Stat 315
Spring 2022
Practice Exam 1
2/25/2022
Time Limit: 50 Minutes

Name (Print): Solutions

Student ID Number: _____

This exam contains 7 pages (including this cover page) and 4 problems.

You may use one two-sided 4" x 6" note card and a calculator. You may *not* use any other material.

You are required to show your work on each problem on this exam. The following rules apply:

- **Show all your work**. You may check your answers using calculator functions, but you must show every step of your calculations to receive full credit.
- **Organize your work** in a reasonably neat and coherent way in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.

Do not write in the table to the right.

Problem	Points	Score
1	10	
2	10	
3	10	
4	10	
Total:	40	

1. Products produced by a machine have a 4% probability of being defective. Random variable X represents the number of products inspected until a defective product is found. (Think about finding a defect as being a "success" in this problem.)

   (a) (3 points) State what random variable that is described in the problem along with its parameter(s).

   Geometric ($p = 0.04$)

   (b) (3 points) What is the probability that the 5^{th} product found is defective?

   $P(X=k) = (1-p)^{k-1} \cdot p$ (PMF of a geo. RV)

   $P(X=5) = (1-0.04)^{5-1} \cdot 0.04 = \underline{0.034}$

   (c) (3 points) What is the probability that it will take more that 3 inspections to find the first defective product?

   $P(X>3) = p(4)+p(5)+p(6)+\ldots$

   $= 1 - P(X \leq 3)$ "comp. rule"

   $= 1 - [p(1)+p(2)+p(3)]$

   $= 1 - [0.04 + 0.04(0.96) + 0.04(0.96)^2]$

   $= 1 - 0.115$

   $= \underline{0.885}$

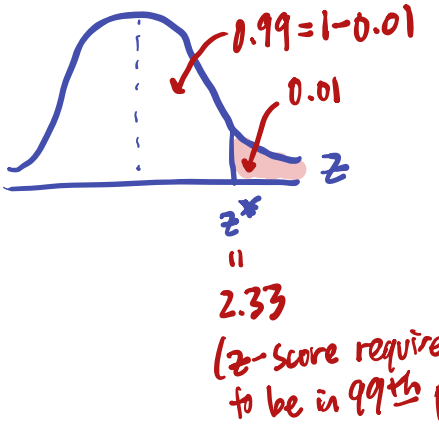
   (d) (1 point) True / (False) X is a continuous random variable.

2. Suppose that the heights of women in the United States are approximately normally distributed with mean $\mu = 64$ inches and standard deviation $\sigma = 4$ inches.

(a) (2 points) According to the Empirical Rule, between what two bounds do approximately 95% of American female heights lie?

$\approx 95\%$ between $\mu \pm 2\sigma \rightarrow 64 \pm 2(4) \rightarrow (56, 72)$

(b) (3 points) What is the minimum height required for an American female to be in the top 1% of female heights? Draw an accompanying figure.



$z = \frac{x-\mu}{\sigma}$

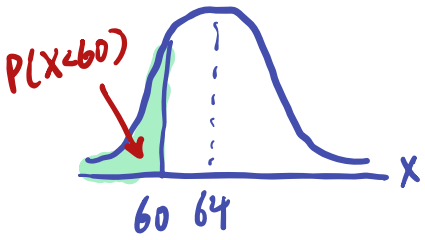
$\downarrow$

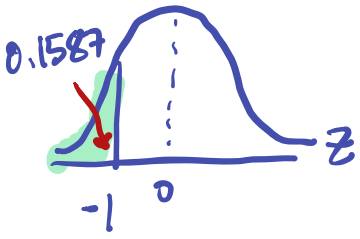
$x = \sigma z + \mu$

$= 4(2.33) + 64$

$= 73.32$ inches

(c) (3 points) What is the probability a randomly selected American female will be shorter than 60 inches? Draw an accompanying figure.





$\hookrightarrow z = \frac{x-\mu}{\sigma} = \frac{60-64}{4} = -1$

$\rightarrow P(X<60) = P(z<-1)$

$= 0.1587$

(d) (2 points) True / False: Approximately 95% of the values generated from any random variable are within two standard deviations of the mean.

↳ true specifically for normal RV

3. (a) (6 points) Six students take an statistics exam and their scores are: 72, 80, 96, 95, 83, 78. Calculate: (i) Sample mean, (ii) Sample standard deviation, (iii) Sample median

x_i	72	80	96	95	83	78	$\rightarrow \sum x_i = 504$
$x_i - \bar{x}$	-12	-4	12	11	-1	-6	$\rightarrow \sum (x_i - \bar{x}) = 0$ ✓
$(x_i - \bar{x})^2$	144	16	144	121	1	36	$\rightarrow \sum (x_i - \bar{x})^2 = 462$

$\bar{x} = \frac{\sum x_i}{n} = \frac{504}{6} = 84 = \tilde{x}$; $s^2 = \frac{\sum (x_i - \bar{x})^2}{n-1} = \frac{462}{6-1} = 92.4$

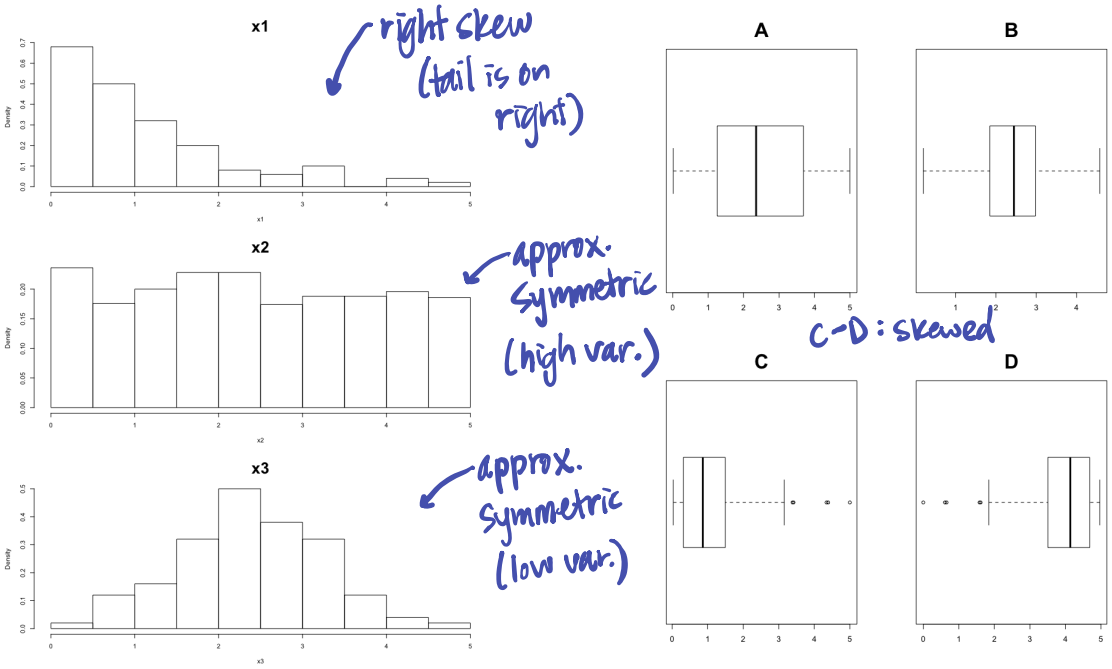
↳ $s = \sqrt{s^2} = \sqrt{92.4} = 9.61 = s$

ordered data: 72, 78, 80, 83, 95, 96

↳ $\tilde{x} = \frac{80+83}{2} = 81.5 = \tilde{x}$

(b) (4 points) Three histograms are given below. Match each with the corresponding box plot. Note: one box plot will not be used.

A-B: symmetric



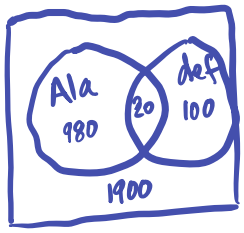
(i) Histogram of x1 matches with box plot letter: C
(ii) Histogram of x2 matches with box plot letter: A
(iii) Histogram of x3 matches with box plot letter: B

4. An automobile manufacturer has two production plants, one in Alabama and one in South Carolina. Over a given month, 1,000 cars are manufactured in Alabama (of which 20 have defective airbags) and 2,000 cars are manufactured in South Carolina (of which 100 have defective airbags). Suppose a car is chosen at random (with equal probability) from the 3,000 cars.

   (a) (2 points) What is the probability that the randomly selected car has a defective airbag?

$$P(\text{defect}) = \frac{\#\text{defects}}{\#\text{total}} = \frac{20+100}{3000} = 0.04$$

   (b) (3 points) What is the probability that the randomly selected car was made in Alabama and does not have a defective airbag?



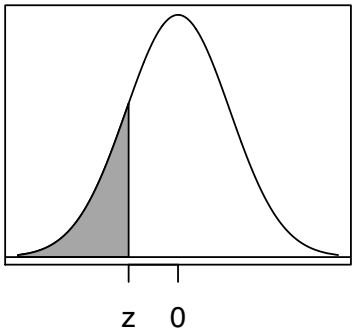
$$P(\text{Alabama} \cap \text{no defect}) = \frac{\#\text{Ala} \cap \text{no def}}{\#\text{total}} = \frac{1000-20}{3000} = 0.327$$

   (c) (5 points) Given that the randomly selected car has a defective airbag, what is the probability that it was manufactured in Alabama?

$$P(\text{Ala} \mid \text{defect}) = \frac{\#\text{Ala} \cap \text{defect}}{\#\text{defect}} = \frac{20}{100+20} = 0.167$$

def of cond. prob.

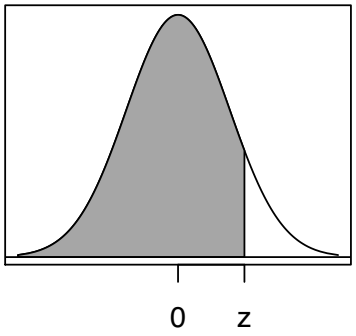
Standard Normal Distribution



Cumulative probabilities for **NEGATIVE** z-values are shown in the following table:

Z	**.00**	**.01**	**.02**	**.03**	**.04**	**.05**	**.06**	**.07**	**.08**	**.09**
−3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
−3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
−3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
−3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
−3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
−2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
−2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
−2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
−2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
−2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
−2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
−2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
−2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
−2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
−2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
−1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
−1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
−1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
−1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
−1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
−1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
−1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
−1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
−1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
−1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
−.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
−.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
−.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
−.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
−.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
−.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
−.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
−.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
−.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
−.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

Standard Normal Distribution



Cumulative probabilities for **POSITIVE** z-values are shown in the following table:

Z	**.00**	**.01**	**.02**	**.03**	**.04**	**.05**	**.06**	**.07**	**.08**	**.09**
.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998