STAT 315 Chapter 4 Review Questions

Scenario A

A tree farm grows trees in batches of size 1,000. The youngest batch is 1 year old, has been affected by drought conditions, and tree farmers are interested in the impact of drought on tree height and diameter.

1. Based on previous batches, the farmers believe 1 year old tree height follows a normal distribution with mean $\mu = 70$ cm and standard deviation $\sigma = 9$ cm (under normal growing conditions). A tree in the youngest batch has a height of 56cm. What is the probability of growing a tree with a height less or equal to 56 centimeters? Based on this, do you think the new batch of trees follows the same distribution as previous batches? (We will revisit this logic when discussing hypothesis testing).

2. Technically, normal random variables can be negative, but tree heights cannot. Do you think it's appropriate to use a normal distribution in this case? You could try computing the probability of getting a negative tree height from the farmers' model to justify your answer.

3. After 5 years, the farmers open up their trees for sale. In order to estimate revenue, they look at the time between tree sales. They find that it follows an exponential distribution with mean $1/\lambda = 3$ days . What is the probability that they will have to wait more than 1 week before their next sale?

4. According to the farmers, tree diameter (at the base of the tree) follows a normal distribution with mean $\mu = 5$ cm and standard deviation $\sigma = 0.5$ cm. With the young trees, the mean is now $\mu = 3.8$ cm with the same standard deviation. In a normal year, the farmers would flag the bottom 5% of trees to receive special attention in future years. If the same cutoff diameter is used in the drought year, what percent of trees would the farmers have to flag?

Scenario B

Traffic engineers are considering installing a photo system to catch speeding cars on a stretch of roadway. Based on a month of preliminary data, they believe the distribution of speeds is given by:

$$f(x) = \begin{cases} kx & 0 < x \leq 25 \\ \frac{k}{4}(x-35)^2 & 25 \leq x \leq 35 \\ 0 & \text{otherwise} \end{cases}$$

5. Find the value of k that makes this a valid PDF.

6. Find the average speed of cars for this section of road.

7. Suppose the speed limit is 35mph, and engineers wish to issue photo-tickets for speeds above 37mph. What percentage of drivers should they expect to ticket?

Assume X is a continuous random variable with pdf $f(x)$ and CDF $F(x)$, and that $a < b < c < d$.
Indicate whether the following are always true, always false, or if the answer cannot be determined

8.	$f(x)$ gives the probability that X takes on the value x	true	false	cannot be determined
9.	$\int_a^a f(x)\,dx = 0$	true	false	cannot be determined
10.	$F(b) < F(a)$	true	false	cannot be determined
11.	$P(X = a) = 0$	true	false	cannot be determined
12.	$P(a < X < b) < P(a \leq X \leq b)$	true	false	cannot be determined
13.	$f(x)$ cannot be negative	true	false	cannot be determined
14.	$P(a < X < b) \leq P(a < X < c)$	true	false	cannot be determined
15.	$P(a < X < c) < P(b < X < d)$	true	false	cannot be determined
16.	$\int_c^d f(x)\,dx = 0$	true	false	cannot be determined
17.	X cannot be negative	true	false	cannot be determined
18.	X cannot be a single value because the probability is zero	true	false	cannot be determined