1. We use the notation as in the class. You are given $A, \vec{b}, \vec{c}$, $\vec{x}_B$, B and B^{-1}.

$$[A|I] = \begin{array}{c} \begin{array}{ccccc} x_1 & x_2 & x_3 & x_4 & x_5 \end{array} \\ \begin{bmatrix} 2 & 1 & 1 & 0 & 0 \\ 1 & 2 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix} \end{array}, \qquad \vec{b} = \begin{bmatrix} 4 \\ 3 \\ 1 \end{bmatrix}, \qquad \vec{x}_B = \begin{bmatrix} x_3 \\ x_1 \\ x_2 \end{bmatrix}$$

$$\vec{c}^T = \begin{array}{c} \begin{array}{ccccc} x_1 & x_2 & x_3 & x_4 & x_5 \end{array} \\ \begin{bmatrix} 3 & 2 & 0 & 0 & 0 \end{bmatrix} \end{array}, \quad B = \begin{array}{c} \begin{array}{ccc} x_3 & x_1 & x_2 \end{array} \\ \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \end{array}, \qquad B^{-1} = \begin{bmatrix} 1 & -2 & 3 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$$

Let $\vec{x}^*$ be the basic solution of the corresponding primal dictionary, and let $\vec{y}^*$ be the corresponding dual basic solution (i.e. the basic solution of the corresponding dual dictionary).

(a) Find $x_1^*, x_2^*, x_3^*, x_4^*, x_5^*$ and $y_1^*, y_2^*, y_3^*, y_4^*, y_5^*$.

(b) Is $\vec{x}^*$ optimal? To earn credit, you have to justify your answer.

2. Exercise 11.3 (Warning: The notation $\vec{x}$ and $\vec{y}$ are the reverse the notation used in lectures. Also the payoff matrix used in the question is the payoff matrix for the column player unlike the convention used in class).

3. Find the value of the zero-sum game given by the following payoff matrix for the row player and determine **all** optimal strategies for both players:

$$\begin{bmatrix} 3 & 0 \\ 0 & 3 \\ 2 & 2 \end{bmatrix}.$$