The Sun-planets interaction and its correlation to the solar cycle

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Abstract

We simulate the solar system's gravitational interaction over last 400 years using Newton's law of universal gravitation and record the Sun's distance, speed and acceleration relative to the solar system's barycenter. Obtained data is compared to the solar sunspot number in search of correlation between the two. We found that the average distance of the Sun from the barycenter over a cycle duration may correlate to the solar cycle's grand minima and maxima.

Contents

1 Introduction

The aim of this project was to simulate the motion of main bodies of the solar system and look for a correlation between the Sun's movement and the annual sunspot number from Solar Influences Data Analysis Center[1]. The programs objectives were

- To accurately model the trajectories of the main bodies of the solar system back in time,
- To calculate Sun's distance, speed and acceleration relative to the barycenter,
- To calculate the moving average of all results over the mean length of a solar cycle.

2 Background

Sunspots are visibly observable dark regions on the surface of the Sun. They arise due to the magnetic field concentration increase in the region which cools the surface giving rise to dark spots. The number of sunspots varies with time but appears to be cyclic with a period of around 11 years. The peak of each cycle is different, but there are periods in time when the peak of every cycle is very high or very low, known as the grand solar maxima and minima respectively. Sunspots are correlated with solar flare and coronal massive ejection events[2] which produce solar wind. Due to that effect, during grand solar maxima humans at high altitudes are at risk of exposure to high radiation. There is also some evidence that maxima and minima might cause some minor climate change on Earth[3].

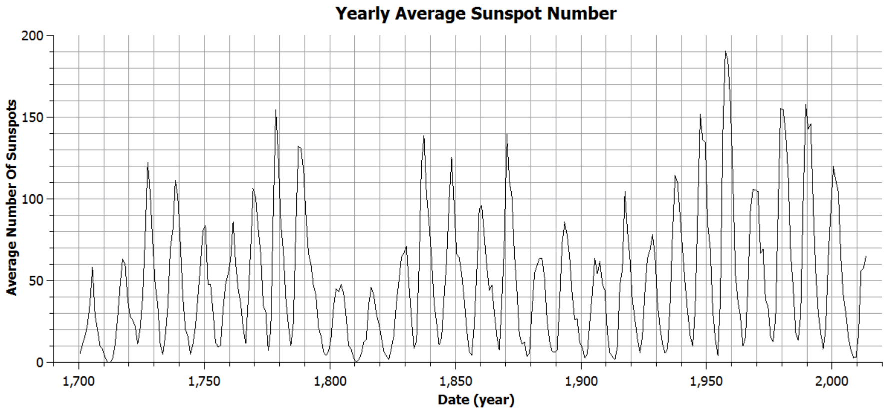

Figure 1: Yearly average sunspot number obtained from Solar Influences Data Analysis Center.

3 Methodology

3.1 Overview

The simulation was written in Python 2.7. It took into account the Sun, 8 planets of the solar system and the largest object in the Kuiper belt, Pluto. Mass, position and velocity of each object was obtained via JPL HORIZONS system[4]. The start time was A.D. 2014-Dec-15 17:00:00 CT and with negative time step it was run to A.D. 1600-Jan-01 00:02:00 CT. This period contains the Maunder minimum (1640-1720), Dalton minimum (1790-1830) and the modern maximum (1944-1985)[5]. The position of the barycenter relative to the Sun was calculated at 3 hour intervals. We only considered Newton's second law of motion and Newton's law of universal gravitation, yielding the equation

$$\sum_{j\neq i=1}^{n} G\frac{m_i m_j(\vec{r}_i - \vec{r}_j)}{|\vec{r}_i - \vec{r}_j|^3} = m_j\vec{a}_j. \tag{1}$$

Here n is the number of bodies, G is the gravitational constant, m_i and m_j are mass of the i'th and j'th body, r_i and r_j are the positions of the i'th and j'th body and $\vec{a}_j$ is the acceleration of the j'th body. Mercury's actual orbit is not described well by this model which raised some concerns, however as Fig. 2 shows, the presence of mercury in the simulation is not necessary. The two data sets are equal for all intents and purposes due to Mercury's relatively low mass and proximity to the barycenter. All data calculated and analysed will contain Mercury's interactions in this model. Velocity and acceleration of the barycenter were obtained by numeric differentiation. We are only concerned with the magnitudes of these vectors so by considering distance, speed or acceleration of barycenter relative to the sun we are equivalently considering them for the Sun relative to the barycenter. All values were arbitrarily mapped between 0 and 200 to be plotted easily with the yearly average sunspot number. A simple moving average was also found for all data with time intervals of roughly 6 years either side so that it coincides with the sunspot cycle period.

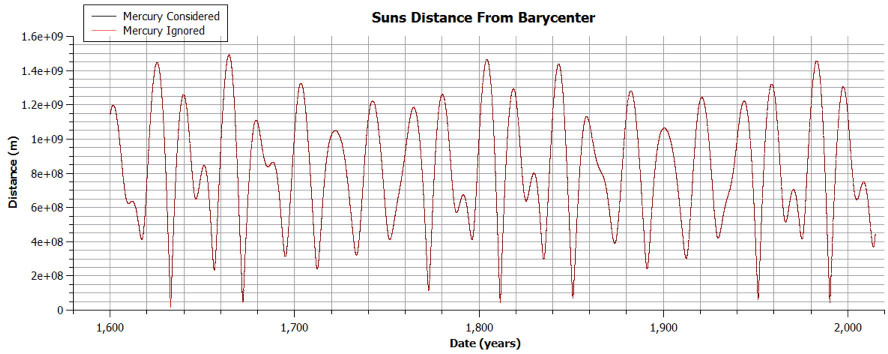

Figure 2: Variation of the Sun's distance to the barycenter as calculated with and without Mercury's interactions. For reference, the Sun's radius is 7x10^8 m.

3.2 Simulation

Firstly we defined a class Body with name, mass, position, velocity and acceleration variables. An update function was also defined in this class to update the values of velocity and then position at each time-step $dt = -3\ hours$:

$$\vec{v}_{n+1} = \vec{v}_n + \vec{a}_n dt, \tag{2}$$

$$\vec{r}_{n+1} = \vec{r}_n + \vec{v}_{n+1}. \tag{3}$$

There are two time variables, Current_Simulation_Time corresponding to A.D. 2014-Dec-15 17:00:00.0000 CT and Stop_Time corresponding to A.D. 1600-Jan-01 00:00:00.0000 CT. The main loop of the program starts by decrementing Current_Simulation_Time by three hours, then proceeds to calculate the acceleration of each body using eq.1, after which the update function of each body is called which uses this acceleration for eq.2. When the positions are updated, the program calculates the position of the barycenter relative to the Sun using

$$\vec{r}_b = \frac{\sum_{i=1}^{n} m_i \vec{r}_i}{\sum_{i=1}^{n} m_i} - \vec{r}_{Sun}. \tag{4}$$

The main loop stops when Current_Simulation_Time is smaller than Stop_Time. When the simulation finishes two sets of data are saved. One contains the Sun-barycenter distance and time to be used for plotting, the other contains the position of the barycenter relative to the Sun and time to be used for numerical differentiation. To ensure stability of the program, we added additional code that used Visual Python module to show the positions of the bodies as the simulation was run. It showed that all the objects were orbiting properly and none of them showed unexpected trajectories, however it did not check the accuracy of the simulation. To do that, we calculated the barycenter's distance from the Sun at A.D. 1914-Dec-15 17:00:00.0000 CT using data obtained from JPL HHORIZONS and compared it to our value from the simulation. JPL HORIZONS' distance was calculated to be 6057301169.549 meters, while our simulation gave it as 595723209.965 meters. After 100 years of simulation the error was 1.65%, which is reasonable considering only 10 bodies were simulated using Newtonian mechanics.

3.3 Numerical differentiation

A separate program was written to perfom numerical differentiation in order to obtain speed and acceleration values. Due to the enormous number of data points, only a simple equation was used:

$$\vec{v}(t_n) = \frac{\vec{x}(t_{n+1}) - \vec{x}(t_n)}{\Delta t}, \tag{5}$$

where $\Delta t = t_{n+1} - t_n = 3\ hours$. This program first loaded the position data obtained from the simulation and differentiated it once to obtain velocity, then differentiated velocity using the same formula to obtain acceleration. Only the magnitudes of these vectors were saved along with their corresponding time. The speed data had one less data point compared to the distance data and similarly the acceleration data had one less data point compared to the speed data, because of eq.5 requiring one point forward from the initial point of calculation. This was not an issue because the distance data had 1212479 data points. To ensure that the program was working correctly, a random point in time was taken and differentiated numerically by hand. The result completely agreed with the program output.

3.4 Arbitrary variables

Another program was written to shift all data to values between 0 and 200 while preserving the shape of the plot. This was done in order to plot the data with the yearly mean total sunspot number (fig.1). To do so we considered the equation $y = mx + c$, where x was the input variable and y was the output variable, so m and c are unknowns. The program loads the data and first finds the minimum and maximum values. With those we have two equations:

$$0 = mx_{min} + c, \tag{6}$$

$$200 = mx_{max} + c. \tag{7}$$

Two equations and as many unknowns. The program first finds m using

$$m = \frac{x_{max} - x_{min}}{200}, \tag{8}$$

then finds c using

$$c = 200 - mx_{max}. \tag{9}$$

This equation is then applied to every point, producing the same shape over a different codomain. This program was applied to the distance, speed and acceleration data. To ensure the program ran properly, we took three random points from the original distance data and the corresponding points from the shifted data. Two of the points were used to work out m and c, then the third point was used to check that the equality still held.

3.5 Moving average

The purpose of the final program we wrote was to calculate the moving average of our data. It used the formula

$$\overline{x}_i = \frac{1}{n} \sum_{j=i-\frac{n}{2}}^{i+\frac{n}{2}} x_j. \tag{10}$$

This formula requires n to be even, so we used n=12 for the yearly average sunspot number. For the data obtained from the simulation we used n=33000, roughly equal to 12 years in conjunction with 12 years used for sunspot number. The formula was applied to only every 2000th data point after 16500th data point in order to reduce the numbers of points to be plotted. It still contained enough points to make a smooth graph. We checked whether the program executed correctly by letting QtiPlot work out the moving average for the yearly average sunspot number, then compared it to our curve which was identical. We decided to make a separate program for our data because QtiPlot was crashing when trying to calculate it for the vast amount of data points, hence the choice to also reduce their amount. We could not reduce the number of data points before applying the moving average due to the acceleration data which is discussed in results section.

4 Results

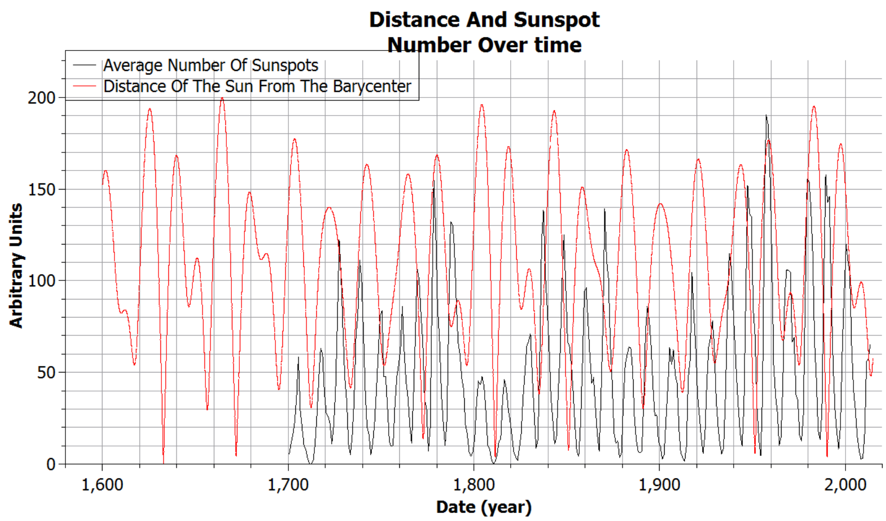

Figure 3: Yearly average sunspot number and distance of the Sun from the barycenter of the solar system.

While the variation of distance appears to be roughly cyclic, it does not correlate well with the sunspot distance. While there are some troughs in distance that also correspond to troughs in sunspot number, year 1710 or 1810 for example, there are also peaks in distance with troughs in sunspot number, year

1900 or 1998. Similarly there are peaks in distance corresponding to both peaks and troughs in sunspot number, years 1959 and 1998. There also seems to be nothing unusual happening during the Maunder minimum, Dalton minimum or the modern maximum.

Fig.4 does not seem to give any insight. It does not correlate with the sunspot

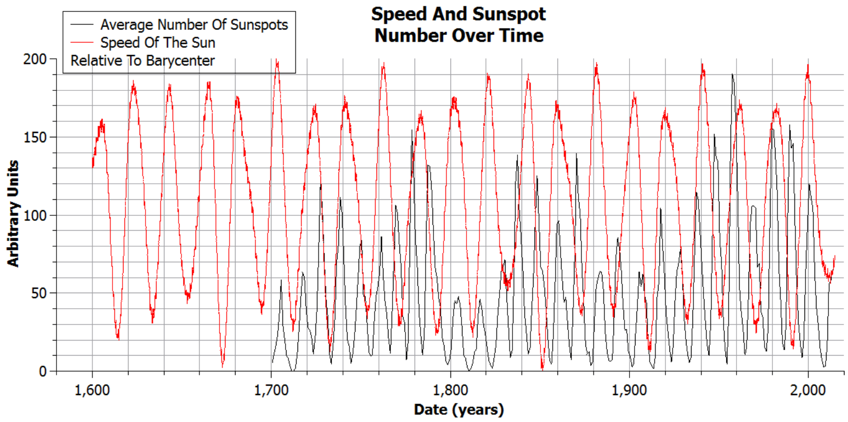

Figure 4: Yearly average sunspot number and speed of the Sun relative to the barycenter of the solar system.

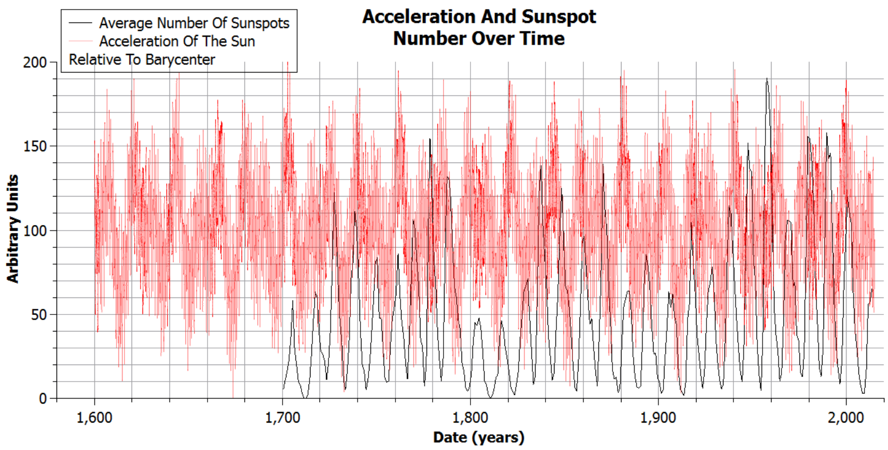

Figure 5: Yearly average sunspot number and acceleration of the Sun relative to the barycenter of the solar system.

number, or show any variation that could help correlate distance to sunspot number.

Fig.5 shows that the value jumps drastically and very fast. We think this might be due to perturbations from Mercury's orbit or inaccuracies of the simulation. Nevertheless some structure can still be observed, but as with the previous two graphs, it does not help in finding any correlation between the Sun's motion and the sunspot number. This hectic distribution however prompted us to consider moving average of the data.

When the moving average is plotted for distance (fig.6) some correlation appears.

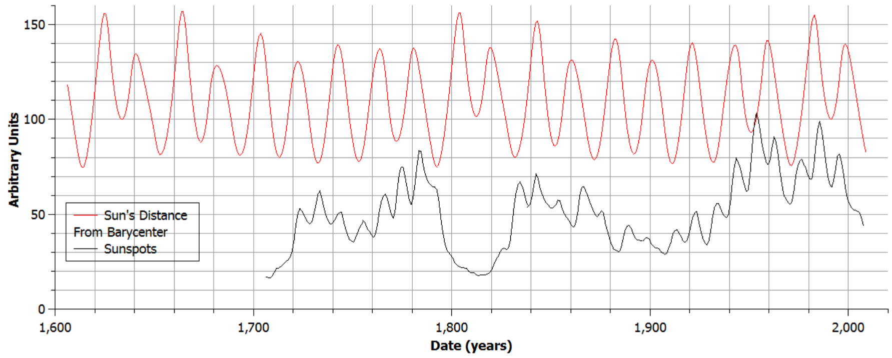

Figure 6: Moving average sunspot number and moving average distance of the Sun to the barycenter of the solar system.

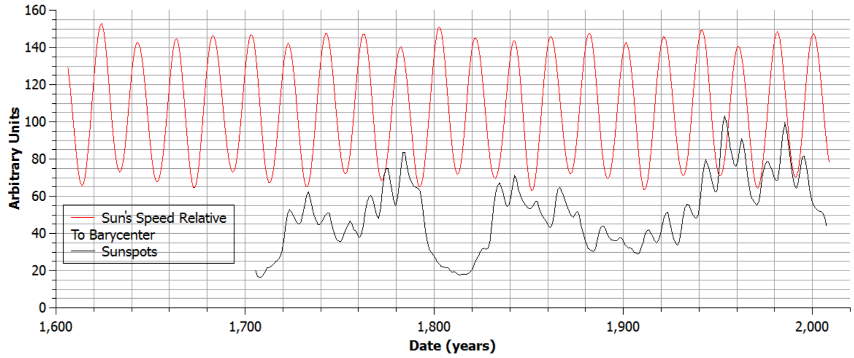

Figure 7: Moving average sunspot number and moving average speed of the Sun relative to the barycenter of the solar system.

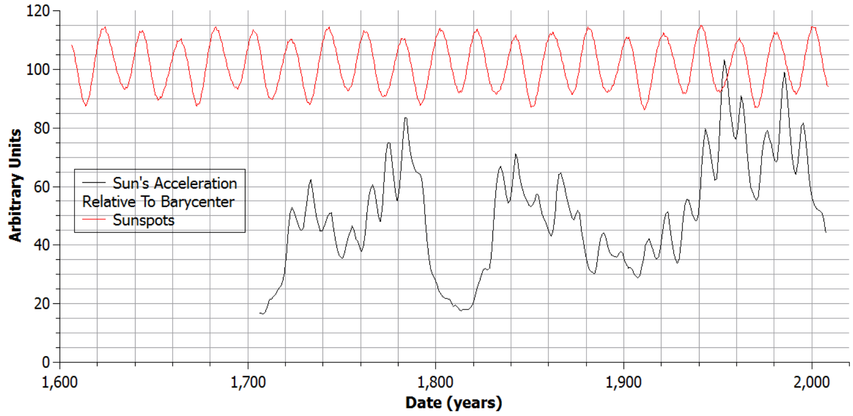

Figure 8: Moving average sunspot number and moving average acceleration of the Sun relative to the barycenter of the solar system.

For the rest of this section, the fact that we are talking about moving average values will be implied. Firstly we define that when the peaks of distance are at about 140 arbitrary units and troughs at about 80 arbitrary units the sunspot number is stable. During the Maunder minimum both peaks and troughs of distance are quite high, indicating that on average the Sun was further away from the barycenter during that time. Then the rise of sunspot number happens with a lower peak in distance in the early 1720s. Then the sunspot number stabilizes for about 40 years, then starts rising again with a high trough in distance in about 1770. During the Dalton minimum, the distance peaks and troughs are above normal again. The period between Dalton minimum and modern maximum is fairly stable, with two lower peaks in distance corresponding to lowest peaks in sunspot number for 20 years either side of them. During the modern maximum, the highest peak in sunspot number occurs with a very high trough in distance. While this might seem like correlation, it is not proof of causation. We can see that the distance data is most likely cyclic with a period of about 180 years, a longer simulation period would probably confirm this. As for moving average for speed and acceleration data, no link can be established with neither sunspot number or impact on Fig. 6.

5 Conclusions

We set out to answer whether the motion of the Sun affects the sunspot number and an affirmative answer can not be drawn. Our data showed some correspondence between drastic changes in moving average solar number and moving average distance straying from oscillating between 80 and 140 arbitrary units. The sunspot number has only been properly documented with the advent of the telescope, so we do not have enough data to go further back to look for a more clear answer. We can however make two hypotheses based on our findings. First one is that grand solar minima and maxima are cyclic events with period of 180 years. It is most likely not true as there would be more "Little Ice Age" periods recorded, which happened during the Maunder minimum[3]. The second hypothesis is that sudden jumps from "stable" oscillations of moving average distance can cause quite noticeable changes in the Sun's activity, which might actually be true if modelled and analysed in more detail[6]. The Sun itself is a complex object with many variable mechanisms, and sunspots are inherently electromagnetic phenomena, so a purely gravitational simulation is simply insufficient and other factors must be considered.

References

[1] Solar Influences Data Analysis Center: http://sidc.oma.be/silso/datafiles Yearly mean total sunspot number [1700 - now]

[2] '*Comprehensive analysis of coronal mass ejection mass and energy properties over a full solar cycle*' Vourlidas, A. , Howard, R. A. , Esfandiari, E. , Patsourakos, S. , Yashiro, S. , Michalek, G., The Astrophysical Journal, 2010, Vol.722(2), pp.1522-1538

[3] '*Synchronized Northern Hemisphere climate change and solar magnetic cycles during the Maunder Minimum*' Yasuhiko T. Yamaguchi, Yusuke Yokoyama, Hiroko Miyahara, Kenjiro Sho, Takeshi Nakatsuka and Konrad A. Hughen, Proceedings of the National Academy of Sciences of the United States of America Vol. 107, No. 48 (November 30, 2010), pp. 20697-20702

[4] NASA's on-line solar system data and ephemeris computation service: http://ssd.jpl.nasa.gov/?horizons

[5] '*The Sunspot Activity in the Last Two Millenia on the Basis of Indirect and Instrumental Indexes: Time Series Models and Their Extrapolations for the 21st Century*' Komitov, Boris; Kaftan, Vladimir Proceedings of the International Astronomical Union, 2004, Vol.2004(IAUS223), pp.113-114

[6] '*A phenomenological study of the timing of solar activity minima of the last millennium through a physical modeling of the SunPlanets Interaction*' Cionco, Rodolfo Gustavo; Soon, Willie New Astronomy, 2015, Vol.34, pp.164-171

6 Appendix

6.1 Arbitrary units

The minimum and maximum distances were 15023656.1588 and 1493037042.29 meters respectively. For conversion to arbitrary units, m=1.3531677174x10^{-7}, c=-2.03295265114.
The minimum and maximum speeds were 8.49392691907 and 16.2167353388 meters per second respectively. For conversion to arbitrary units, m=25.8973146982, c=-219.969898447.
The minimum and maximum accelerations were 1.18201385503x10^{-7} and 3.01641564273x10^{-7} meters per second per second respectively. For conversion to arbitrary units, m=1090273686.72, c=-128.871860348.

6.2 Actual value plots

The graph of distance is given in Fig.2.

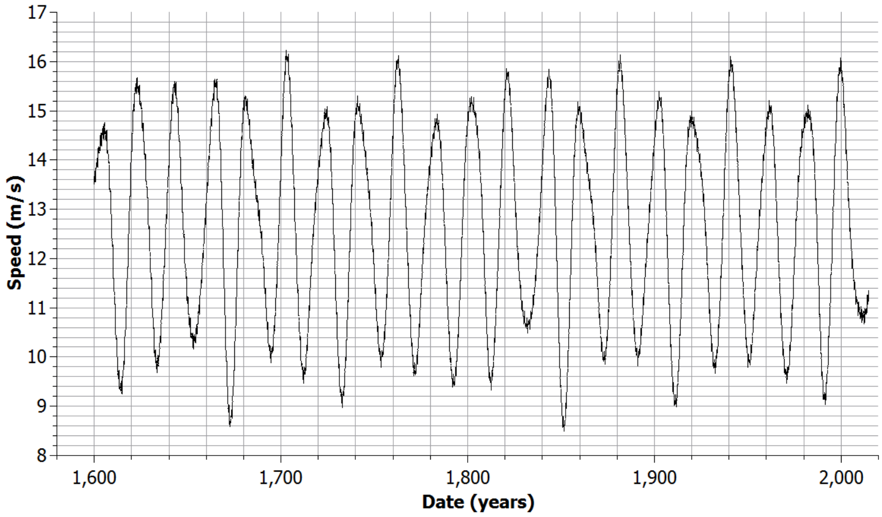

Figure 9: Speed of the Sun relative to the barycenter of the solar system.

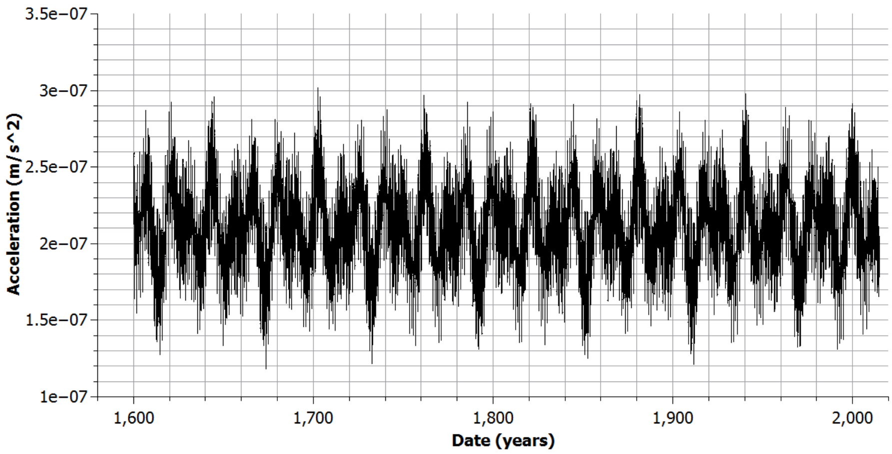

Figure 10: Acceleration of the Sun relative to the barycenter of the solar system.