**Stat 315** **Name (Print):** Solutions

**~~Fall 2021~~** Spring 2024

**Practice Exam 2**

**~~11/12/2021~~** Exam 2: Fri, April 12 (in class)

**Time Limit: 50 minutes** Section: ____________

This exam contains 12 pages (including this cover page) and 5 problems.

You may use a two-sided 4"x6" notecard of formulas/notes, etc and a calculator. You may *not* use any other material including the internet, other people, or other reference books. Violators of this provision will receive a zero.

You are required to show your work on each problem on this exam. The following rules apply:

- **Show all your work**. You may check your answers using calculator functions, but you must show every step of your calculations to receive full credit.
- **Organize your work** in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.

Do not write in the table to the right.

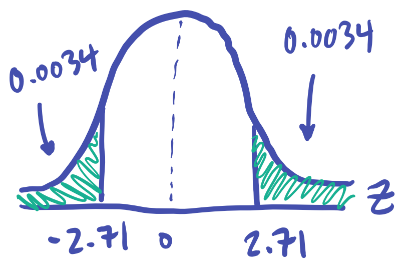
| Problem | Points | Score |
|---|---|---|
| 1 | 10 | |
| 2 | 10 | |
| 3 | 10 | |
| 4 | 10 | |
| 5 | 10 | |
| Total: | 50 | |

$p = 0.11$

(ch8) 1. (10 points) It is claimed that 11% of bridges in the United States are structurally deficient. To test this claim, inspections of 200 randomly selected bridges found that 34 were structurally deficient. At $\alpha = 0.025$, test the claim that the proportion of bridges that are structurally deficient differs from 0.11. Also, explain which type of error may have been committed.

$p \neq 0.11$ $\left.\begin{matrix} n=200 \\ x=34 \end{matrix}\right\} \rightarrow \bar{p} = \frac{34}{200} = \underline{0.17}$

two-sided test $\left\{\begin{matrix} ① \; H_0: p = 0.11 \\ H_a: p \neq 0.11 \end{matrix}\right.$ ② $\alpha = 0.025$ ③ $z_{test} = \frac{\bar{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{0.17 - 0.11}{\sqrt{\frac{0.11(0.89)}{200}}} = \underline{2.71}$

④ 


$\rightarrow$ p-value $= 2 \cdot 0.0034$

$\rightarrow$ <u>p-value $= 0.0068$</u>

⑤ p-value $= 0.0068 < \alpha = 0.025 \rightarrow$ <u>Reject $H_0$</u>

⑥ At $\alpha = 0.025$, there is sufficient evidence to conclude that the true proportion of structurally deficient bridges in the US differs from 0.11.

<u>Follow up</u>: Type I error: reject $H_0$ when $H_0$ is true $\rightarrow$ possible we made a Type I error since we rejected $H_0$.

Type II error: FTR $H_0$ when $H_0$ is false diff = after - before → $H_a: \mu_d > 0$

(ch 9)

2. 5 students took two Math exams: one exam before tutoring and one exam after tutoring. It is of interest if tutoring helps the students score higher. Below are the exam scores before and after. Assume the test scores are approximately normally distributed.

after-before → "point gain"

| | 1 | 2 | 3 | 4 | 5 | |
|---|---|---|---|---|---|---|
| After | 90 | 80 | 95 | 70 | 75 | |
| Before | 85 | 80 | 90 | 55 | 50 | |
| $d_i$ | 5 | 0 | 5 | 15 | 25 | $\Sigma d_i = 50$ |
| $d_i - \bar{d}$ | -5 | -10 | -5 | 5 | 15 | $\Sigma(d_i - \bar{d}) = 0$ ✓ |
| $(d_i - \bar{d})^2$ | 25 | 100 | 25 | 25 | 225 | $\Sigma(d_i - \bar{d})^2 = 400$ |

(a) (3 points) Complete the above table

(b) (1 point) Calculate $\bar{d} = \frac{\Sigma d_i}{n} = \frac{50}{5} = \underline{10 = \bar{d}}$

(c) (1 point) Calculate $s_d$

$$s_d^2 = \frac{1}{n-1} \Sigma (d_i - \bar{d})^2 = \frac{1}{5-1} \cdot 400 = 100 = s_d^2 \rightarrow \underline{s_d = \sqrt{100} = 10}$$

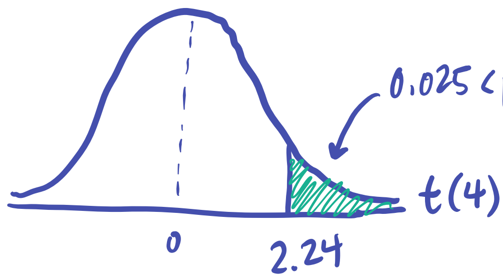
(d) (5 points) Complete a hypothesis test at $\alpha = 0.05$. Show all six steps.

① $H_0: \mu_d = 0$
$H_a: \mu_d > 0$

② $\alpha = 0.05$

③ $t_{test} = \frac{\bar{d} - \mu_{d_0}}{s_d/\sqrt{n}} = \frac{10 - 0}{10/\sqrt{5}} = \underline{2.24}$ (df $= n-1 = 4$)

④




⑤ Reject $H_0$

⑥ At $\alpha=0.05$, there is sufficient evidence to conclude that students, on average, score higher after tutoring than before.

(ch6)

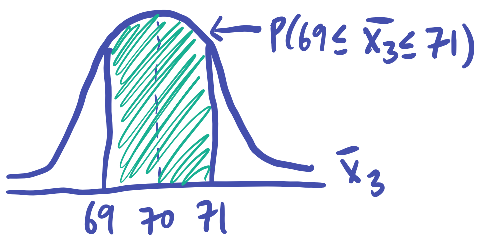
3. Suppose that adult male heights are normally distributed with mean 70 inches and standard deviation 4 inches. A random sample of three male heights is drawn.

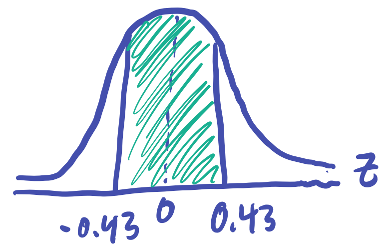
(a) (3 points) What is the sampling distribution of $\bar{X}_3 = \frac{X_1+X_2+X_3}{3}$?

$X_i \sim N(\mu = 70, \sigma = 4)$

$$\boxed{\bar{X}_3 \sim N\left(\mu = 70, \sigma = \frac{4}{\sqrt{3}}\right)}$$

(b) (3 points) Calculate the probability the sample mean is within one inch of the population mean.




$z_1 = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}} = \frac{69-70}{4/\sqrt{3}} = -0.43$

$z_2 = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}} = \frac{71-70}{4/\sqrt{3}} = 0.43$

$P(69 \le \bar{X}_3 \le 71) = P(-0.43 \le Z \le 0.43)$

$= P(Z \le 0.43) - P(Z \le -0.43)$

$= 0.6664 - 0.3336 = \boxed{0.3328}$

(c) (4 points) Suppose instead of using the sample mean of the three men, i.e. $\bar{X}_3 = \frac{X_1+X_2+X_3}{3}$, we estimate the population mean by calculating the statistic $\gamma = \frac{X_1}{2} + \frac{X_2}{4} + \frac{X_3}{4}$. Determine if $\gamma$ is an unbiased estimator for $\mu$ and find $SE(\gamma)$.

$\gamma$ is an unbiased est. of $\mu$ if $E[\gamma] = \mu$

$E[\gamma] = E\left[\frac{X_1}{2} + \frac{X_2}{4} + \frac{X_3}{4}\right] = \frac{E[X_1]}{2} + \frac{E[X_2]}{4} + \frac{E[X_3]}{4} = \frac{70}{2} + \frac{70}{4} + \frac{70}{4} = 70 = \mu$

$\rightarrow$ $\gamma$ is an unbiased est. of $\mu$

$SE[\gamma] = \sqrt{Var(\gamma)}$

$Var(\gamma) = Var\left(\frac{X_1}{2} + \frac{X_2}{4} + \frac{X_3}{4}\right) \overset{\text{by indep.}}{=} \frac{Var(X_1)}{2^2} + \frac{Var(X_2)}{4^2} + \frac{Var(X_3)}{4^2}$

$= \frac{16}{4} + \frac{16}{16} + \frac{16}{16} = 6$

$\rightarrow$ $SE[\gamma] = \sqrt{Var(\gamma)} = \sqrt{6}$

(ch9)

4. Suppose that a sample of six Mathematics majors found they spend on average 10 hours weekly on homework with a sample standard deviation of 4 hours. Also, suppose that a sample of eight Electrical Engineering majors found they spend an average of 8.5 hours on homework weekly with a sample standard deviation of 3.8 hours. Finally, assume that the two populations are approximately normally distributed. Use $df = 10$ if needed.

(two independent samples, $\sigma$ is unknown → use t-dist, $df=10$)

(a) (8 points) Create a 99% confidence interval for the difference in mean homework time spent between Mathematics and Electrical Engineering majors.

Math (pop #1): $\bar{x}_1 = 10$, $s_1 = 4$, $n_1 = 6$

EE (pop #2): $\bar{x}_2 = 8.5$, $s_2 = 3.8$, $n_2 = 8$

$\underbrace{99\%\ CI\ (df=10)}$ → $t_{c.v.} = 3.169$

CI: $(\bar{x}_1 - \bar{x}_2) \pm t_{c.v.} \cdot \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} \rightarrow (10-8.5) \pm 3.169 \cdot \sqrt{\frac{4^2}{6} + \frac{3.8^2}{8}}$

$\rightarrow 1.5 \pm 6.7$

$\rightarrow \boxed{-5.2 < \mu_1 - \mu_2 < 8.2}$

(b) (2 points) What is power and how could it be increased in this example?

power = prob. of rejecting Ho when Ho is false

- can increase power by increasing sample size.

No chapter 10 on this exam



(ch10)

7. Jim is analyzing the amount of mercury in three types of fish: bass, rainbow trout, and brown trout. It is of interest to determine if all three types of fish have the same mean mercury content. He collects 8 fish of each type and measures their mercury content (in ppb, parts per billion). Some results are given in the R output below.

$N = 8 \cdot 3 = 24$

$k = 3$

```
            Df  Sum Sq Mean Sq F value Pr(>F)
fish        2   172
Residuals   21  7396
```

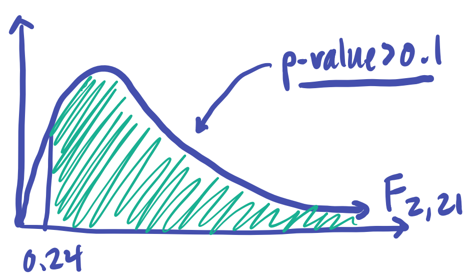
(a) (2 points) Calculate $MSTR$ and $MSE$.

$$MSTR = \frac{SSTR}{df_{TR}} = \frac{172}{2} = \underline{86 = MSTR}$$

$$MSE = \frac{SSE}{df_E} = \frac{7396}{21} = \underline{352.2 = MSE}$$

(b) (4 points) Calculate the relevant test statistic, find the p-value, and plot the distribution for this ANOVA F-test.

$$F_{test} = \frac{MSTR}{MSE} = \frac{86}{352.2} = \underline{0.24} \quad (df_1 = 2,\ df_2 = 21)$$




(FTR $H_0$)

(c) (2 points) State the conclusion of your ANOVA F-test in your own words.

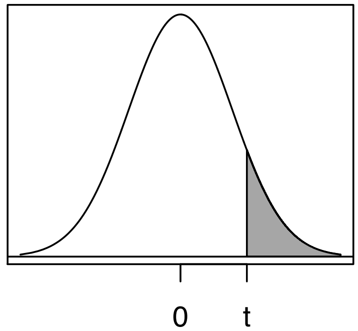
At $\alpha = 0.05$, there is not sufficient evidence to conclude the average mercury content differs among the three types of fish.

(d) (2 points) Suppose we want to compare all pairwise difference in mercury content for the three types of fish. How many pairwise comparisons are there?

$$\binom{3}{2} = \frac{3!}{2!\,1!} = \underline{3}$$

$$\left.\begin{matrix} AB \\ AC \\ BC \end{matrix}\right\} 3$$

# t Distribution

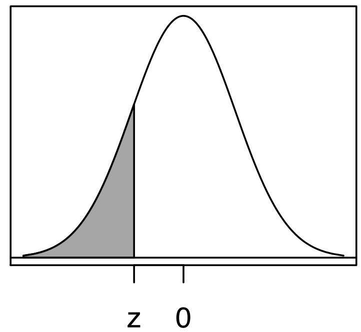



p-value > 0.

t-values for selected UPPER TAIL probabilities are shown in the following table:

| | | 90% | 95% | | 99% | ← For **this** CI |
|---|---|---|---|---|---|---|
| **df** | **.10** | **.05** | **.025** | **.01** | **.005** | ← Upper tail probability |
| 1 | 3.078 | 6.314 | 12.706 | 31.821 | 63.657 | |
| 2 | 1.886 | 2.920 | 4.303 | 6.965 | 9.925 | |
| 3 | 1.638 | 2.353 | 3.182 | 4.541 | 5.841 | |
| 4 | 1.533 | 2.132 | 2.776 | 3.747 | 4.604 | |
| | | 2.24 | | | | |
| 5 | 1.476 | 2.015 | 2.571 | 3.365 | 4.032 | |
| 6 | 1.440 | 1.943 | 2.447 | 3.143 | 3.707 | |
| 7 | 1.415 | 1.895 | 2.365 | 2.998 | 3.499 | |
| 8 | 1.397 | 1.860 | 2.306 | 2.896 | 3.355 | |
| 9 | 1.383 | 1.833 | 2.262 | 2.821 | 3.250 | |
| 10 | 1.372 | 1.812 | 2.228 | 2.764 | 3.169 | |
| 11 | 1.363 | 1.796 | 2.201 | 2.718 | 3.106 | |
| 12 | 1.356 | 1.782 | 2.179 | 2.681 | 3.055 | |
| 13 | 1.350 | 1.771 | 2.160 | 2.650 | 3.012 | |
| 14 | 1.345 | 1.761 | 2.145 | 2.624 | 2.977 | |
| 15 | 1.341 | 1.753 | 2.131 | 2.602 | 2.947 | |
| 16 | 1.337 | 1.746 | 2.120 | 2.583 | 2.921 | |
| 17 | 1.333 | 1.740 | 2.110 | 2.567 | 2.898 | |
| 18 | 1.330 | 1.734 | 2.101 | 2.552 | 2.878 | |
| 19 | 1.328 | 1.729 | 2.093 | 2.539 | 2.861 | |
| 20 | 1.325 | 1.725 | 2.086 | 2.528 | 2.845 | |
| 21 | 1.323 | 1.721 | 2.080 | 2.518 | 2.831 | |
| 22 | 1.321 | 1.717 | 2.074 | 2.508 | 2.819 | |
| 23 | 1.319 | 1.714 | 2.069 | 2.500 | 2.807 | |
| 24 | 1.17 1.318 | 1.711 | 2.064 | 2.492 | 2.797 | |
| 25 | 1.316 | 1.708 | 2.060 | 2.485 | 2.787 | |
| 26 | 1.315 | 1.706 | 2.056 | 2.479 | 2.779 | |
| 27 | 1.314 | 1.703 | 2.052 | 2.473 | 2.771 | |
| 28 | 1.313 | 1.701 | 2.048 | 2.467 | 2.763 | |
| 29 | 1.311 | 1.699 | 2.045 | 2.462 | 2.756 | |
| 30 | 1.310 | 1.697 | 2.042 | 2.457 | 2.750 | |
| 40 | 1.303 | 1.684 | 2.021 | 2.423 | 2.704 | |
| 50 | 1.299 | 1.676 | 2.009 | 2.403 | 2.678 | |
| 60 | 1.296 | 1.671 | 2.000 | 2.390 | 2.660 | |
| 70 | 1.294 | 1.667 | 1.994 | 2.381 | 2.648 | |
| 80 | 1.292 | 1.664 | 1.990 | 2.374 | 2.639 | |
| 90 | 1.291 | 1.662 | 1.987 | 2.368 | 2.632 | |
| 100 | 1.290 | 1.660 | 1.984 | 2.364 | 2.626 | |
| ∞ | 1.282 | 1.645 | 1.960 | 2.326 | 2.576 | ← Same as **z-values** |

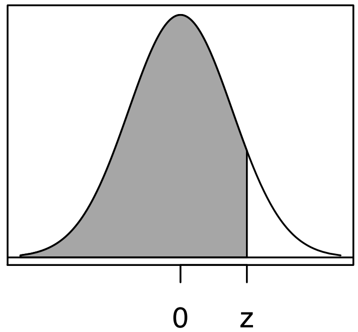
Standard Normal Distribution




Cumulative probabilities for **NEGATIVE** z-values are shown in the following table:

| Z | .00 | .01 | .02 | .03 | .04 | .05 | .06 | .07 | .08 | .09 |
|---|---|---|---|---|---|---|---|---|---|---|
| −3.4 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0002 |
| −3.3 | 0.0005 | 0.0005 | 0.0005 | 0.0004 | 0.0004 | 0.0004 | 0.0004 | 0.0004 | 0.0004 | 0.0003 |
| −3.2 | 0.0007 | 0.0007 | 0.0006 | 0.0006 | 0.0006 | 0.0006 | 0.0006 | 0.0005 | 0.0005 | 0.0005 |
| −3.1 | 0.0010 | 0.0009 | 0.0009 | 0.0009 | 0.0008 | 0.0008 | 0.0008 | 0.0008 | 0.0007 | 0.0007 |
| −3.0 | 0.0013 | 0.0013 | 0.0013 | 0.0012 | 0.0012 | 0.0011 | 0.0011 | 0.0011 | 0.0010 | 0.0010 |
| −2.9 | 0.0019 | 0.0018 | 0.0018 | 0.0017 | 0.0016 | 0.0016 | 0.0015 | 0.0015 | 0.0014 | 0.0014 |
| −2.8 | 0.0026 | 0.0025 | 0.0024 | 0.0023 | 0.0023 | 0.0022 | 0.0021 | 0.0021 | 0.0020 | 0.0019 |
| −2.7 | 0.0035 | 0.0034 | 0.0033 | 0.0032 | 0.0031 | 0.0030 | 0.0029 | 0.0028 | 0.0027 | 0.0026 |
| −2.6 | 0.0047 | 0.0045 | 0.0044 | 0.0043 | 0.0041 | 0.0040 | 0.0039 | 0.0038 | 0.0037 | 0.0036 |
| −2.5 | 0.0062 | 0.0060 | 0.0059 | 0.0057 | 0.0055 | 0.0054 | 0.0052 | 0.0051 | 0.0049 | 0.0048 |
| −2.4 | 0.0082 | 0.0080 | 0.0078 | 0.0075 | 0.0073 | 0.0071 | 0.0069 | 0.0068 | 0.0066 | 0.0064 |
| −2.3 | 0.0107 | 0.0104 | 0.0102 | 0.0099 | 0.0096 | 0.0094 | 0.0091 | 0.0089 | 0.0087 | 0.0084 |
| −2.2 | 0.0139 | 0.0136 | 0.0132 | 0.0129 | 0.0125 | 0.0122 | 0.0119 | 0.0116 | 0.0113 | 0.0110 |
| −2.1 | 0.0179 | 0.0174 | 0.0170 | 0.0166 | 0.0162 | 0.0158 | 0.0154 | 0.0150 | 0.0146 | 0.0143 |
| −2.0 | 0.0228 | 0.0222 | 0.0217 | 0.0212 | 0.0207 | 0.0202 | 0.0197 | 0.0192 | 0.0188 | 0.0183 |
| −1.9 | 0.0287 | 0.0281 | 0.0274 | 0.0268 | 0.0262 | 0.0256 | 0.0250 | 0.0244 | 0.0239 | 0.0233 |
| −1.8 | 0.0359 | 0.0351 | 0.0344 | 0.0336 | 0.0329 | 0.0322 | 0.0314 | 0.0307 | 0.0301 | 0.0294 |
| −1.7 | 0.0446 | 0.0436 | 0.0427 | 0.0418 | 0.0409 | 0.0401 | 0.0392 | 0.0384 | 0.0375 | 0.0367 |
| −1.6 | 0.0548 | 0.0537 | 0.0526 | 0.0516 | 0.0505 | 0.0495 | 0.0485 | 0.0475 | 0.0465 | 0.0455 |
| −1.5 | 0.0668 | 0.0655 | 0.0643 | 0.0630 | 0.0618 | 0.0606 | 0.0594 | 0.0582 | 0.0571 | 0.0559 |
| −1.4 | 0.0808 | 0.0793 | 0.0778 | 0.0764 | 0.0749 | 0.0735 | 0.0721 | 0.0708 | 0.0694 | 0.0681 |
| −1.3 | 0.0968 | 0.0951 | 0.0934 | 0.0918 | 0.0901 | 0.0885 | 0.0869 | 0.0853 | 0.0838 | 0.0823 |
| −1.2 | 0.1151 | 0.1131 | 0.1112 | 0.1093 | 0.1075 | 0.1056 | 0.1038 | 0.1020 | 0.1003 | 0.0985 |
| −1.1 | 0.1357 | 0.1335 | 0.1314 | 0.1292 | 0.1271 | 0.1251 | 0.1230 | 0.1210 | 0.1190 | 0.1170 |
| −1.0 | 0.1587 | 0.1562 | 0.1539 | 0.1515 | 0.1492 | 0.1469 | 0.1446 | 0.1423 | 0.1401 | 0.1379 |
| −.9 | 0.1841 | 0.1814 | 0.1788 | 0.1762 | 0.1736 | 0.1711 | 0.1685 | 0.1660 | 0.1635 | 0.1611 |
| −.8 | 0.2119 | 0.2090 | 0.2061 | 0.2033 | 0.2005 | 0.1977 | 0.1949 | 0.1922 | 0.1894 | 0.1867 |
| −.7 | 0.2420 | 0.2389 | 0.2358 | 0.2327 | 0.2296 | 0.2266 | 0.2236 | 0.2206 | 0.2177 | 0.2148 |
| −.6 | 0.2743 | 0.2709 | 0.2676 | 0.2643 | 0.2611 | 0.2578 | 0.2546 | 0.2514 | 0.2483 | 0.2451 |
| −.5 | 0.3085 | 0.3050 | 0.3015 | 0.2981 | 0.2946 | 0.2912 | 0.2877 | 0.2843 | 0.2810 | 0.2776 |
| −.4 | 0.3446 | 0.3409 | 0.3372 | 0.3336 | 0.3300 | 0.3264 | 0.3228 | 0.3192 | 0.3156 | 0.3121 |
| −.3 | 0.3821 | 0.3783 | 0.3745 | 0.3707 | 0.3669 | 0.3632 | 0.3594 | 0.3557 | 0.3520 | 0.3483 |
| −.2 | 0.4207 | 0.4168 | 0.4129 | 0.4090 | 0.4052 | 0.4013 | 0.3974 | 0.3936 | 0.3897 | 0.3859 |
| −.1 | 0.4602 | 0.4562 | 0.4522 | 0.4483 | 0.4443 | 0.4404 | 0.4364 | 0.4325 | 0.4286 | 0.4247 |
| −.0 | 0.5000 | 0.4960 | 0.4920 | 0.4880 | 0.4840 | 0.4801 | 0.4761 | 0.4721 | 0.4681 | 0.4641 |

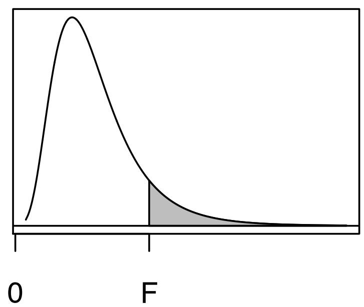
Standard Normal Distribution




Cumulative probabilities for **POSITIVE** z-values are shown in the following table:

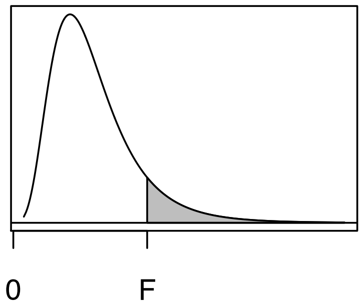
| **Z** | **.00** | **.01** | **.02** | **.03** | **.04** | **.05** | **.06** | **.07** | **.08** | **.09** |
|---|---|---|---|---|---|---|---|---|---|---|
| .0 | 0.5000 | 0.5040 | 0.5080 | 0.5120 | 0.5160 | 0.5199 | 0.5239 | 0.5279 | 0.5319 | 0.5359 |
| .1 | 0.5398 | 0.5438 | 0.5478 | 0.5517 | 0.5557 | 0.5596 | 0.5636 | 0.5675 | 0.5714 | 0.5753 |
| .2 | 0.5793 | 0.5832 | 0.5871 | 0.5910 | 0.5948 | 0.5987 | 0.6026 | 0.6064 | 0.6103 | 0.6141 |
| .3 | 0.6179 | 0.6217 | 0.6255 | 0.6293 | 0.6331 | 0.6368 | 0.6406 | 0.6443 | 0.6480 | 0.6517 |
| .4 | 0.6554 | 0.6591 | 0.6628 | 0.6664 | 0.6700 | 0.6736 | 0.6772 | 0.6808 | 0.6844 | 0.6879 |
| .5 | 0.6915 | 0.6950 | 0.6985 | 0.7019 | 0.7054 | 0.7088 | 0.7123 | 0.7157 | 0.7190 | 0.7224 |
| .6 | 0.7257 | 0.7291 | 0.7324 | 0.7357 | 0.7389 | 0.7422 | 0.7454 | 0.7486 | 0.7517 | 0.7549 |
| .7 | 0.7580 | 0.7611 | 0.7642 | 0.7673 | 0.7704 | 0.7734 | 0.7764 | 0.7794 | 0.7823 | 0.7852 |
| .8 | 0.7881 | 0.7910 | 0.7939 | 0.7967 | 0.7995 | 0.8023 | 0.8051 | 0.8078 | 0.8106 | 0.8133 |
| .9 | 0.8159 | 0.8186 | 0.8212 | 0.8238 | 0.8264 | 0.8289 | 0.8315 | 0.8340 | 0.8365 | 0.8389 |
| 1.0 | 0.8413 | 0.8438 | 0.8461 | 0.8485 | 0.8508 | 0.8531 | 0.8554 | 0.8577 | 0.8599 | 0.8621 |
| 1.1 | 0.8643 | 0.8665 | 0.8686 | 0.8708 | 0.8729 | 0.8749 | 0.8770 | 0.8790 | 0.8810 | 0.8830 |
| 1.2 | 0.8849 | 0.8869 | 0.8888 | 0.8907 | 0.8925 | 0.8944 | 0.8962 | 0.8980 | 0.8997 | 0.9015 |
| 1.3 | 0.9032 | 0.9049 | 0.9066 | 0.9082 | 0.9099 | 0.9115 | 0.9131 | 0.9147 | 0.9162 | 0.9177 |
| 1.4 | 0.9192 | 0.9207 | 0.9222 | 0.9236 | 0.9251 | 0.9265 | 0.9279 | 0.9292 | 0.9306 | 0.9319 |
| 1.5 | 0.9332 | 0.9345 | 0.9357 | 0.9370 | 0.9382 | 0.9394 | 0.9406 | 0.9418 | 0.9429 | 0.9441 |
| 1.6 | 0.9452 | 0.9463 | 0.9474 | 0.9484 | 0.9495 | 0.9505 | 0.9515 | 0.9525 | 0.9535 | 0.9545 |
| 1.7 | 0.9554 | 0.9564 | 0.9573 | 0.9582 | 0.9591 | 0.9599 | 0.9608 | 0.9616 | 0.9625 | 0.9633 |
| 1.8 | 0.9641 | 0.9649 | 0.9656 | 0.9664 | 0.9671 | 0.9678 | 0.9686 | 0.9693 | 0.9699 | 0.9706 |
| 1.9 | 0.9713 | 0.9719 | 0.9726 | 0.9732 | 0.9738 | 0.9744 | 0.9750 | 0.9756 | 0.9761 | 0.9767 |
| 2.0 | 0.9772 | 0.9778 | 0.9783 | 0.9788 | 0.9793 | 0.9798 | 0.9803 | 0.9808 | 0.9812 | 0.9817 |
| 2.1 | 0.9821 | 0.9826 | 0.9830 | 0.9834 | 0.9838 | 0.9842 | 0.9846 | 0.9850 | 0.9854 | 0.9857 |
| 2.2 | 0.9861 | 0.9864 | 0.9868 | 0.9871 | 0.9875 | 0.9878 | 0.9881 | 0.9884 | 0.9887 | 0.9890 |
| 2.3 | 0.9893 | 0.9896 | 0.9898 | 0.9901 | 0.9904 | 0.9906 | 0.9909 | 0.9911 | 0.9913 | 0.9916 |
| 2.4 | 0.9918 | 0.9920 | 0.9922 | 0.9925 | 0.9927 | 0.9929 | 0.9931 | 0.9932 | 0.9934 | 0.9936 |
| 2.5 | 0.9938 | 0.9940 | 0.9941 | 0.9943 | 0.9945 | 0.9946 | 0.9948 | 0.9949 | 0.9951 | 0.9952 |
| 2.6 | 0.9953 | 0.9955 | 0.9956 | 0.9957 | 0.9959 | 0.9960 | 0.9961 | 0.9962 | 0.9963 | 0.9964 |
| 2.7 | 0.9965 | 0.9966 | 0.9967 | 0.9968 | 0.9969 | 0.9970 | 0.9971 | 0.9972 | 0.9973 | 0.9974 |
| 2.8 | 0.9974 | 0.9975 | 0.9976 | 0.9977 | 0.9977 | 0.9978 | 0.9979 | 0.9979 | 0.9980 | 0.9981 |
| 2.9 | 0.9981 | 0.9982 | 0.9982 | 0.9983 | 0.9984 | 0.9984 | 0.9985 | 0.9985 | 0.9986 | 0.9986 |
| 3.0 | 0.9987 | 0.9987 | 0.9987 | 0.9988 | 0.9988 | 0.9989 | 0.9989 | 0.9989 | 0.9990 | 0.9990 |
| 3.1 | 0.9990 | 0.9991 | 0.9991 | 0.9991 | 0.9992 | 0.9992 | 0.9992 | 0.9992 | 0.9993 | 0.9993 |
| 3.2 | 0.9993 | 0.9993 | 0.9994 | 0.9994 | 0.9994 | 0.9994 | 0.9994 | 0.9995 | 0.9995 | 0.9995 |
| 3.3 | 0.9995 | 0.9995 | 0.9995 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9997 |
| 3.4 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9998 |

# F Distribution




$F$-values for selected UPPER TAIL probabilities are shown in the following table:

| Denom. df | Upper tail area | Numerator df | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| 19 | 0.10 | 2.99 | 2.61 | 2.40 | 2.27 | 2.18 | 2.11 | 2.06 | 2.02 | 1.98 | 1.96 | 1.93 |
| | 0.05 | 4.38 | 3.52 | 3.13 | 2.90 | 2.74 | 2.63 | 2.54 | 2.48 | 2.42 | 2.38 | 2.34 |
| | 0.025 | 5.92 | 4.51 | 3.90 | 3.56 | 3.33 | 3.17 | 3.05 | 2.96 | 2.88 | 2.82 | 2.76 |
| | 0.01 | 8.18 | 5.93 | 5.01 | 4.50 | 4.17 | 3.94 | 3.77 | 3.63 | 3.52 | 3.43 | 3.36 |
| 20 | 0.10 | 2.97 | 2.59 | 2.38 | 2.25 | 2.16 | 2.09 | 2.04 | 2.00 | 1.96 | 1.94 | 1.91 |
| | 0.05 | 4.35 | 3.49 | 3.10 | 2.87 | 2.71 | 2.60 | 2.51 | 2.45 | 2.39 | 2.35 | 2.31 |
| | 0.025 | 5.87 | 4.46 | 3.86 | 3.51 | 3.29 | 3.13 | 3.01 | 2.91 | 2.84 | 2.77 | 2.72 |
| | 0.01 | 8.10 | 5.85 | 4.94 | 4.43 | 4.10 | 3.87 | 3.70 | 3.56 | 3.46 | 3.37 | 3.29 |
| 21 | 0.10 | 2.96 | 2.57 | 2.36 | 2.23 | 2.14 | 2.08 | 2.02 | 1.98 | 1.95 | 1.92 | 1.90 |
| | 0.05 | 4.32 | 3.47 | 3.07 | 2.84 | 2.68 | 2.57 | 2.49 | 2.42 | 2.37 | 2.32 | 2.28 |
| | 0.025 | 5.83 | 4.42 | 3.82 | 3.48 | 3.25 | 3.09 | 2.97 | 2.87 | 2.80 | 2.73 | 2.68 |
| | 0.01 | 8.02 | 5.78 | 4.87 | 4.37 | 4.04 | 3.81 | 3.64 | 3.51 | 3.40 | 3.31 | 3.24 |
| 22 | 0.10 | 2.95 | 2.56 | 2.35 | 2.22 | 2.13 | 2.06 | 2.01 | 1.97 | 1.93 | 1.90 | 1.88 |
| | 0.05 | 4.30 | 3.44 | 3.05 | 2.82 | 2.66 | 2.55 | 2.46 | 2.40 | 2.34 | 2.30 | 2.26 |
| | 0.025 | 5.79 | 4.38 | 3.78 | 3.44 | 3.22 | 3.05 | 2.93 | 2.84 | 2.76 | 2.70 | 2.65 |
| | 0.01 | 7.95 | 5.72 | 4.82 | 4.31 | 3.99 | 3.76 | 3.59 | 3.45 | 3.35 | 3.26 | 3.18 |
| 23 | 0.10 | 2.94 | 2.55 | 2.34 | 2.21 | 2.11 | 2.05 | 1.99 | 1.95 | 1.92 | 1.89 | 1.87 |
| | 0.05 | 4.28 | 3.42 | 3.03 | 2.80 | 2.64 | 2.53 | 2.44 | 2.37 | 2.32 | 2.27 | 2.24 |
| | 0.025 | 5.75 | 4.35 | 3.75 | 3.41 | 3.18 | 3.02 | 2.90 | 2.81 | 2.73 | 2.67 | 2.62 |
| | 0.01 | 7.88 | 5.66 | 4.76 | 4.26 | 3.94 | 3.71 | 3.54 | 3.41 | 3.30 | 3.21 | 3.14 |
| 24 | 0.10 | 2.93 | 2.54 | 2.33 | 2.19 | 2.10 | 2.04 | 1.98 | 1.94 | 1.91 | 1.88 | 1.85 |
| | 0.05 | 4.26 | 3.40 | 3.01 | 2.78 | 2.62 | 2.51 | 2.42 | 2.36 | 2.30 | 2.25 | 2.22 |
| | 0.025 | 5.72 | 4.32 | 3.72 | 3.38 | 3.15 | 2.99 | 2.87 | 2.78 | 2.70 | 2.64 | 2.59 |
| | 0.01 | 7.82 | 5.61 | 4.72 | 4.22 | 3.90 | 3.67 | 3.50 | 3.36 | 3.26 | 3.17 | 3.09 |
| 25 | 0.10 | 2.92 | 2.53 | 2.32 | 2.18 | 2.09 | 2.02 | 1.97 | 1.93 | 1.89 | 1.87 | 1.84 |
| | 0.05 | 4.24 | 3.39 | 2.99 | 2.76 | 2.60 | 2.49 | 2.40 | 2.34 | 2.28 | 2.24 | 2.20 |
| | 0.025 | 5.69 | 4.29 | 3.69 | 3.35 | 3.13 | 2.97 | 2.85 | 2.75 | 2.68 | 2.61 | 2.56 |
| | 0.01 | 7.77 | 5.57 | 4.68 | 4.18 | 3.85 | 3.63 | 3.46 | 3.32 | 3.22 | 3.13 | 3.06 |
| 26 | 0.10 | 2.91 | 2.52 | 2.31 | 2.17 | 2.08 | 2.01 | 1.96 | 1.92 | 1.88 | 1.86 | 1.83 |
| | 0.05 | 4.23 | 3.37 | 2.98 | 2.74 | 2.59 | 2.47 | 2.39 | 2.32 | 2.27 | 2.22 | 2.18 |
| | 0.025 | 5.66 | 4.27 | 3.67 | 3.33 | 3.10 | 2.94 | 2.82 | 2.73 | 2.65 | 2.59 | 2.54 |
| | 0.01 | 7.72 | 5.53 | 4.64 | 4.14 | 3.82 | 3.59 | 3.42 | 3.29 | 3.18 | 3.09 | 3.02 |
| 27 | 0.10 | 2.90 | 2.51 | 2.30 | 2.17 | 2.07 | 2.00 | 1.95 | 1.91 | 1.87 | 1.85 | 1.82 |
| | 0.05 | 4.21 | 3.35 | 2.96 | 2.73 | 2.57 | 2.46 | 2.37 | 2.31 | 2.25 | 2.20 | 2.17 |
| | 0.025 | 5.63 | 4.24 | 3.65 | 3.31 | 3.08 | 2.92 | 2.80 | 2.71 | 2.63 | 2.57 | 2.51 |
| | 0.01 | 7.68 | 5.49 | 4.60 | 4.11 | 3.78 | 3.56 | 3.39 | 3.26 | 3.15 | 3.06 | 2.99 |




$F$-values for selected UPPER TAIL probabilities are shown in the following table:

| Denom. df | Upper tail area | Numerator df | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| 28 | 0.10 | 2.89 | 2.50 | 2.29 | 2.16 | 2.06 | 2.00 | 1.94 | 1.90 | 1.87 | 1.84 | 1.81 |
| | 0.05 | 4.20 | 3.34 | 2.95 | 2.71 | 2.56 | 2.45 | 2.36 | 2.29 | 2.24 | 2.19 | 2.15 |
| | 0.025 | 5.61 | 4.22 | 3.63 | 3.29 | 3.06 | 2.90 | 2.78 | 2.69 | 2.61 | 2.55 | 2.49 |
| | 0.01 | 7.64 | 5.45 | 4.57 | 4.07 | 3.75 | 3.53 | 3.36 | 3.23 | 3.12 | 3.03 | 2.96 |
| 29 | 0.10 | 2.89 | 2.50 | 2.28 | 2.15 | 2.06 | 1.99 | 1.93 | 1.89 | 1.86 | 1.83 | 1.80 |
| | 0.05 | 4.18 | 3.33 | 2.93 | 2.70 | 2.55 | 2.43 | 2.35 | 2.28 | 2.22 | 2.18 | 2.14 |
| | 0.025 | 5.59 | 4.20 | 3.61 | 3.27 | 3.04 | 2.88 | 2.76 | 2.67 | 2.59 | 2.53 | 2.48 |
| | 0.01 | 7.60 | 5.42 | 4.54 | 4.04 | 3.73 | 3.50 | 3.33 | 3.20 | 3.09 | 3.00 | 2.93 |
| 30 | 0.10 | 2.88 | 2.49 | 2.28 | 2.14 | 2.05 | 1.98 | 1.93 | 1.88 | 1.85 | 1.82 | 1.79 |
| | 0.05 | 4.17 | 3.32 | 2.92 | 2.69 | 2.53 | 2.42 | 2.33 | 2.27 | 2.21 | 2.16 | 2.13 |
| | 0.025 | 5.57 | 4.18 | 3.59 | 3.25 | 3.03 | 2.87 | 2.75 | 2.65 | 2.57 | 2.51 | 2.46 |
| | 0.01 | 7.56 | 5.39 | 4.51 | 4.02 | 3.70 | 3.47 | 3.30 | 3.17 | 3.07 | 2.98 | 2.91 |
| 40 | 0.10 | 2.84 | 2.44 | 2.23 | 2.09 | 2.00 | 1.93 | 1.87 | 1.83 | 1.79 | 1.76 | 1.74 |
| | 0.05 | 4.08 | 3.23 | 2.84 | 2.61 | 2.45 | 2.34 | 2.25 | 2.18 | 2.12 | 2.08 | 2.04 |
| | 0.025 | 5.42 | 4.05 | 3.46 | 3.13 | 2.90 | 2.74 | 2.62 | 2.53 | 2.45 | 2.39 | 2.33 |
| | 0.01 | 7.31 | 5.18 | 4.31 | 3.83 | 3.51 | 3.29 | 3.12 | 2.99 | 2.89 | 2.80 | 2.73 |
| 60 | 0.10 | 2.79 | 2.39 | 2.18 | 2.04 | 1.95 | 1.87 | 1.82 | 1.77 | 1.74 | 1.71 | 1.68 |
| | 0.05 | 4.00 | 3.15 | 2.76 | 2.53 | 2.37 | 2.25 | 2.17 | 2.10 | 2.04 | 1.99 | 1.95 |
| | 0.025 | 5.29 | 3.93 | 3.34 | 3.01 | 2.79 | 2.63 | 2.51 | 2.41 | 2.33 | 2.27 | 2.22 |
| | 0.01 | 7.08 | 4.98 | 4.13 | 3.65 | 3.34 | 3.12 | 2.95 | 2.82 | 2.72 | 2.63 | 2.56 |
| 100 | 0.10 | 2.76 | 2.36 | 2.14 | 2.00 | 1.91 | 1.83 | 1.78 | 1.73 | 1.69 | 1.66 | 1.64 |
| | 0.05 | 3.94 | 3.09 | 2.70 | 2.46 | 2.31 | 2.19 | 2.10 | 2.03 | 1.97 | 1.93 | 1.89 |
| | 0.025 | 5.18 | 3.83 | 3.25 | 2.92 | 2.70 | 2.54 | 2.42 | 2.32 | 2.24 | 2.18 | 2.12 |
| | 0.01 | 6.90 | 4.82 | 3.98 | 3.51 | 3.21 | 2.99 | 2.82 | 2.69 | 2.59 | 2.50 | 2.43 |
| 1000 | 0.10 | 2.71 | 2.31 | 2.09 | 1.95 | 1.85 | 1.78 | 1.72 | 1.68 | 1.64 | 1.61 | 1.58 |
| | 0.05 | 3.85 | 3.00 | 2.61 | 2.38 | 2.22 | 2.11 | 2.02 | 1.95 | 1.89 | 1.84 | 1.80 |
| | 0.025 | 5.04 | 3.70 | 3.13 | 2.80 | 2.58 | 2.42 | 2.30 | 2.20 | 2.13 | 2.06 | 2.01 |
| | 0.01 | 6.66 | 4.63 | 3.80 | 3.34 | 3.04 | 2.82 | 2.66 | 2.53 | 2.43 | 2.34 | 2.27 |