Intro to real numbers

Chapter 1

★ $\frac{1}{2} = 0.5 = 50\% = 2^{-1} = 1:2$

fraction decimal percentage exponent ratio

↳ $2^{3/4} = \sqrt[4]{2^3} = (\sqrt[4]{2})^3$

★ $\frac{1}{a} \pm \frac{1}{b} = \frac{a \pm b}{ab}$

↳ $\left(\frac{1}{a} \times \frac{b}{b}\right) \pm \left(\frac{1}{b} \times \frac{a}{a}\right) = \frac{b}{ab} \pm \frac{a}{ab} = \frac{b \pm a}{ab}$

↳ $\frac{1}{a} \overset{②}{\underset{①}{\times}} \overset{③}{\pm} \frac{1}{b} = \frac{a \pm b}{ab}$

★ $\frac{1}{a} \xrightarrow{\times} \frac{1}{b} = \frac{1}{ab}$

★ $\frac{a}{\frac{b}{c}} = a \div \frac{b}{c} = a \times \frac{c}{b} = \frac{ac}{b}$

★ $\frac{a}{b} = \frac{c}{d} \Rightarrow ad = bc \Rightarrow d = \frac{bc}{a}$

★ $a(b+c) \overset{\longleftrightarrow}{=} ab + ac$ (works vise-versa) distrubutive properties

★ $a(bc) = c(ab) = b(ad)$

Exponents

★ $(a^b)^c \neq ab^c = a^{bc}$

★ $a^b + a^b \neq 2ab \neq ab^2 \neq 2a^{2b} \neq a^{2b} \neq a^{b+b} = 2a^b$

↳ $x^2 + x^2 = 2x^2$

★ $a^b \cdot a^b = a^{b+b} = a^{2b}$

↳ $2^3 \times 3^2 \neq 6^6$ or $6^5 = 2^3 \times 3^2$

↳ $2^3 \times 2^2 = 2^5$

$(2)(2)(2)(2)(2)$

★ $\frac{a^b}{a^c} = a^{b-c}$

↳ $\frac{2^5}{2^3} = 2^{5-3} = 2^2 \Rightarrow \frac{(2)(2)(2)(2)(2)}{(2)(2)(2)}$

★ $a^{\pm\frac{b}{c}}$ ← c is the index of the root $= \overset{\text{if +ve}}{\sqrt[c]{a^b}} = \overset{\text{if -ve}}{\frac{1}{\sqrt[c]{a^b}}}$

↳ $x^{\frac{-3}{5}} = \frac{1}{\sqrt[5]{x^3}}$

★ $\sqrt{x^2} = \pm x = |x|$

★ $|x| = \pm x \Rightarrow \begin{cases} x \text{ if } x \geq 0 \\ -x \text{ if } x < 0 \end{cases}$

Common Exponents

$2^1 = 2$	$3^1 = 3$	$5^1 = 5$
$2^2 = 4$	$3^2 = 9$	$5^2 = 25$
$2^3 = 8$	$3^3 = 27$	$5^3 = 125$
$2^4 = 16$	$3^4 = 81$	$5^4 = 625$
$2^5 = 32$	$3^5 = 243$	$5^5 = 3125$
$2^6 = 64$	$3^6 = 729$	$5^6 = 15625$

* Never distribute the exponent for + or − only for x or ÷.
  - L> $(xy)^2 = x^2y^2$
  - L> $(x+y)^2 \neq x^2+y^2$ => foil it out
    $(x+y)(x+y) = x^2 + 2xy + y^2$

* Assume $3^x = y$
  $y+y+y = 3y$; $3y => 3^1(3^x)$
  $= 3^{1+x}$ $\neq 3^x$ $\neq 9^x$

* Use process of elimination when stuck on a question.
* use/find Lowest Common Multiple (LCM)

** Easy multiplication

↳ $12 \times 12 \Rightarrow 12(10+2)$

$120 + 24 = 144$

** multiplying between square roots

↳ $\sqrt{24} = \sqrt{6 \times 4} = \sqrt{2 \times 12} = \sqrt{3 \times 8}$

$= \sqrt{6}\sqrt{4} = \sqrt{2}\sqrt{3}\sqrt{4} = \sqrt{3}\sqrt{2}\sqrt{4}$

$= 2\sqrt{6} \quad = 2\sqrt{6} \quad = 2\sqrt{6}$

Conjugate

* $\frac{1}{\sqrt{a}} \times \frac{\sqrt{a}}{\sqrt{a}}$ ← conjugate

↳ $= \frac{\sqrt{a}}{a}$ ← simplest form

* $\frac{1}{\sqrt{a} \pm \sqrt{b}} \times \frac{\sqrt{a} \mp \sqrt{b}}{\sqrt{a} \mp \sqrt{b}}$

↳ if -ve then x by +ve

↳ if +ve then x by -ve

↳ $= \frac{\sqrt{a} \mp \sqrt{b}}{a \mp b}$

↳ $(\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b})$

$= a - \cancel{\sqrt{a}\sqrt{b}} + \cancel{\sqrt{a}\sqrt{b}} - b = a \pm b$

↳ $a - b$ ← difference of squares

Subtraction (easy method)

ex

$25 - 17 = 8$

25 → 5 → 20 → 3 → 17 > 8

Intro the Polynomials

Chapter 2

☆ Reorganize algebraically

1. cross multiply
2. Seperate variables
3. common factor
4. Seperate variables
5. if answer is not in choices, the multiply by $\frac{-1}{-1}$.

☆ Keyword : substitution

↳ if stuck, substitution works

↳ Plug a number that can confirm or eliminate two of the choices. Then plug another number to confirm the final answer.

General form of the quadratic equation

★ $ax^2 + bx + c = 0$

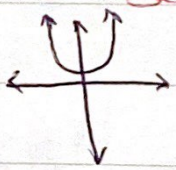
↳ solution to quadratic equation (quadratic formula)

★ $x = \frac{-b \pm \sqrt{\Delta}}{2a}$, $\Delta = b^2 - 4ac$

★ (1) $\Delta < 0$, there is no real solution

ex $(x^2 + 4) = 0$

$= 0 - 4(1)(4) = -16$

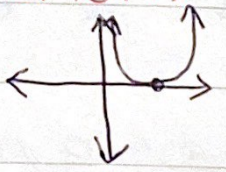


(2) $\Delta = 0$, there is one real solution

aka: two similar solutions

ex $(x-2)^2 = 0$

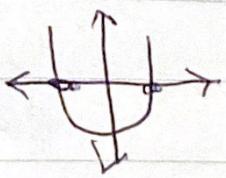
↳ complete / perfect square



(3) $\Delta > 0$, there are two real solutions

ex $(x^2 - 4) = 0$

↳ difference of two squares



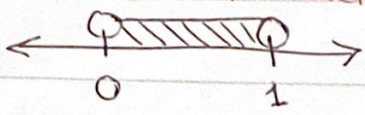
↳ $(x-2)(x+2) = 0 \Rightarrow x = -2, 2$

Intro to Inequalities

Chapter 3

Intervals

★ $(\square, \bigcirc)$ ← not included

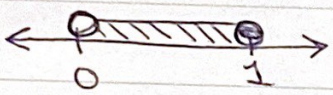
ex: $(0,1)$ 

$0 < x < 1$

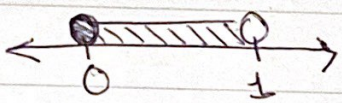
★ $[\square, \bigcirc]$ ← included

ex: $[0,1]$ 

$0 \leq x \leq 1$

★ $(0,1]$ 

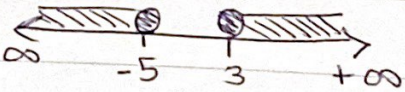
$0 < x \leq 1$

★ $[0,1)$ 

$0 \leq x < 1$

union

★ $(\triangle, \star] \cup [\square, \bigcirc)$

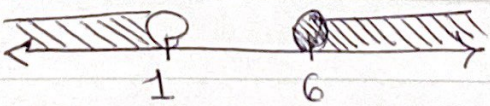
ex: $(-\infty, -5] \cup [3, \infty)$ 

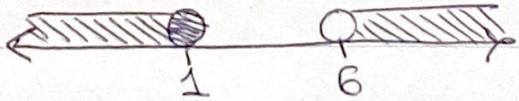
all real numbers

★ $R \setminus [\square, \bigcirc)$ ← when you plug switch to the opposite

↳ except

→ all real number except $\square$ to $\bigcirc$ but still including $\bigcirc$.

ex: $R\,[1,6)$ 

not including 

Guidelines for Solving Inequalities

- ☆ Eliminate zeros of denominator (if applicable).
- ☆ choose a unique number.
- ☆ substitute the number in the inequality.
- ☆ If number validates inequality, eliminate intervals not including chosen number.
- ☆ If number does not validate inequality eliminate intervals including chosen number.
- ☆ repeat until one interval is left.
- ☆ If none of the above is one of the choices, check boundaries of the other intervals.

↳ one side is zero, other side factored out

↳ Then: ① draw a number line; place zeros of numerator and denominator; choose a number and plug in inequality; place sign in alternating intervals. ② include zeros of numerator if $\geq \leq$ in inequality. ③ never include infinity $(-\infty, \infty)$ and zero of denominator. ④ place a union in between.

Intro to Absolute Value

Chapter 4

* $|x| = n$
  - ↳ $x = \pm n$

ex: $|x| = 5 \rightarrow x = 5, x = -5$

* $|x| < n$
  - ↳ $-n < n < n$
  - * for negative flip < to this >.

ex: $|x| < 5 \rightarrow -5 < x < 5$

Chapter 5

Intro to functions

- discontinuous → holes
- continuous → continuous

functions

- continuous
- discontinuous
  - zero in denominator
  - negative under even root

- composite functions
  - ↳ $f(x) = x^2 + 1$ domain $(-\infty, \infty)$

    $f(5) = 5^2 + 1 = 26$

    $f(y) = y^2 + 1$
  - ↳ $g(x) = \frac{1}{x}$

    $g(5) = \frac{1}{5}$

    $g(y) = \frac{1}{y}$

    $\left(\frac{1}{x}\right)^2 + 1$

    ↑

    $\frac{1}{x}$
  - ↳ $f \circ g(x) \rightarrow f(g(x)) \rightarrow f\left(\frac{1}{x}\right)$

* $f(x) = ax^2 + bx + c$
* $f'(x) = 2ax + b$ set $f'(x) = 0$

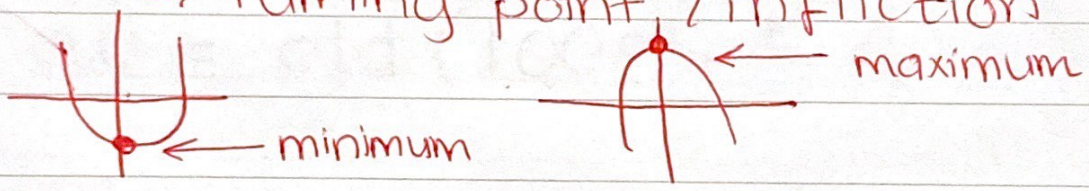
$2ax + b = 0$

$2ax = -b$

$$x = \frac{-b}{2a}$$

Critical point formula

* Critical point : minimum or maximum
  - turning point / infliction



Percent

Chapter 6

* $\% = \frac{\text{Part}}{\text{whole}} \times 100$

* $\underset{\text{(change)}}{\Delta\%} = \frac{\text{new} - \text{old}}{\text{old}} \times 100$

* $\text{new} = \text{old}(100\% \pm \Delta\%)$

* $\text{old} = \frac{\text{new}}{100\% \pm \Delta\%}$

Units

* $1\,km = 1000\,m = 100000\,cm = 1000000\,mm$

* $1\,hr = 60\,min = 3600\,sec$

* $\text{rates} = \frac{\text{work}}{\text{time}}$

Proportions

Proportions

- Direct
- Indirect (inverse)

↳ Direct: (cross multiply)

* if quantity a increases, then quantity b increases.
* if quantity a decreases, then quantity b decreases.

↳ Indirect: (inverse) (horizonally multiply)

* if quantity a decreases, then quantity b increases.
* if quantity a increases, then quantity b decreases.

ex (1) 1 bottle of water costs 100 fils. How many would 2 KD purchase? (Direct)

bottles		Money
1	↗	100
x	↘	2000

$\frac{100x}{100} = \frac{2000}{100}$

$x = 20$

ex ② 10 workers can build a mosque in 10 months, how long would it take for 20 workers?

(Indirect)

workers		time
10	⟶	10
20	⟶	x

$10 \times 10 = 20x$

$\frac{100}{20} = \frac{20x}{20}$

$x = 5$

Steps :

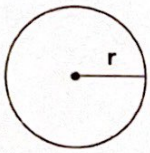
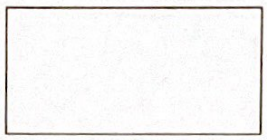
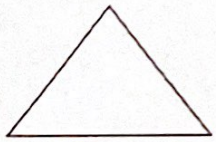
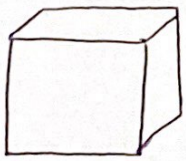
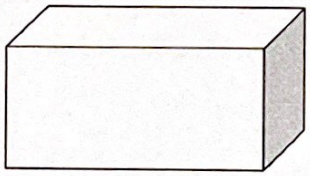
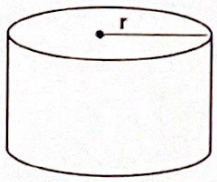
* check units
* line variables correctly
* choose correct proportion
* solve

Formulas to Remember :

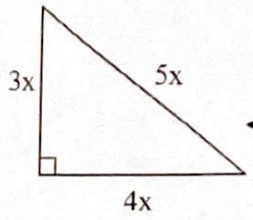
* $(a^3+b^3) \rightarrow (a+b)(a^2-ab+b^2)$

* $(a^3-b^3) \rightarrow (a-b)(a^2+ab+b^2)$

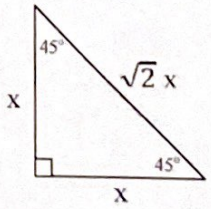
Perimeters, Areas, & Volumes

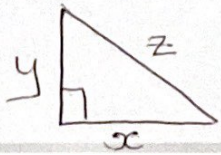
Shape			
Perimeter	$2\pi r$	2(length + width)	Sum of all sides
Area	πr^2	length × width	$\frac{1}{2}$ base × height
Shape			
Surface Area	$6(L^2)$	Sum of all side areas	$2\pi r^2$ + (height × $2\pi r$)
Volume	L^3	length × width × height	πr^2 × height

Right Triangles



◀ **Lengths** **Angles** ▶





$z^2 = x^2 + y^2 \iff z = \sqrt{x^2+y^2}$