# chapter 5 practice problem

(a) (2 points) Let $X, Y$ have a joint probability mass function as follows. Fill in the value missing in the table.

X and Y with joint pmf $p(x,y)$ are independent if

$p(x,y) = p_X(x) \cdot p_Y(y)$ (for all $x,y$)

$\downarrow$

$p(1,2) = 0$

$p_X(1) = 0.5$

$p_Y(2) = 0.1$

$p_X(1) \cdot p_Y(2) = 0.05$

$\neq$

$p(2,1) = 0$

$\downarrow$

dependent

|  | Y=0 | Y=1 | Y=2 |  |
|---|---|---|---|---|
| X=0 | 0.3 | 0.1 | 0.1 | 0.5 |
| X=1 | 0.1 | 0.4 | 0 | 0.5 |
|  | 0.4 | 0.5 | 0.1 |  |

$p(x,y)$ is valid if

(i) $p(x,y) \geq 0$

(ii) $\sum_x \sum_y p(x,y) = 1$

(b) (2 points) Using the missing value in the table, calculate $E[Y]$.
*(If you can't find the answer in (a), use a "missing value" of 0 in your calculation.)*

$$E[Y] = \sum y \cdot p_Y(y) = 0(0.4) + 1(0.5) + 2(0.1) = \boxed{0.7}$$

(c) (1 point) Which of the following best describes the relationship between $X$ and $Y$ in (a)? Circle the correct answer.

(i) Independent (b) Not independent (c) Not enough information

(d) (4 points) Let $X, Y$ have a joint probability density function as follows. Compute $c$ so that $f(x,y)$ is a valid pdf. $\rightarrow$ $1 = \iint f(x,y)\,dy\,dx$

$$f_{X,Y}(x,y) = \begin{cases} c \cdot (x^2 + y^2) & 0 \leq x \leq 1, 0 \leq y \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

$$1 = \int_0^1 \int_0^1 c\,(x^2+y^2)\,dy\,dx = c\int_0^1 \left[x^2 y + \frac{y^3}{3}\right]_0^1 dx = c\int_0^1 \left(x^2 + \frac{1}{3}\right)dx$$

$$= c\left[\frac{x^3}{3} + \frac{x}{3}\right]_0^1 = \frac{2}{3}c = 1 \rightarrow \boxed{c = \frac{3}{2}}$$

Thm: $X, Y$ cont. RVs then X and Y are independent if $f(x,y) = g(x) \cdot h(y)$ for some $g(x), h(y)$

(e) (1 point) Which of the following best describes the relationship between $X$ and $Y$ in (d)? Circle the correct answer.

(i) Independent (b) Not independent (c) Not enough information