8 Hypothesis Tests for One Population Parameter

Chapter Preview

In the previous chapter, we looked at doing Statistical Inference using confidence intervals. In this chapter, we will examine an alternative method of Statistical Inference using hypothesis tests.

Reading

Please read chapter 8 of the textbook (Devore).

8.1: Hypothesis and Test Procedures

Example Question: Suppose it is claimed that employees at a particular company spend 5 hours on Facebook during the work week. How do we test this claim?

(1) One way to test this claim would be to collect a sample and see if 5 is contained in the confidence interval for μ.

(2) Another approach to testing this claim is to again collect a sample and see how consistent that sample is with initial hypothesis of $\mu = 5$. This is called hypothesis testing.

Goal of Hypothesis Testing: Test competing claims about a population parameter.

Def: The null hypothesis, denoted H_0, is the claim that is initially assumed to be true.

Def: The alternative hypothesis, denoted H_a, is the assertion contradictory to H_0 ("the opposite of H_0").

Note: The null hypothesis will be rejected in favor of the alternative hypothesis if the sample evidence suggests that H_0 is false.

Note: The two possible conclusions of a hypothesis test are Reject H_0 or Fail to Reject H_0.

Formulating Hypotheses

Rules for formulating hypotheses
1. H_0 contains "=".
2. H_a contains "$\neq$", ">", "<".
3. Usually, the claim we are attempting to show is more plausible is H_a.

Example: Joe claims that homework for this class takes, on average, six hours per week. Aaron doesn't think the average is six hours, so he collects a random sample of eight students and records the time it takes them to complete the homework.

Example: Brenda claims that newly graduated electrical engineers have an average salary of \$63,000, but Rawand believes it is smaller.

Example: Steve believes less than 50% of college students have consumed alcohol. You collect a random sample of 20 students to test the claim.

Different Types of Hypothesis Tests

There are three different types of hypothesis tests.

1. Left-tailed test

2. Right-tailed test

3. Two-tailed test

Errors in Hypothesis Testing

After collecting a sample, we will decide which hypothesis is supported by the data.

Since we don't know the true parameter value, there is a chance we will make an error.

Two Types of Error
1. Type I error (α): Reject H_0 when H_0 is true.
2. Type II error (β): Fail to reject H_0 when H_0 is false.

Note: α is also called the level of significance for a test.

Example: Jury trials, H_0 : Defendant is innocent vs. H_A : Defendant is guilty

Six Steps for Hypothesis Testing

Different textbooks will give a different number of steps or outline on how to complete a hypothesis test. For our class, stick to the following method.

Step 1: State the hypotheses.

Step 2: Determine the level of significance.

Step 3: Compute a test statistic.

Step 4: Calculate a p-value.

Step 5: Make a statistical decision.

Step 6: Interpret the statistical decision (in the context of the problem).

8.2: Tests About a Population Mean, μ

σ known

If σ is known, then the test statistic is the following:

$$\boxed{z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}}$$

Example: Suppose it is claimed that the average lifespan of houseflies is 28 days. Additionally, suppose we know that housefly lifespans are approximately normally distributed with population variance 56 days. A sample of 12 houseflies finds a sample mean of 29 days. Complete a six-step hypothesis test to test the claim that houseflies live on average 28 days at $\alpha = 0.05$.

Example: It is claimed that the average tax refund is greater than \$3,000. Suppose that tax refunds are approximately normally distributed with a population standard deviation of \$700. A sample of 22 tax refunds found an average tax refund of \$3,200. Complete a six-step hypothesis test for the claim that the average tax refund is greater than \$3,000 at a significance level of $\alpha = 0.10$. Also, state what Type I and Type II errors are in the context of the problem.

σ unknown

If σ is unknown, then the test statistic is the following:

$$\boxed{t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}, df = n - 1}$$

Example: It is claimed that the weight of a penny is 2.5 grams. A sample of 20 pennies found an average weight of 2.4 grams with a sample standard deviation of 0.3 grams. Complete a six-step hypothesis test to test the claim at $\alpha = 0.01$.

Example: It is claimed that college students spend less than $500 on average books on textbooks each semester. A sample of 20 college students found that they spend an average of $452 on textbooks with a sample standard deviation of $112. Test the claim at $\alpha = 0.05$.

8.3: Tests Concerning a Population Proportion, p

We can test a population proportion using the following formula:

$$\boxed{z = \frac{\bar{p} - p_0}{\sqrt{p_0(1-p_0)/n}}}$$

Example: One news source claims that Congress has the support more than 30% of Republicans nationally. Suppose we sample 99 Republicans and find 24 of them approve of Congress. Test the claim that Congress's Repubican support is greater than 30% at a significance level of $\alpha = 0.05$.