

Scenario A: Developers for a shopping website are conducting an "A/B test" to determine which page layout results in higher sales. Each visitor to the webpage is randomly assigned either layout A or B, and the website then records whether the visitor made a purchase. Below are the results of the data collected:

	Number of Visitors	Number of visitors who made a purchase
Layout A	612	56
Layout B	586	62

1. Can the developers conclude that one website layout produces more sales? Support your answer with an appropriate statistical test.

$$H_0: p_1 - p_2 = 0$$

$$\bar{p}_1 = \frac{56}{612} = 0.092 \quad \bar{p}_2 = \frac{62}{586} = 0.106 \quad D_0 = 0$$

$$\hat{p} = \frac{612}{586+612} \cdot 0.092 + \frac{586}{586+612} \cdot 0.106 = 0.047 + 0.052 = 0.099$$

$$z = \frac{(0.092 - 0.106) - 0}{\sqrt{0.099(1-0.099)\left(\frac{1}{612} + \frac{1}{586}\right)}} = -0.930$$

two sided test, $\alpha = 0.05$

$$p = 0.3974 \times 2 = 0.7948 > 0.05$$

Fail to reject H_0

we cannot conclude the layouts produce different sales.

Scenario B: An political campaign wants to determine whether their paid advertisements affect someone's favorability for their candidate. They sample 12 people from the community to form a focus group. First, they ask the participants to rate their favorability of the candidate on a 0-100 scale. Then they show the participants an advertisement for their candidate and ask them the same question again. The data are below:

Favorability	1	2	3	4	5	6	7	8	9	10	11	12
Before	2	20	65	80	15	50	100	40	0	6	13	60
After	2	25	60	81	35	50	100	50	10	9	20	64

2. Is this paired or unpaired data?

paired

3. Write out the null hypothesis the campaign probably wants to test

$$H_0: \mu_d = 0$$

4. Compute a test statistic, appropriate p-value, and determine whether to Reject or Fail-To-Reject the null hypothesis, using $\alpha = 0.05$

$$\bar{d} = 4.58 \quad s_d = 6.56$$

$$t = \frac{4.58 - 0}{6.56 / \sqrt{12}} = 2.42$$

$$p = 0.017 \times 2 = 0.034$$

Reject.

We conclude the ad had an effect on favorability ratings.

5. Compute a confidence interval for the mean change in favorability before vs. after the advertisement.

$$\bar{d} \pm t_{\alpha/2, 11} \cdot s_d / \sqrt{n} = 4.58 \pm 2.20 \cdot \left(\frac{6.56}{\sqrt{12}} \right) \quad (\text{assuming 95\% confidence})$$

$$\underline{0.414 \leq \mu_d \leq 8.746}$$

Scenario C: Alex's family likes to play Settlers of Catan (a popular board game) with a group of friends as well as with his family. He wants to know whether he gets higher scores on average when playing against his family, or his friends. So he started collecting data on his games. Here are Alex's scores at the end of 20 different games (10 with his friends and 10 with his family):

Group	Game 1	Game 2	Game 3	Game 4	Game 5	Game 6	Game 7	Game 8	Game 9	Game 10
Friends	5	2	1	8	4	5	6	3	5	7
Family	7	8	8	7	5	6	8	7	9	10

6. Is this paired or unpaired data? *unpaired.*

7. Write out the null hypothesis that Alex is testing

$$H_0: \mu_{\text{Friends}} - \mu_{\text{Family}} = 0$$

8. Compute a test statistic, appropriate p-value, and determine whether to Reject or Fail-To-Reject the null hypothesis, using $\alpha = 0.05$

$$\bar{x}_1 = 4.6 \quad \bar{x}_2 = 7.5 \quad s_1 = 2.17 \quad s_2 = 1.43$$

$$v \approx 15.60 \rightarrow 15$$

$$t = \frac{\bar{x}_1 - \bar{x}_2 - 0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = -3.52$$

$$P = 0.0015 \times 2 = 0.003 < 0.05 \quad \text{Reject}$$

We conclude Alex's scores are different w/ friends vs. family

9. Compute a confidence interval for the difference in Alex's mean score between the two groups

$$\bar{x}_1 - \bar{x}_2 \pm t_{\alpha/2, v} \cdot \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} =$$

$$-4.65 < \mu_1 - \mu_2 < -1.15$$

(assuming 95% confidence)

In the following, imagine we are comparing two group means and we've taken one sample from each group. Which of the following statements are always true?

1. Cohen's d measures how far apart the two group means are, relative to how spread out they are
2. If the sample means were further apart, Cohen's d would be larger
3. If both samples were more spread out, Cohen's d would be larger
4. A high powered test is likely to reject the null hypothesis if it's false.
5. A high powered test is likely to not reject the null hypothesis if it's true.
6. A low powered test is not likely to reject the null hypothesis if it's false.
7. A low powered test is likely to not reject the null hypothesis if it's true.

Power only tells us about
what's likely if H_0 is
false