\ N$$ Eq. 4

$$F_{R,r} = F_{t,r} \cdot \tan(20°) = 66.3 \cdot 0.364 = 24.1\ N$$ Eq. 2

$$T_{p_1} = F_{t,r} \cdot r_r \cdot n_p = 66.3 \cdot 0.045 \cdot 3 = 8.95\ N \cdot m$$

$$S = \frac{F_{t,r}}{m_r \cdot b_r \cdot Y_r} = \frac{66.3}{2 \cdot 6 \cdot 0.415} = 13.3\ MPa$$ Eq. 11

## Velocity and RPM

The pitch line velocity for the sun gear and carrier is found by converting $RPM$ to $rad/s$ and multiplying by the radius.

$$N_c = 23.32\ RPM \qquad N_s = 185.03\ RPM$$

$$v_s = \frac{\pi \cdot d_s \cdot N_s}{60} = \frac{\pi \cdot 0.03293 \cdot 185.03}{60} = 0.3190\frac{m}{s} = 319.0\frac{mm}{s}$$ Eq. 16

$$v_c = \frac{\pi \cdot d_s \cdot N_s}{60} = \frac{\pi \cdot 0.090 \cdot 23.32}{60} = 0.1099\frac{m}{s} = 109.9\frac{mm}{s}$$

These velocities are then used to find the $RPM$ of the planet gears. $N_{p,\alpha}$ is the planet gear $RPM$ based on the mating between the sun gear and planet gear. $N_{p,\beta}$ is the $RPM$ based on the mating of the ring and planet gear. Both equations are used as a check, and if they are not equal then an error has been made.

$$N_{p,\alpha} = \frac{1}{d_{p_1}}(d_c \cdot N_c - d_s \cdot N_s)$$ Eq. 18

$$N_{p,\alpha} = \frac{1}{0.05707}(0.090 \cdot 23.32 - 0.03293 \cdot 185.03) = -69.99\ RPM$$

$$N_{p,\beta} = N_c \frac{d_c}{d_{p_2}} = -23.32 \frac{0.090}{0.030} = -69.96\ RPM \quad \text{Eq. 20}$$

It is then shown that the $RPM$ of both planet gears is equal, as expected since they are connected.

$N_{p,\alpha} = N_{p,\beta}$, within a rounding error.

Using the $RPM$ calculated, the pitch line velocity components from the planet gears rotation can be calculated.

$$v'_{p_1} = \frac{\pi \cdot d_{p_1} \cdot N_{p_1}}{60} = \frac{\pi \cdot 0.05707 \cdot 69.99}{60} = 0.2091 \frac{m}{s} = 209.1 \frac{mm}{s}$$

$$v'_{p_1} = v_s - v_c = 319.0 - 109.9 = 209.1 \frac{mm}{s} \quad \text{Eq. 17}$$

$$v'_{p_2} = \frac{\pi \cdot d_{p_2} \cdot N_{p_2}}{60} = \frac{\pi \cdot 0.030 \cdot 69.96}{60} = 0.1099 \frac{m}{s} = 109.9 \frac{mm}{s}$$

$$v'_{p_2} = v_c \quad \text{Eq. 19}$$

The power going through each gear can be calculated, with either the torque or tangential force can be used. If using tangential force for the sun gear and carrier, the force needs to be multiplied by the number of planet gears. Note that the planet gear power is per planet.

$$\dot{W}_s = T_s \cdot \omega_s = T_s \cdot \frac{2 \cdot \pi \cdot N_s}{60} = 3.69 \cdot \frac{2 \cdot \pi \cdot 185.03}{60} = 71.5\ W$$

$$\dot{W}_c = T_c \cdot \omega_c = T_c \cdot \frac{2 \cdot \pi \cdot N_c}{60} = 20 \cdot \frac{2 \cdot \pi \cdot 23.32}{60} = 48.4\ W$$

$$\dot{W}_{p_1} = F_{t,p_1} \cdot v_{p_1} = 51.0 \cdot 0.2091 = 10.66\ W$$

$$\dot{W}_{p_2} = F_{t,p_2} \cdot v_{p_2} = 97.1 \cdot 0.1099 = 10.67\ W$$

$\dot{W}_{p_1} = \dot{W}_{p_2}$, within a rounding error.

# Stage 2 Planet Gear Shaft Analysis

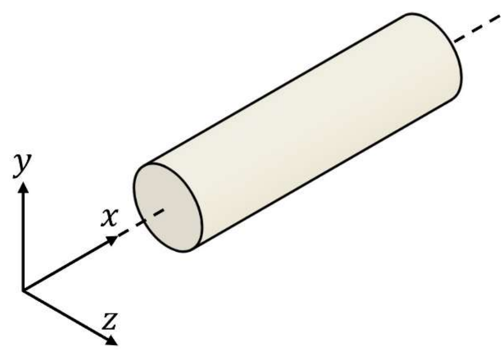


*Figure 3:Stage 2 Planet Gear Shaft with axis.*

$$F_{t,p_1} = 51.1\ N$$
$$F_{R,p_1} = 18.6\ N$$
$$F_{t,p_2} = 97.1\ N$$
$$F_{R,p_2} = 35.3\ N$$
$$d_{shaft} = 10\ mm = 0.010\ m$$
$$A_{shaft} = \frac{\pi}{4} \cdot 0.010^2 = 7.854 \cdot 10^{-5}\ m^2$$
$$I = I_y = I_z = \frac{\pi}{64} \cdot d_{shaft}^2$$
$$I = \frac{\pi}{64} \cdot 0.010^4 = 4.909 \cdot 10^{-10}\ m^4$$
$$J = I_y + I_z = 9.817 \cdot 10^{-10}\ m^4$$

## $x - y$ Plane

$$\Sigma F_y = 0 = R_y + F_{R,p_1} - F_{R,p_2}$$
$$R_y = -F_{R,p_1} + F_{R,p_2} = -18.6 + 35.3$$
$$R_y = 16.7\ N$$
$$R_x = 0\ N$$
$$\Sigma M_{z,A} = 0 = -F_{R,p_2} \cdot 0.003 + F_{R,p_2} \cdot 0.015 + R_y \cdot 0.034 + M_{D,z}$$
$$M_{D,z} = F_{R,p_1} \cdot 0.003 - F_{R,p_2} \cdot 0.015 - R_y \cdot 0.034$$
$$M_{D,z} = 35.3 \cdot 0.003 - 18.6 \cdot 0.015 - 16.7 \cdot 0.034$$
$$M_{D,z} = -0.741\ N \cdot m$$

## $x - z$ Plane

$$\Sigma F_z = 0 = R_z + F_{t,p_1} + F_{t,p_2}$$
$$R_z = -F_{t,p_1} - F_{t,p_2} = -51.1 - 97.1$$
$$R_z = -148.2\ N$$
$$R_x = 0\ N$$
$$\Sigma M_{z,A} = -F_{t,p_2} \cdot 0.003 - F_{t,p_1} \cdot 0.015 - R_z \cdot 0.034 + M_{D,y}$$
$$M_{D,y} = F_{t,p_2} \cdot 0.003 + F_{t,p_1} \cdot 0.015 + R_z \cdot 0.034$$
$$M_{D,y} = 97.1 \cdot 0.003 + 51.1 \cdot 0.015 + (-148.2) \cdot 0.034$$
$$M_{D,y} = -3.98\ N \cdot m$$

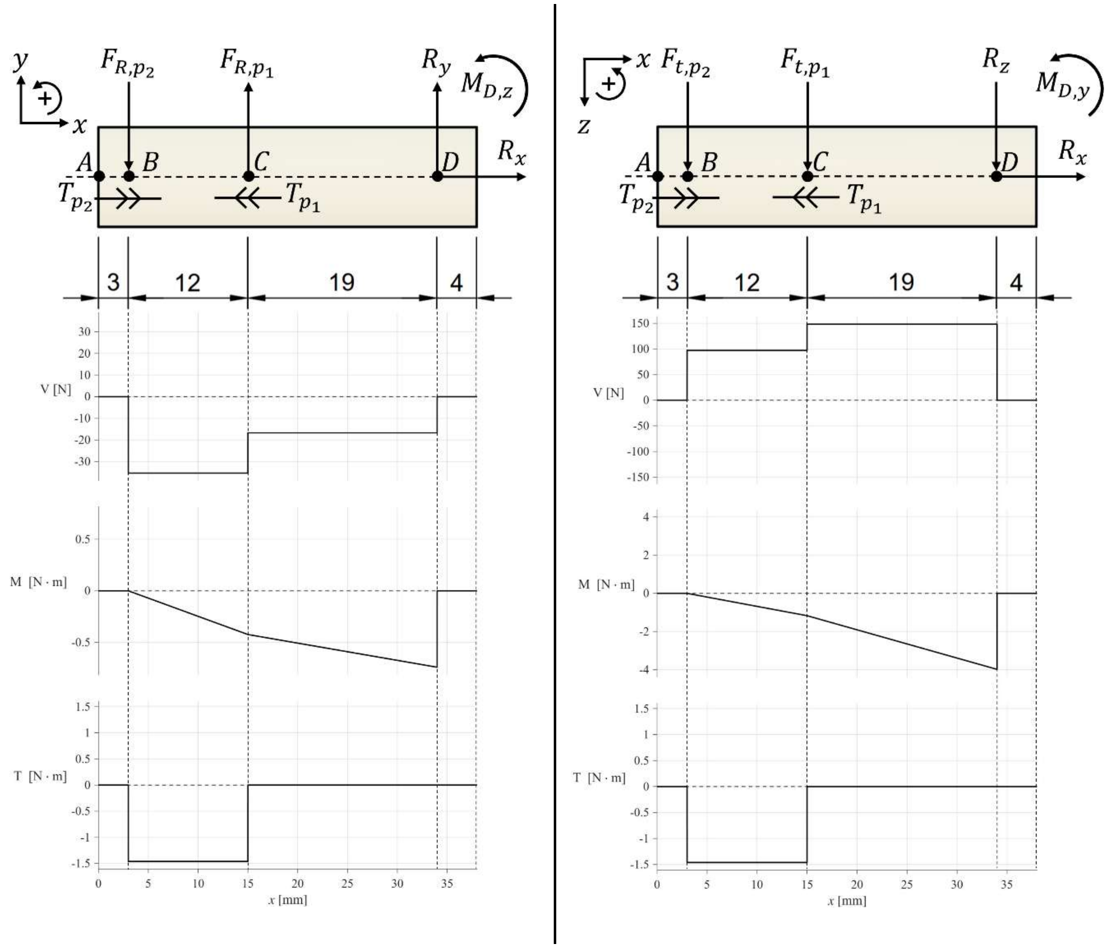


*Figure 4: Shear force, bending moment, and torsion diagrams for the stage planet gear shaft in the $x-y$ and $x-z$ planes.*

The two potential critical points will be the left side of point $C$ and left side of point $D$.

## Point C

$$M_{C,z} = -0.424\ N \cdot m \qquad M_{C,z} = -1.17\ N \cdot m \qquad T_{C,x} = -1.46\ N \cdot m$$

$$M = \sqrt{M_{C,z}^2 + M_{C,z}^2} = \sqrt{(-0.424\ )^2 + (-1.17)^2} = 1.24\ N \cdot m$$

$$S_B = \frac{M \cdot c}{I} = \frac{1.24 \cdot 0.005}{4.909 \cdot 10^{-10}} = 12.6\ MPa$$

$$\tau_T = \frac{T \cdot c}{J} = \frac{1.46 \cdot 0.005}{9.817 \cdot 10^{-10}} = 7.44\ MPa$$

$$S' = \sqrt{S_B^2 + 3 \cdot \tau^2} = \sqrt{12.6^2 + 3 \cdot 7.44^2} = 18.0\ MPa$$

### Point D

$M_{D,z} = -0.741\ N \cdot m$ $\qquad$ $M_{D,z} = -3.98\ N \cdot m$

$$M = \sqrt{M_{D,z}^2 + M_{D,z}^2} = \sqrt{(-0.741\ )^2 + (-3.98)^2} = 4.05\ N \cdot m$$

$$S_B = \frac{M \cdot c}{I} = \frac{4.05 \cdot 0.005}{4.909 \cdot 10^{-10}} = 41.3\ MPa$$

$$S' = \sqrt{S_B^2} = 41.3\ MPa$$

## Shaft Stress Summary

The stage 1 planet gear shaft was analysed in a similar manner to stage 2, with the difference being the positions of the two planet gears are swapped. The shafts on the main axis only have a net torque force, since the axial symmetry causes the other forces to cancel out. For both planet gear shafts the critical point is at the bearing where the bending moment is highest.

Table 4 gives a summary of the forces and stresses in the shafts, showing they all have a factor of safety greater than 2. The shafts are all aluminium, with the shafts being the alloy 6063-T5 [2, p. 2], except for the output shaft which is Alloy 6061-T6 [3].

*Table 4: Shaft forces and stresses summary table*

| Shaft | Input | Middle | Output | Planet 1 | Planet 2 |
|---|---|---|---|---|---|
| $S_B$ | 0 | 0 | 0 | 0.720 | 3.98 |
| $T$ | 0.643 | 3.69 | 20 | 0 | 0 |
| $S'(MPa)$ | 5.67 | 32.6 | 44.1 | 7.33 | 41.3 |
| $S_y\ (MPa)$ | 110 | 110 | 240 | 110 | 110 |
| FOS | 19.4 | 3.37 | 5.44 | 15.0 | 2.66 |

# References


[1] D. Jelaska, Gears and Gear Drives, West Sussex: John Wiley & Sons Ltd, 2012.

[2] Atlas Steel, “Aluminium Alloy 6063 Data Sheet,” 11 August 2021. [Online]. Available: https://atlassteels.com.au/wp-content/uploads/2021/08/Aluminium-Alloy-6063-Data-Sheet-11-08-21.pdf. [Accessed 20 October 2025].

[3] BlueScope Distribution, “Aluminium 6061 Data Sheet,” [Online]. Available: https://www.bluescopedistribution.com.au/products/aluminium/aluminium-6061-data-sheet/. [Accessed 24 October 2025].

[4] R. C. Juvinall and K. M. Marshek, Fundamentals of Machine Component Design, Hoboken: John Wiley & Sons, Inc. , 2020.

[5] S. Bell, *Planetary Gearbox Theory Lecture Notes,* Brisbane: Queensland University of Technology, 2025.

# Appendix: Gear Analysis and Equations

## Sun Gear Forces and Radial Force

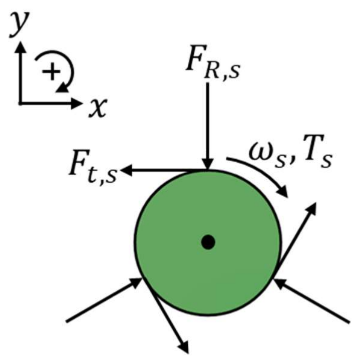


*Figure 5: Sun gear forces diagram.*

$$\Sigma M_x = 0 = T_s - n_p \cdot r_s \cdot F_{t,s}$$

$$F_{t,s} = \frac{T_s}{n_p \cdot r_s} \quad \text{Eq. 1}$$

$$F_R = F_t \cdot \tan(20°) \quad \text{Eq. 2}$$

## Carrier Forces

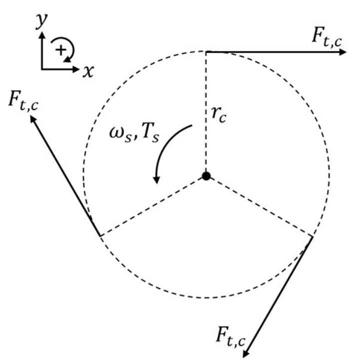


*Figure 6: Carrier forces diagram.*

$$\Sigma M_x = 0 = -T_c + n_p \cdot r_c \cdot F_{t,c}$$

$$F_{t,c} = \frac{T_c}{n_p \cdot r_c} \quad \text{Eq. 3}$$

The forces act at the centre of the planet gear bearings and are tangential to the bearings pitch circle.

## Ring Gear Forces

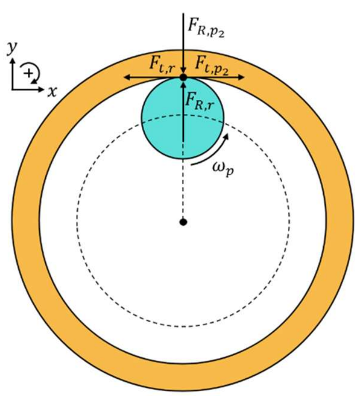


*Figure 7: Ring gear forces diagram.*

$$F_{t,r} = F_{t,p_2} \cdot \eta_{actual} \quad \text{Eq. 4}$$

## Planet Gears Forces

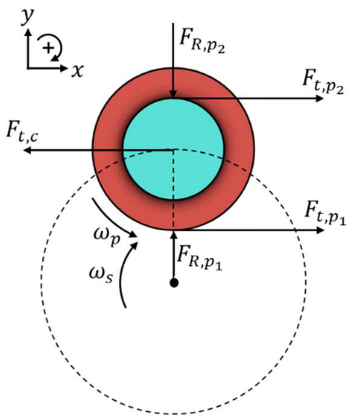


*Figure 8: Planet gear forces diagram.*

The two planet gears are fixed to the same shaft, therefore $\omega_{p_1} = \omega_{p_2} = \omega_p$

$$\Sigma M_O = 0 = F_{t,p_1} \cdot r_{p_1} - F_{t,p_2} \cdot r_{p_2}$$

$$F_{t,p_1} \cdot r_{p_1} = F_{t,p_2} \cdot r_{p_2}$$

$$F_{t,p_1} = F_{t,p_2} \cdot \frac{r_{p_2}}{r_{p_1}} \qquad \text{Eq. 5}$$

$$\Sigma F_x = 0 = F_{t,p_1} + F_{t,p_2} - F_{t,c}$$

$$F_{t,c} = F_{t,p_1} + F_{t,p_2} = F_{t,p_2} \cdot \frac{r_{p_2}}{r_{p_1}} + F_{t,p_2}$$

$$F_{t,c} = F_{t,p_2}\left(1 + \frac{r_{p_2}}{r_{p_1}}\right)$$

$$F_{t,p_2} = \frac{F_{t,c}}{\left(1 + \frac{r_{p_2}}{r_{p_1}}\right)} \qquad \text{Eq. 6}$$

## Gear Ratio

$$i = 1 + \frac{z_{p_1} \cdot z_r}{z_{p_2} \cdot z_s} \qquad \text{[1, pp. 343-347]} \qquad \text{Eq. 7}$$

$$N_s = N_c \cdot i \qquad N_c = \frac{N_s}{i}$$

## Module Ratio

Since the planet gears are coaxial, and the number of gear teeth and sun gear module is set, the module of the ring gear is constrained.

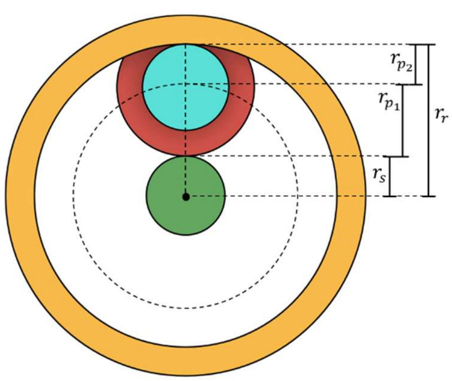


*Figure 9: Module ratio diagram.*

$$r_c = r_s + r_{p_1} \qquad \text{Eq. 8}$$

$$r_s + r_{p_1} = r_r - r_{p_2}$$

$$\frac{m_s \cdot z_s}{2} + \frac{m_s \cdot z_{p_1}}{2} = \frac{m_r \cdot z_r}{2} - \frac{m_r \cdot z_{p_2}}{2}$$

$$m_s(z_s + z_{p_1}) = m_r(z_r - z_{p_2})$$

$$\frac{m_s}{m_r} = \frac{z_r - z_{p_2}}{z_s + z_{p_1}}$$

$$m_r = m_s \cdot \frac{z_s + z_{p_1}}{z_r - z_{p_2}} \qquad \text{Eq. 9}$$

## Assembly Condition

Teeth of all the planet gears must mesh with the central gear at the same time, to avoid any tooth penetrating any tooth of the mating gear. This can be checked using the equation

$$k = \frac{z_s \cdot z_{p_2} - z_r \cdot z_{p_1}}{n_p \cdot \gcd\left(z_{p_1}, z_{p_2}\right)} \qquad \text{Eq. 10}$$

where $k$ must be a whole number [1, pp. 343, Eq. 6.20].

## Gear Stresses

The Lewis bending stress equation is used to calculate the maximum bending stress on a gear tooth [4, pp. 455, Eq. 15.16a].

$$S_{allow} = \frac{F_t}{m \cdot b \cdot Y} \qquad \text{Eq. 11}$$

## Output Torque and Efficiency

Since the output torque is known, the required input torque can be calculated by using the input and output, along with an initial guess at the efficiency {Cite planetary gear calculations example}.

$$T_s = \frac{-T_c \cdot \beta_r}{\eta_0 \cdot \beta_s - \beta_r} \qquad [5]$$

$$\beta_s = N_s - N_c$$

$$\beta_r = N_r - N_c = 0 - N_c = -N_c \qquad N_r = 0$$

$$T_s = \frac{-T_c \cdot (-N_c)}{\eta_0 \cdot (N_s - N_c) - (-N_c)}$$

$$T_s = \frac{T_c \cdot N_c}{\eta_0 \cdot (N_s - N_c) + N_c} \qquad \text{Eq. 12}$$

To calculate the initial guess for the efficiency

$$\eta_0 = \left(1 - L_{s-p}\right)\left(1 - L_{p-r}\right) = \left(1 - L_{ext}\right)\left(1 - \frac{z_r/z_s - 1}{z_r/z_s + 1} \cdot L_{ext}\right) \qquad \text{Eq. 13}$$

$$L_{s-p} = L_{ext} \qquad L_{p-r} = L_{int} = \frac{R-1}{R+1} \cdot L_{ext} \qquad R = \frac{z_r}{z_s}$$

Using $R = \frac{z_{p_1} \cdot z_r}{z_{p_2} \cdot z_s}$ will give a higher efficiency, but since there is uncertainty that this should be used instead, using $R = \frac{z_r}{z_s}$ is more conservative in terms of calculating maximum gear stress. The actual efficiency can be calculated by the input and output power

$$\eta_{actual} = \frac{T_c \cdot N_c}{T_s \cdot N_s} = \frac{\dot{W}_c}{\dot{W}_s} \qquad \text{Eq. 14}$$

## Sun Gear Module

The module of the sun gear can be solved for given the output torque from the carrier, gear ratio, sun gear parameters and yield stress of the gear material.

Rearranging the Lewis bending stress equation for the tangential force. A factor of safety is applied to the materials yield stress to determine the maximum allowable stress.

$$S_{allow} = \frac{F_{t,s}}{m_s \cdot b_s \cdot Y_s} \Rightarrow F_{t,s} = S_{allow} \cdot m_s \cdot b_s \cdot Y_s$$

$$S_{allow} = \frac{S_y}{FOS}$$

$$F_{t,s} = \frac{S_y}{FOS} \cdot m_s \cdot b_s \cdot Y_s \qquad \text{Eq. A}$$

Taking the equation for the tangential force on the sun gear and substituting the radius for the module and number of teeth.

$$F_{t,s} = \frac{T_s}{n_p \cdot r_s} = \frac{2 \cdot T_s}{n_p \cdot m_s \cdot z_s} \qquad \text{Eq. B}$$

$$r_s = \frac{m_s \cdot z_s}{2}$$

Simplifying the equation for the torque at the sun gear.

$$T_s = \frac{T_c \cdot N_c}{\eta_0 \cdot (N_s - N_c) + N_c} = \frac{T_c \cdot \frac{N_s}{i}}{\eta_0 \cdot \left(N_s - \frac{N_s}{i}\right) + \frac{N_s}{i}} = \frac{T_c}{\eta_0 \cdot (i-1) + 1} \qquad \text{Eq. C}$$

$$N_c = \frac{N_s}{i}$$

Equate equations A and B

$$\frac{2 \cdot T_s}{n_p \cdot m_s \cdot z_s} = \frac{S_y}{FOS} \cdot m_s \cdot b_s \cdot Y_s$$

$$T_s = \frac{1}{2} \frac{S_y}{FOS} \cdot m_s^2 \cdot b_s \cdot Y_s \cdot n_p \cdot z_s \qquad \text{Eq. D}$$

Equate equations C and D, then solve for the module.

$$\frac{1}{2} \frac{S_y}{FOS} \cdot m_s^2 \cdot b_s \cdot Y_s \cdot n_p \cdot z_s = \frac{T_c}{\eta_0 \cdot (i-1) + 1}$$

$$\boxed{m_s = \sqrt{\frac{2 \cdot T_c}{[\eta_0 \cdot (i-1) + 1] \cdot \frac{S_y}{FOS} \cdot b_s \cdot Y_s \cdot n_p \cdot z_s}} \qquad \text{Eq. 15}}$$

## Pitch Line Velocity

The pitch line velocities of mating gears must equal under the no-slip condition {cite reference}. Since the planet gears move along the carriers pitch circle, the linear velocity at their pitch lines will be the sum of the velocity components from their own rotation and the rotation of the carrier. Since the planet gears are attached to the same shaft, the two equations for $N_p$ must be equal. This can be used to verify calculations.

$$\omega = \frac{2 \cdot \pi \cdot N}{60}$$

$$v = \omega \cdot r = \frac{2 \cdot \pi \cdot N}{60} \cdot \frac{d}{2} \quad \Rightarrow \quad v = \frac{\pi \cdot d \cdot N}{60} \qquad \text{Eq. 16}$$

### *Sun-Planet*

Since the sun gear is stationary its pitch line velocity only comes from its rotation. The planet gears rotate around the sun gear, so their pitch line velocity is the sum of the components from its own rotation and the rotation of the carrier.

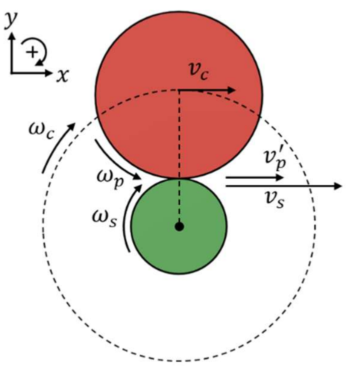


*Figure 10: Planet gear 1 velocity diagram.*

$$v_s = v_p = v_{p_1}' + v_c$$

$$v_{p_1}' = v_s - v_c \qquad \text{Eq. 17}$$

$$-\omega_p \cdot r_{p_1} = \omega_s \cdot r_s - \omega_c \cdot r_c$$

$$\omega_p \cdot r_{p_1} = \omega_c \cdot r_c - \omega_s \cdot r_s$$

$$\frac{\pi \cdot d_{p_1} \cdot N_p}{60} = \frac{\pi \cdot d_c \cdot N_c}{60} - \frac{\pi \cdot d_s \cdot N_s}{60}$$

$$N_{p,\alpha} = \frac{1}{d_{p_1}}(d_c \cdot N_c - d_s \cdot N_s) \qquad \text{Eq. 18}$$

### *Ring-Planet*

Since the ring gear is stationary, the absolute velocity of the planet at the pitch line must be zero. This means the velocity component from the rotation of the planet equals the velocity component from the carrier, in the opposite direction.

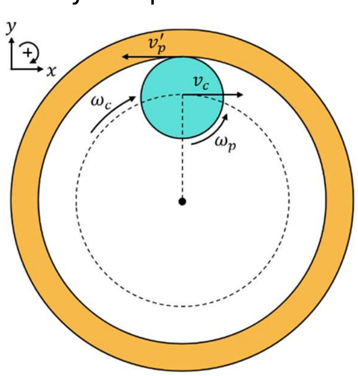


*Figure 11: Planet gear 2 velocity diagram.*

$$v_r = \omega_r \cdot r_r = (0) \cdot r_r = 0$$

$$v_r = v_{p_2} = -v_{p_2}' + v_c = 0$$

$$v_{p_2}' = v_c \qquad \text{Eq. 19}$$

$$d_{p_2} \cdot N_p = d_c \cdot N_c$$

$$N_{p,\beta} = N_c \frac{d_c}{d_{p_2}} \qquad \text{Eq. 20}$$