chapter 5 practice problem

(a) (2 points) Let X, Y have a joint probability mass function as follows. Fill in the value missing in the table.

X and Y with joint post plxy) are independent if

p(x,y)=px(x)·Pyly)	(for all x,y)
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X=0 0.3 0.1 0.1 X=1 0.1 0.4 0	
X=0 0.3 0.1 0.1	0
	0
Y=0 Y=1 Y=2	

$$P_{x}(1) \cdot P_{x}(2) = 0.05$$
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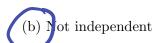
- (b) (2 points) Using the missing value in the table, calculate E[Y]. (If you can't find the answer in (a), use a "missing value" of 0 in your calculation.)
 - E[Y] = Zy. Pyly) = 0(0.4)+1(0.5)+2(0.1) = 0.7
- (c) (1 point) Which of the following best describes the relationship between X and Y in (a)? Circle the correct answer.
 - (i) Independent
- (b) Not independent
- (c) Not enough information
- (d) (4 points) Let X, Y have a joint probability density function as follows. Compute c so that f(x,y) is a valid pdf. \rightarrow |= $\int \int f(x,y) dy dx$

$$f_{X,Y}(x,y) = \begin{cases} c \cdot (x^2 + y^2) & 0 \le x \le 1, 0 \le y \le 1\\ 0 & \text{otherwise} \end{cases}$$

$$|z| = \int_{0}^{1} \int_{0}^{1} c(x^{2}+y^{2}) dy dx = c \int_{0}^{1} \left[x^{2}y + \frac{y^{3}}{3} \right]_{0}^{1} dx = c \int_{0}^{1} \left(x^{2} + \frac{1}{3} \right) dx$$
$$= c \left[\frac{x^{3}}{3} + \frac{x}{3} \right]_{0}^{1} = \frac{z}{3} c = 1 \implies c = \frac{3}{2}$$

Thm: X,Y cont. RVs then X and Y are independent if f(x,y) = g(x)·h(y) for some g(x), h(y)

- (e) (1 point) Which of the following best describes the relationship between X and Y in (d)? Circle the correct answer.
 - (i) Independent



(c) Not enough information