Final exam: Monday, May 6, 9:40-11:40am, Behavioral Sciences A101

Stat 315
Spring 2022
Practice Final Exam
5/10/2022
Time Limit: 120 Minutes

Name (Print): Solutions

Section: ____________

This exam contains 12 pages (including this cover page) and 6 problems.

You may use three double-sided 4" x 6" note cards of formulas/notes/etc and a calculator. You may *not* use any other materials.

You are required to show your work on each problem on this exam. The following rules apply:

- **Show all your work**. You may check your answers using calculator functions, but you must show every step of your calculations to receive full credit.
- **Organize your work** in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.

Do not write in the table to the right.

Problem	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	15	
Total:	65	

1. A factory produces ball bearings, which are measured by their diameter. A sample of 5 ball bearings from one manufacturing process are measured to have diameters 8.1mm, 7.9mm, 8.6mm, 8.2mm, 7.9mm.

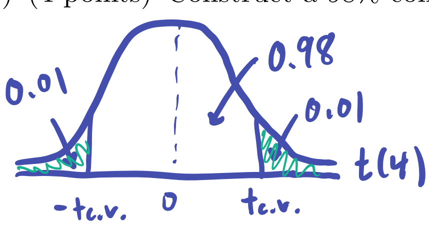
   (a) (4 points) Compute the mean and standard deviation of the ball bearings in this sample.

   Reminder: You must show your work to receive credit.

x_i	8.1	7.9	8.6	8.2	7.9	$\rightarrow \sum x_i = 40.7 \rightarrow \bar{x} = \frac{40.7}{5} = \underline{8.14 = \bar{x}}$
$x_i - \bar{x}$	-0.04	-0.24	0.46	0.06	-0.24	$\rightarrow \sum(x_i - \bar{x}) = 0$ ✓
$(x_i - \bar{x})^2$	0.0016	0.0576	0.2116	0.0036	0.0576	$\rightarrow \sum(x_i - \bar{x})^2 = 0.332$

$$s^2 = \frac{\sum(x_i - \bar{x})^2}{n-1} = \frac{0.332}{5-1} = 0.083 \rightarrow s = \sqrt{0.083} \rightarrow \boxed{s = 0.288}$$

   (b) (4 points) Construct a 98% confidence interval for the mean ball bearing diameter.



$n = 5 \rightarrow df = n-1 = 4 \rightarrow t_{c.v.} = 3.747$

$$\bar{x} \pm t_{c.v.} \cdot \frac{s}{\sqrt{n}} \rightarrow 8.14 \pm 3.747 \cdot \frac{0.288}{\sqrt{5}}$$

$$\rightarrow 8.14 \pm 0.48$$

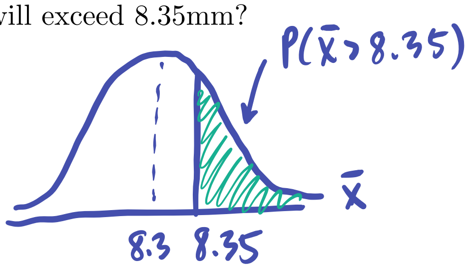
$$\rightarrow \boxed{7.66 < \mu < 8.62}$$

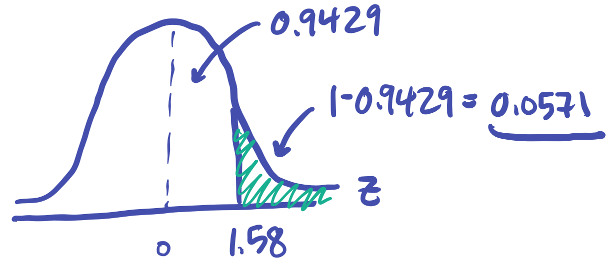
   (c) (2 points) Based on your confidence interval, would you agree or disagree with the statement that the population mean ball bearing diameter is 8.6mm?

Since 8.6 is in the CI, it is a plausible value

2. For the following two parts, assume the diameters of ball bearings follow a normal distribution with mean 8.3mm and standard deviation 0.1mm.

   (a) (5 points) What is the probability that the mean of a random sample of 10 ball bearings will exceed 8.35mm?



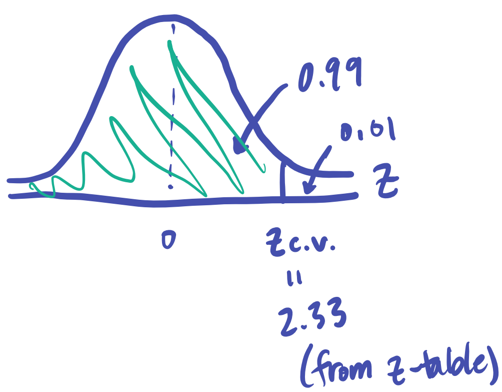


$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

$$= \frac{8.35 - 8.3}{0.1/\sqrt{10}}$$

$$= 1.58$$

   (b) (5 points) Suppose a random sample of 10 ball bearings is drawn. What sample mean would be required to be in the 99$^{\text{th}}$ percentile of all such sample means?



$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

$$\rightarrow 2.33 = \frac{\bar{x} - 8.3}{0.1/\sqrt{10}}$$

$$\rightarrow \underline{\bar{x} = 8.37}$$

3. Blake and Axel want to know if the average starting salary of Mechanical Engineering (M.E.) majors differs from the average starting salary of Computer Science (C.S.) majors. The sample information is given in the following table. Use $df = 30$, if needed.

Major	Sample Size	Sample Mean	Sample SD
Mechanical Eng.	20	59116	3400
Computer Sci.	20	65432	4200

(a) (6 points) Construct a two-sided 95% confidence interval for the difference in mean starting salaries.

$df = 30 \rightarrow t_{c.v.} = 2.042$

$(\bar{x}_1 - \bar{x}_2) \pm t_{c.v.} \sqrt{s_1^2/n_1 + s_2^2/n_2}$

$\rightarrow (59116 - 65432) \pm 2.042 \cdot \sqrt{\frac{3400^2}{20} + \frac{4200^2}{20}}$

$\rightarrow -6316 \pm 2467$

$\rightarrow \boxed{-8783 < \mu_1 - \mu_2 < -3849}$

(b) (2 points) Suppose we are testing $H_0 : \mu_1 - \mu_2 = 0$ vs. $H_A : \mu_1 - \mu_2 \neq 0$. What is a Type II error in the context of this problem?

Type II error: FTR H_0 when H_0 is false

In this context, a type II error would be to conclude that there is no differance in the true mean salaries when, in fact, there is.

(c) (2 points) TRUE / FALSE If we fail to reject H_0, then H_0 must be true.

4. A brewer at New Belgium is brewing the same beer (i.e., same ingredients) using two different methods. It is of interest whether the average amount of bitterness (measured in IBU, international bitterness units) differs between the two methods. The brewer collects random samples of beer from the two methods and calculates the following summary statistics of the bitterness (in IBU).

Method	Sample Mean	Sample Variance	Sample Size
Method 1	60.5	122.5	12
Method 2	53.6	103.4	15

(a) (5 points) Complete a hypothesis test at $\alpha = 0.05$ to test whether the average bitterness differs between the two methods. Use $df = 20$, if needed.

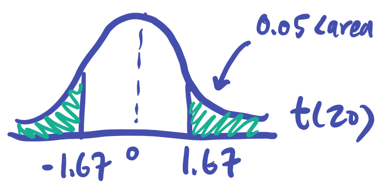
① $H_0: \mu_1 - \mu_2 = 0$

$H_a: \mu_1 - \mu_2 \neq 0$

② $\alpha = 0.05$

③ $t_{test} = \dfrac{(\bar{x}_1 - \bar{x}_2) - D_0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$

$= \dfrac{(60.5 - 53.6) - 0}{\sqrt{122.5/12 + 103.4/15}} = \underline{1.67}$

④ 

↳ $\underline{0.1 < \text{p-value} < 0.2}$

⑤ FTR H_0

⑥ At $\alpha = 0.05$, there is not sufficient evidence to conclude that the true mean bitterness differs between the two methods.

(b) (5 points) Now, suppose we compare three different brewing methods and want to know whether the average bitterness differs across the three methods. Complete an ANOVA F-test at $\alpha = 0.05$ using the R output below.

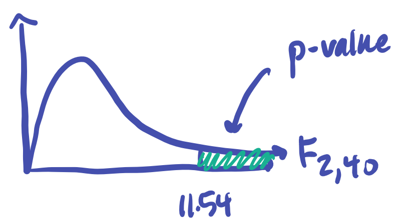
```
            Df  Sum Sq  Mean Sq
method       2    2479   1239.3
Residuals   40    4295    107.4
```

① $H_0: \mu_1 = \mu_2 = \mu_3$

H_a: at least one μ_i differs $(i = 1, 2, 3)$

② $\alpha = 0.05$

③ $F_{test} = \dfrac{MSTR}{MSE} = \dfrac{1239.3}{107.4} = \underline{11.54}$

④ 

⑤ Reject H_0

⑥ At $\alpha = 0.05$, there is sufficient evidence to conclude that one or more of the methods has a different true mean bitterness.

5. A sample of 32 cars was obtained from a 1974 *Motor Trend* magazine and the MPG (miles per gallon) and weight (in 1000s of pounds) were recorded. A simple linear regression model was fit in R using weight as the predictor variable and MPG as the response variable.

```
            Estimate Std. Error t value
(Intercept)  37.2851     1.8776  19.858
weight       -5.3445     0.5591  -9.559
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 3.046 on 30 degrees of freedom
Multiple R-squared:  0.7528,Adjusted R-squared:  0.7446
F-statistic: 91.38 on 1 and 30 DF,  p-value: 1.294e-10
```

(a) (3 points) Write the estimated regression equation and use it to estimate the MPG of a car that weighs $x = 4.155$ (i.e., 4,155 pounds).

$\boxed{\hat{y} = 37.2851 - 5.3445x} \rightarrow \hat{y} = 37.2851 - 5.3445(4.155)$

$= \boxed{15.08 \text{ MPG}}$

(b) (3 points) What proportion of the variation in MPG can be explained by the car's weight?

$R^2 = 0.7528 \rightarrow \underline{75.28\%}$

(c) (4 points) Compute a 95% two-sided confidence interval for the true slope, β_1, and use it to determine if β_1 differs 0.

$b_1 \pm t_{c.v.} \cdot s_{b_1} \rightarrow -5.3445 \pm 2.042 \cdot 0.5591$

$df = n-2 = 30$

$\hookrightarrow t_{c.v.} = 2.042$

$\boxed{-6.49 < \beta_1 < -4.20}$

0 is not in our CI, so we would reject $H_0: \beta_1 = 0$.

6. Following up on question 6, the horsepower of the car is also noted for the 32 cars. Two multiple regression models are being considered. The first model uses weight and horsepower, while the second model uses weight, horsepower, and an interaction term between weight and horsepower.

```
-----------------------------------------------------
# MODEL 1
lm(formula = mpg ~ wt + hp, data = mtcars)
                Estimate Std. Error t value Pr(>|t|)
(Intercept) 37.22727    1.59879  23.285  < 2e-16 ***
wt          -3.87783    0.63273  -6.129 (omitted)
hp          -0.03177    0.00903  -3.519  0.00145 **

Residual standard error: 2.593 on 29 degrees of freedom
Multiple R-squared:  0.8268,Adjusted R-squared:  0.8148
F-statistic: 69.21 on 2 and 29 DF,  p-value: (omitted)
-----------------------------------------------------
# MODEL 2
lm(formula = mpg ~ wt * hp, data = mtcars)
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 49.80842    3.60516  13.816 5.01e-14 ***
wt          -8.21662    1.26971  -6.471 5.20e-07 ***
hp          -0.12010    0.02470  -4.863 4.04e-05 ***
wt:hp        0.02785    0.00742   3.753 0.000811 ***

Residual standard error: 2.153 on 28 degrees of freedom
Multiple R-squared:  0.8848,Adjusted R-squared:  0.8724
F-statistic: 71.66 on 3 and 28 DF,  p-value: 2.981e-13
-----------------------------------------------------
```

(a) (5 points) Based on the R output below, which model is preferred? Give at least two reasons why this model is preferred.

Model 2 is preferred:

① Higher R^2

② Lower residual standard error

③ the interaction term is stat. sig.

Problem 7 (cont.)

(b) (5 points) Complete a hypothesis test at $\alpha = 0.05$ to test if $\beta_1 = 0$ in MODEL 1.

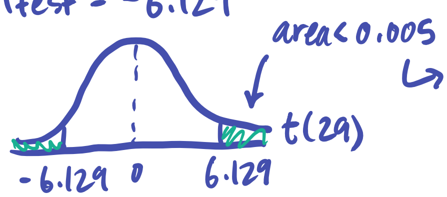
① $H_0: \beta_1 = 0$

$H_a: \beta_1 \neq 0$

② $\alpha = 0.05$

③ $t_{test} = -6.129$

④



↳ p-value<0.01

⑤ Reject H_0

⑥ At $\alpha = 0.05$, there is sufficient evidence to conclude that weight and MPG are linearly related, after adjusting for HP.

(c) (5 points) Complete a hypothesis test at $\alpha = 0.05$ to test if $\beta_1 = \beta_2 = 0$ (i.e., test for overall model) in MODEL 1.

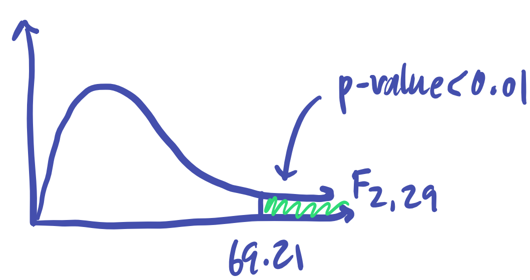
① $H_0: \beta_1 = \beta_2 = 0$

H_a: at least one β_i $(i=1,2)$ differs from 0.

② $\alpha = 0.05$

③ $F_{test} = 69.21$ ($df_1 = 2$, $df_2 = 29$)

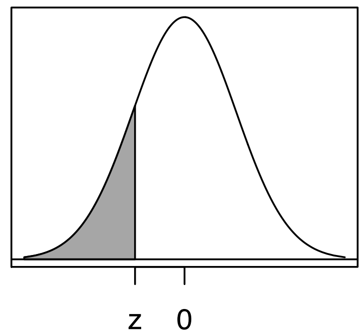
④



⑤ Reject H_0

⑥ At $\alpha = 0.05$, there is sufficient evidence to conclude that wt, hp, or both are linearly related to MPG.

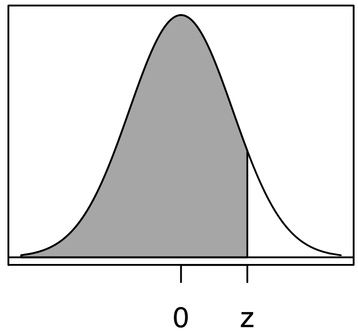
Standard Normal Distribution



Cumulative probabilities for **NEGATIVE** z-values are shown in the following table:

Z	**.00**	**.01**	**.02**	**.03**	**.04**	**.05**	**.06**	**.07**	**.08**	**.09**
−3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
−3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
−3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
−3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
−3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
−2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
−2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
−2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
−2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
−2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
−2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
−2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
−2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
−2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
−2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
−1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
−1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
−1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
−1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
−1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
−1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
−1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
−1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
−1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
−1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
−.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
−.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
−.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
−.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
−.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
−.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
−.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
−.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
−.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
−.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

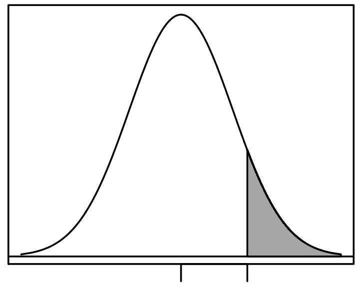
Standard Normal Distribution



Cumulative probabilities for **POSITIVE** z-values are shown in the following table:

Z	**.00**	**.01**	**.02**	**.03**	**.04**	**.05**	**.06**	**.07**	**.08**	**.09**
.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

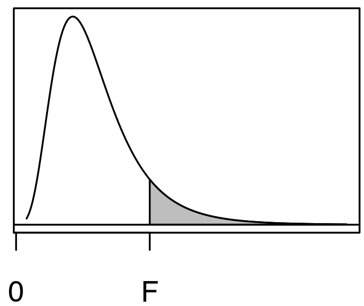
t Distribution



t-values for selected UPPER TAIL probabilities are shown in the following table:

		90%	**95%**		**99%**	← For **this** CI
df	**.10**	**.05**	**.025**	**.01**	**.005**	← Upper tail probability
1	3.078	6.314	12.706	31.821	63.657	
2	1.886	2.920	4.303	6.965	9.925	
3	1.638	2.353	3.182	4.541	5.841	
4	1.533	2.132	2.776	3.747	4.604	
5	1.476	2.015	2.571	3.365	4.032	
6	1.440	1.943	2.447	3.143	3.707	
7	1.415	1.895	2.365	2.998	3.499	
8	1.397	1.860	2.306	2.896	3.355	
9	1.383	1.833	2.262	2.821	3.250	
10	1.372	1.812	2.228	2.764	3.169	
11	1.363	1.796	2.201	2.718	3.106	
12	1.356	1.782	2.179	2.681	3.055	
13	1.350	1.771	2.160	2.650	3.012	
14	1.345	1.761	2.145	2.624	2.977	
15	1.341	1.753	2.131	2.602	2.947	
16	1.337	1.746	2.120	2.583	2.921	
17	1.333	1.740	2.110	2.567	2.898	
18	1.330	1.734	2.101	2.552	2.878	
19	1.328	1.729	2.093	2.539	2.861	
20	1.325	1.725	2.086	2.528	2.845	
21	1.323	1.721	2.080	2.518	2.831	
22	1.321	1.717	2.074	2.508	2.819	
23	1.319	1.714	2.069	2.500	2.807	
24	1.318	1.711	2.064	2.492	2.797	
25	1.316	1.708	2.060	2.485	2.787	
26	1.315	1.706	2.056	2.479	2.779	
27	1.314	1.703	2.052	2.473	2.771	
28	1.313	1.701	2.048	2.467	2.763	
29	1.311	1.699	2.045	2.462	2.756	
30	1.310	1.697	2.042	2.457	2.750	
40	1.303	1.684	2.021	2.423	2.704	
50	1.299	1.676	2.009	2.403	2.678	
60	1.296	1.671	2.000	2.390	2.660	
70	1.294	1.667	1.994	2.381	2.648	
80	1.292	1.664	1.990	2.374	2.639	
90	1.291	1.662	1.987	2.368	2.632	
100	1.290	1.660	1.984	2.364	2.626	
∞	1.282	1.645	1.960	2.326	2.576	← Same as **z-values**

F Distribution



F-values for selected UPPER TAIL probabilities are shown in the following table:

Denom. df	Upper tail area	Numerator df 1	2	3	4	5	6	7	8	9	10	11
28	0.10	2.89	2.50	2.29	2.16	2.06	2.00	1.94	1.90	1.87	1.84	1.81
	0.05	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.15
	0.025	5.61	4.22	3.63	3.29	3.06	2.90	2.78	2.69	2.61	2.55	2.49
	0.01	7.64	5.45	4.57	4.07	3.75	3.53	3.36	3.23	3.12	3.03	2.96
29	0.10	2.89	2.50	2.28	2.15	2.06	1.99	1.93	1.89	1.86	1.83	1.80
	0.05	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18	2.14
	0.025	5.59	4.20	3.61	3.27	3.04	2.88	2.76	2.67	2.59	2.53	2.48
	0.01	7.60	5.42	4.54	4.04	3.73	3.50	3.33	3.20	3.09	3.00	2.93
30	0.10	2.88	2.49	2.28	2.14	2.05	1.98	1.93	1.88	1.85	1.82	1.79
	0.05	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.13
	0.025	5.57	4.18	3.59	3.25	3.03	2.87	2.75	2.65	2.57	2.51	2.46
	0.01	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07	2.98	2.91
40	0.10	2.84	2.44	2.23	2.09	2.00	1.93	1.87	1.83	1.79	1.76	1.74
	0.05	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.04
	0.025	5.42	4.05	3.46	3.13	2.90	2.74	2.62	2.53	2.45	2.39	2.33
	0.01	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89	2.80	2.73
60	0.10	2.79	2.39	2.18	2.04	1.95	1.87	1.82	1.77	1.74	1.71	1.68
	0.05	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.95
	0.025	5.29	3.93	3.34	3.01	2.79	2.63	2.51	2.41	2.33	2.27	2.22
	0.01	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72	2.63	2.56
100	0.10	2.76	2.36	2.14	2.00	1.91	1.83	1.78	1.73	1.69	1.66	1.64
	0.05	3.94	3.09	2.70	2.46	2.31	2.19	2.10	2.03	1.97	1.93	1.89
	0.025	5.18	3.83	3.25	2.92	2.70	2.54	2.42	2.32	2.24	2.18	2.12
	0.01	6.90	4.82	3.98	3.51	3.21	2.99	2.82	2.69	2.59	2.50	2.43
1000	0.10	2.71	2.31	2.09	1.95	1.85	1.78	1.72	1.68	1.64	1.61	1.58
	0.05	3.85	3.00	2.61	2.38	2.22	2.11	2.02	1.95	1.89	1.84	1.80
	0.025	5.04	3.70	3.13	2.80	2.58	2.42	2.30	2.20	2.13	2.06	2.01
	0.01	6.66	4.63	3.80	3.34	3.04	2.82	2.66	2.53	2.43	2.34	2.27