NAVIER-STOKED

Visualizing the Navier-Stokes Equations

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Problem Statement

To design a physical model of laminar flow between flat plates that can be manipulated and observed, to report the fluid velocity at various points as a teaching aid to facilitate learning of the **Navier-Stokes equations, assumptions, and conditions.**

Motivation

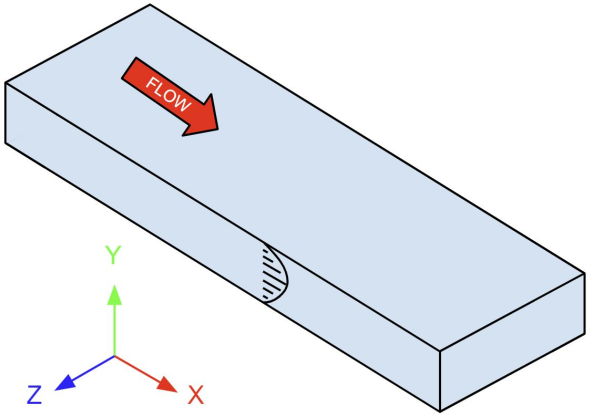
- **Our Client:** Prof. Marguerite Matherne
- The **Navier-Stokes** equations are complex and the conditions in which critical assumptions are valid are hard to conceptualize
- Assumptions and Conditions:
  - Infinite Plates Condition ➡ *v, w* = 0
  - Plane Flow Assumption ➡ *∂/∂z* = 0
  - Steady State Condition ➡ *∂/∂t* = 0
  - Fully Developed Condition ➡ *∂u/∂x* = 0

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

$$x: \rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right) = -\frac{\partial p}{\partial x} + \rho g_x + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right)$$

$$y: \rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right) = -\frac{\partial p}{\partial y} + \rho g_y + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right)$$

$$z: \rho\left(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}\right) = -\frac{\partial p}{\partial z} + \rho g_z + \mu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right)$$



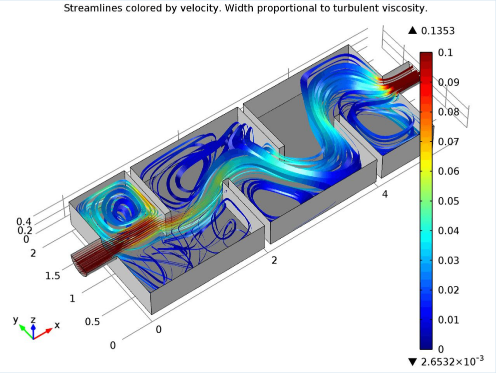
Justification

- Students struggle with identifying appropriate conditions in which the assumptions are valid such as:
  - **Infinite Plates Condition ➡** no wall/edge effects
  - **Plane Flow Assumption ➡** fluid flow predominantly in x-direction, no change in z-direction
- Physical models have been shown to improve learning new concepts [1]

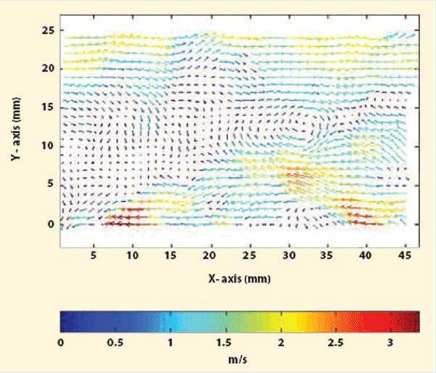
Prior Work

- Existing fluid visualization setups exist, but none specifically tailored for flat plates or experiential classroom learning

Ink injection into fluid pipe flow [2]



Computational visualization [3]

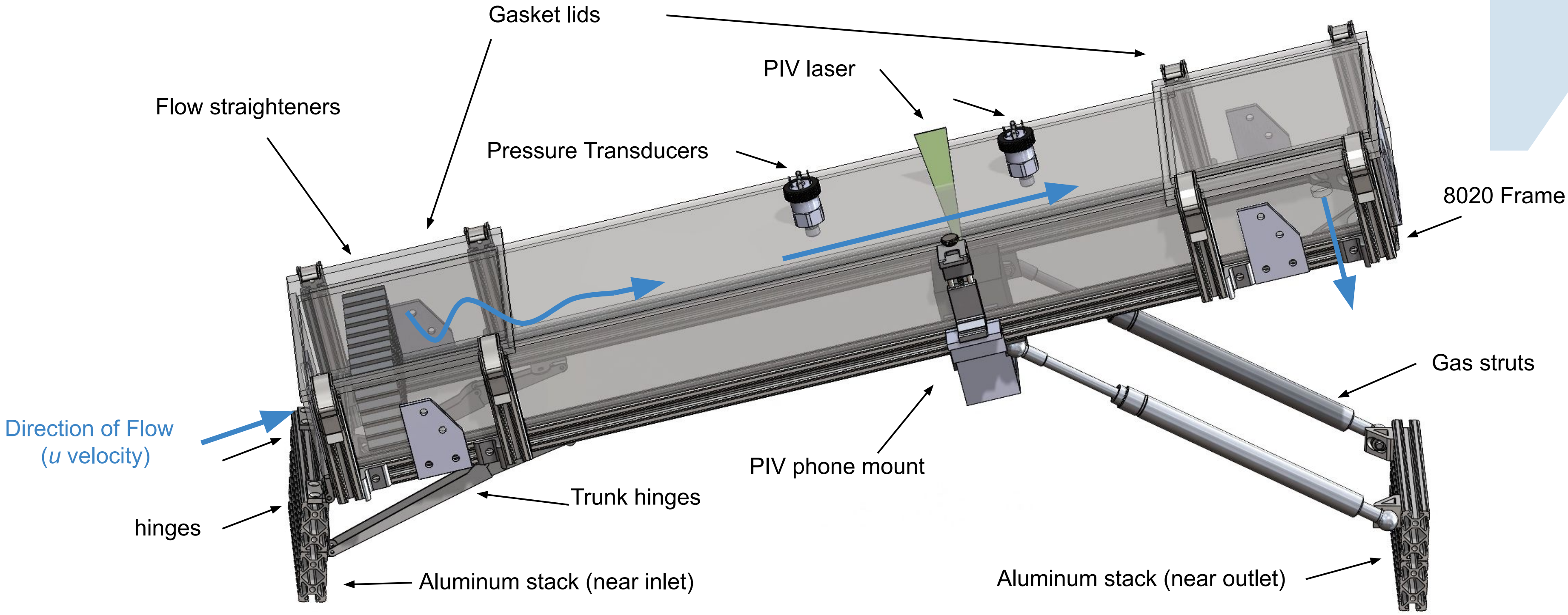


Particle image velocimetry [4]

Team Strategy

- Develop **one final prototype**, iteration through analysis
- **Ongoing testing** to ensure prototype functionality
- Different groups working in conjunction
    - **Measurements** - Flow/CFD, Structural/FEA, Analysis/PIV
    - **Design** - Overall CAD, Piping System

Our Solution



Overall Build

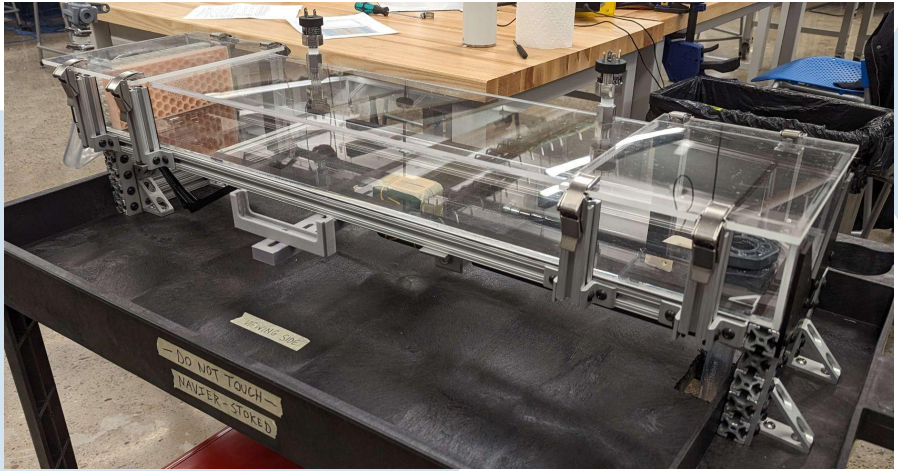


Image of empty tank prior to filling

Testing & Analysis

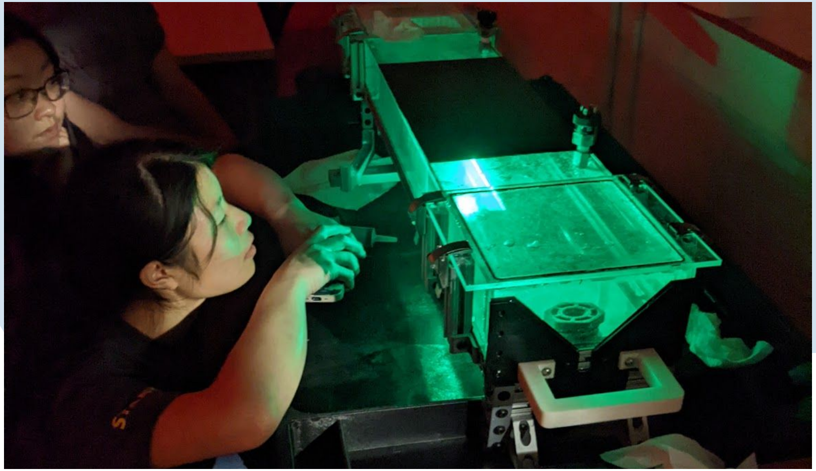
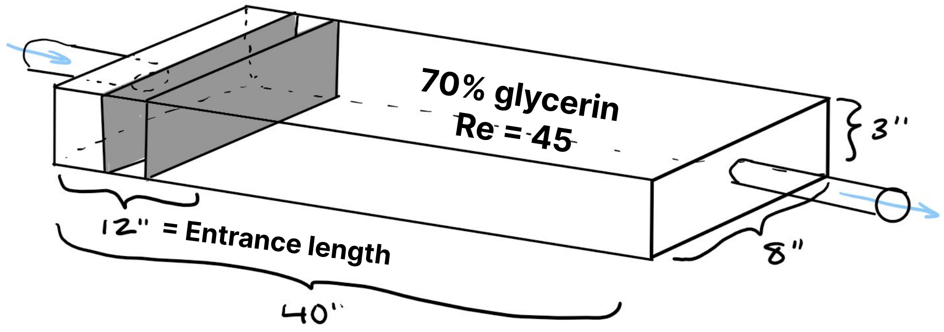


Image of testing PIV software

Iterative Flow Analysis

- Infinite plate approximation vs portability



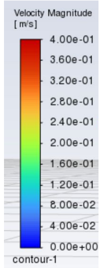
- Pump specifications

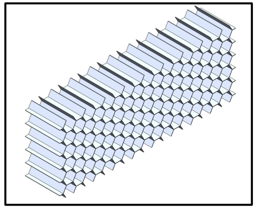
$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + F_{hl} \longrightarrow$$

Pump Delivery Pressure
1.97 ft of head

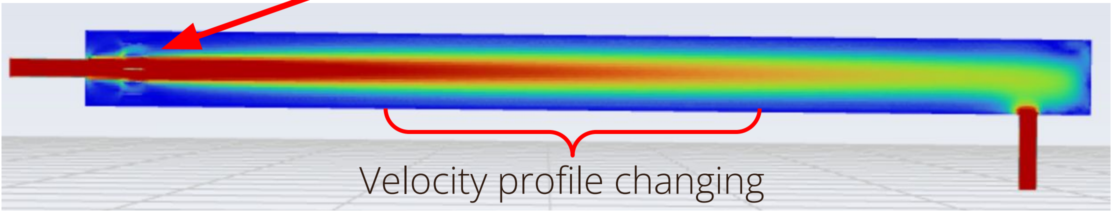
Iterative Flow Analysis (cont.)

Original Flow Straightener



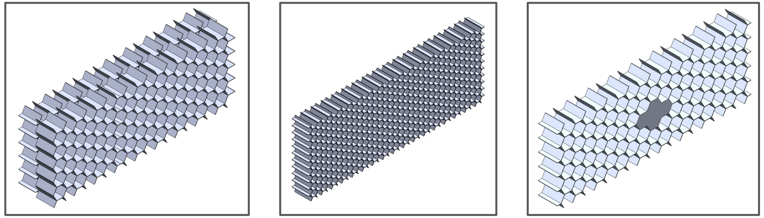


Poorly dispersed

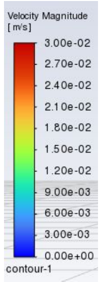


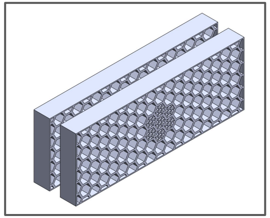
Velocity profile changing

iterations

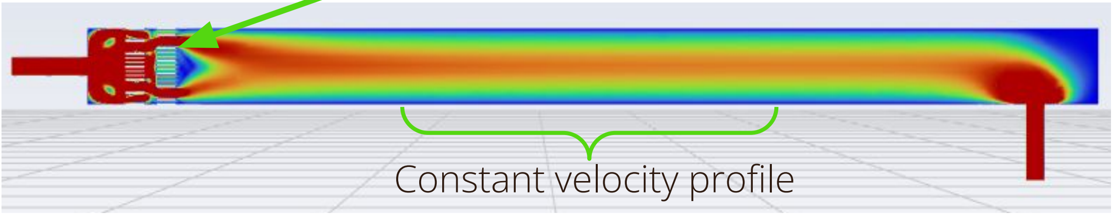


Final Flow Straightener





Well dispersed

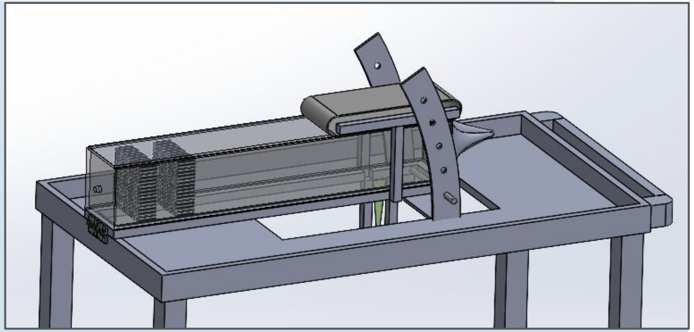


Constant velocity profile

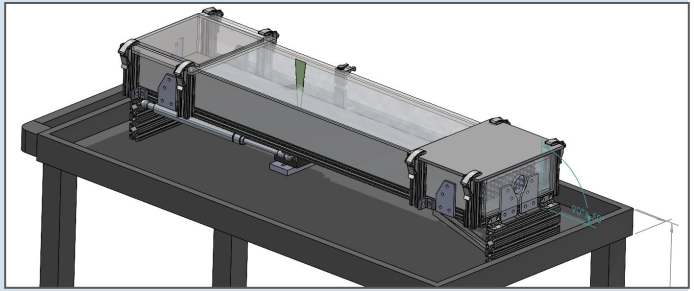
Iterative Structural Analysis

- Performed FEA and hand calculations using ME design theories

Critical results of structural components	
Component	**Critical Results**
Aluminum 8020 Frame	$\eta = 19$
Angling Support Hinges	$\eta = 5.4$
Overall Cart Stability	$\eta = 6.7$
Gas Struts	25000 cycles



Original design

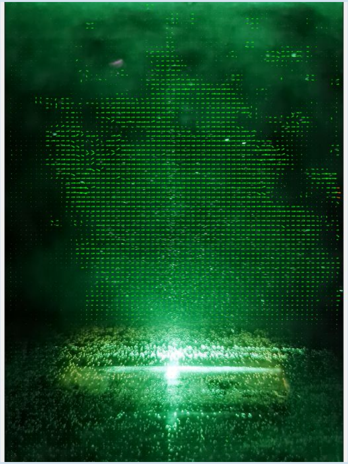


Final design

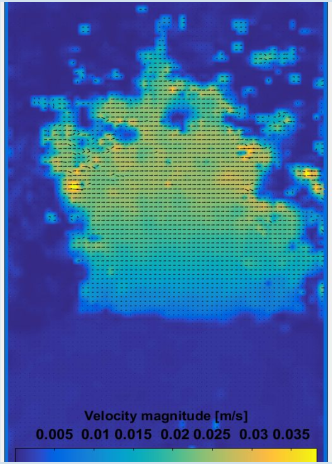
Particle Image Velocimetry (PIV) Analysis

- 20 micron polyamide particles
- Class II laser level
- Processing software: SmartPIV (live), PIVlab (post-process)

Original



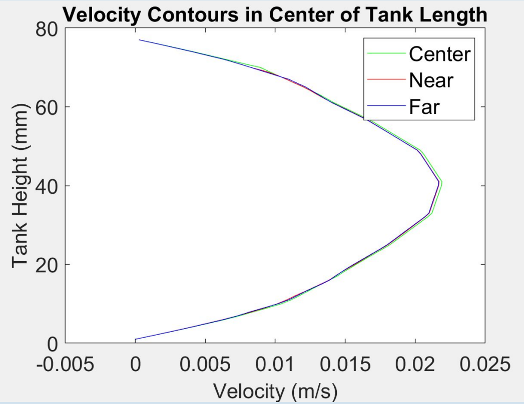
PIVlab: Velocity arrows



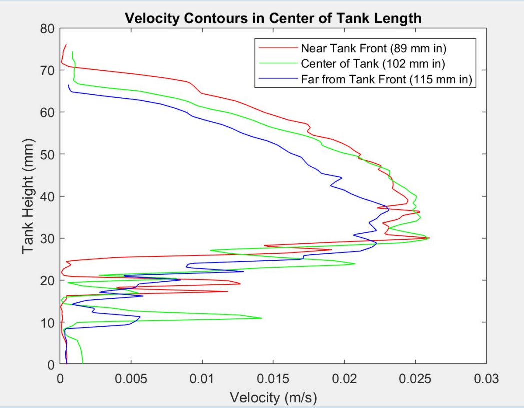
PIVlab: Velocity color map

Key Successes

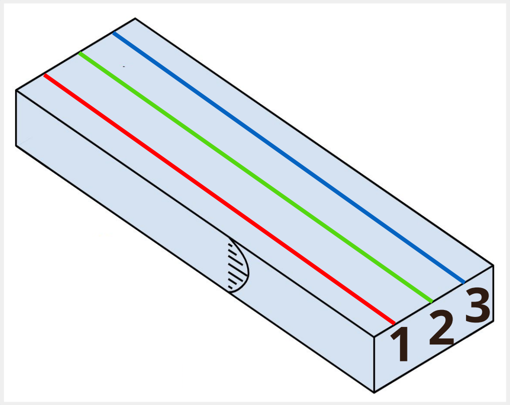
- Theoretical calculations match PIVlab measurements
- Infinite plates, plane flow assumptions are fulfilled



ANSYS Velocity vs. Tank height



PIVlab: Velocity vs. Tank height



Velocity profile measured in 3 positions

Anticipated Impact

- With our physical model, students should have a **stronger conceptual understanding** of the Navier-Stokes assumptions
- Our rig can allow students to associate confusing textbook problems with a tangible model
- Greater understanding should be **reflected by higher scores**

Professor Matherne's exam scores on the Navier-Stokes question		
	Summer 1 2023 Section 1	Summer 1 2023 Section 2
Exam 2	68	54
Final Exam	77	78

Exit Strategy

- We have created a PIV guide and operating procedure to facilitate teaching and handoff
- There are some changes and additions that could be implemented in a **Phase 2** project
    - Add **shear-driven flow** component
    - Improve **maintenance** process
    - Expand the use of **real-time PIV**

Thank You!

Presented by NAVIER-STOKED

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“Developing the flow of knowledge and the knowledge of flow”

References

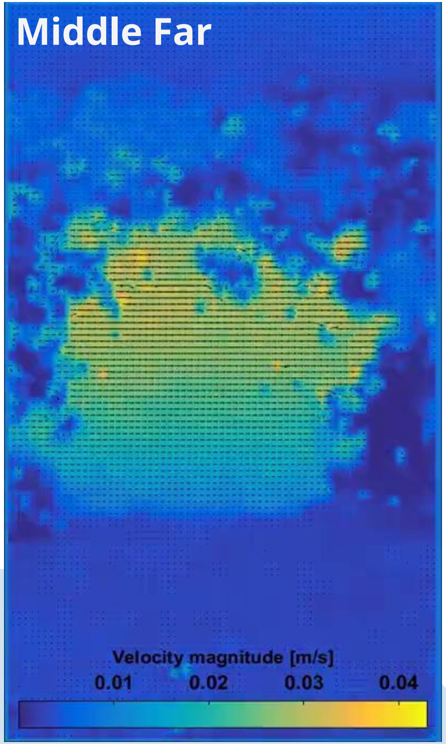
[1] D. L. Newman, M. Stefkovich, C. Clasen, M. A. Franzen, and L. K. Wright, “Physical models can provide superior learning opportunities beyond the benefits of active engagements,” *Biochemistry and Molecular Biology Education*, vol. 46, no. 5, pp. 435–444, 2018, doi: 10.1002/bmb.21159. [Online]. Available: https://onlinelibrary.wiley.com/doi/abs/10.1002/bmb.21159.

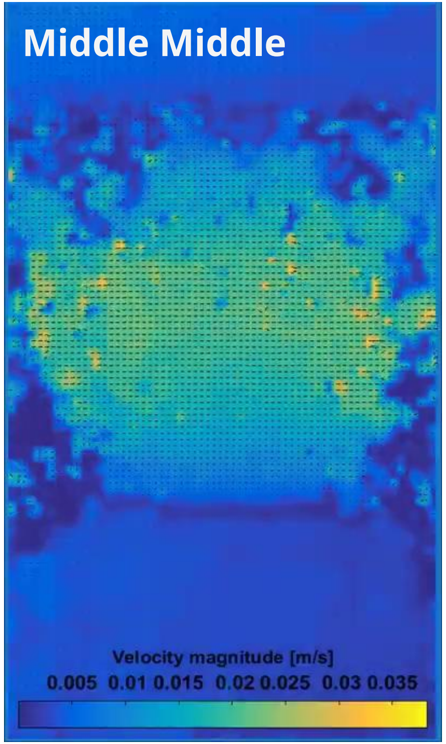
[2] “Water Purification Reactor.” *Comsol*, Comsol Inc. https://www.comsol.de/model/water-purification-reactor-14049

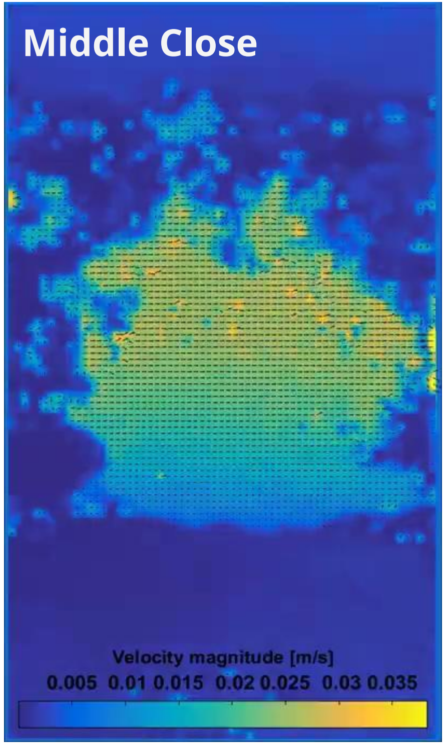
[3] Fluids Explained. “Laminar Flow, Turbulent Flow and Reynolds Number (Lesson 3, Part 2)” *Youtube video*, 2:29. February 5, 2021. https://www.youtube.com/watch?v=vhDaCZZ0Sc4

[4] Photonics. “Particle Image Velocimetry: Basics, Developments and Techniques.” https://www.photonics.com/Articles/Particle_Image_Velocimetry_Basics_Developments/a25121

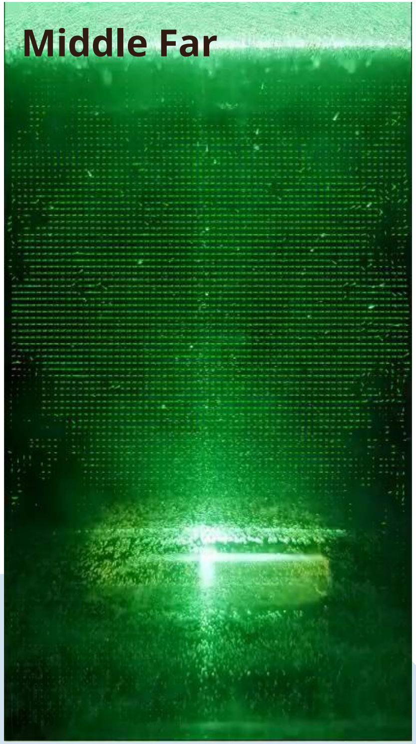
Appendix A - PIV Infinite Plates

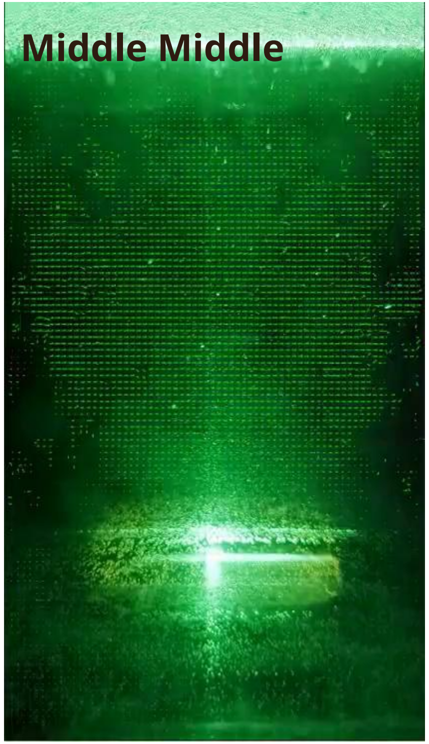


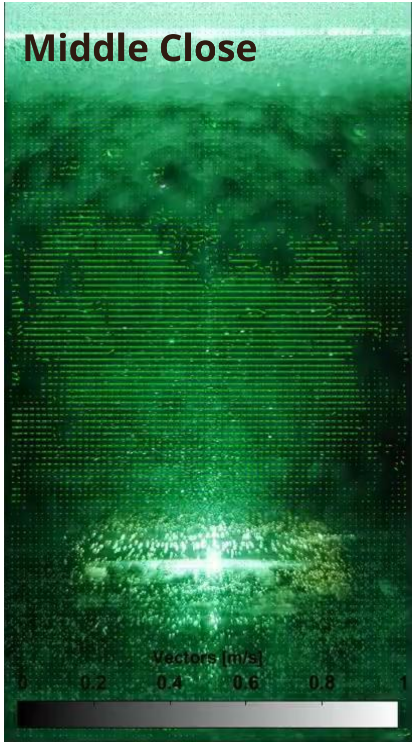




Appendix A - PIV Infinite Plates Arrows



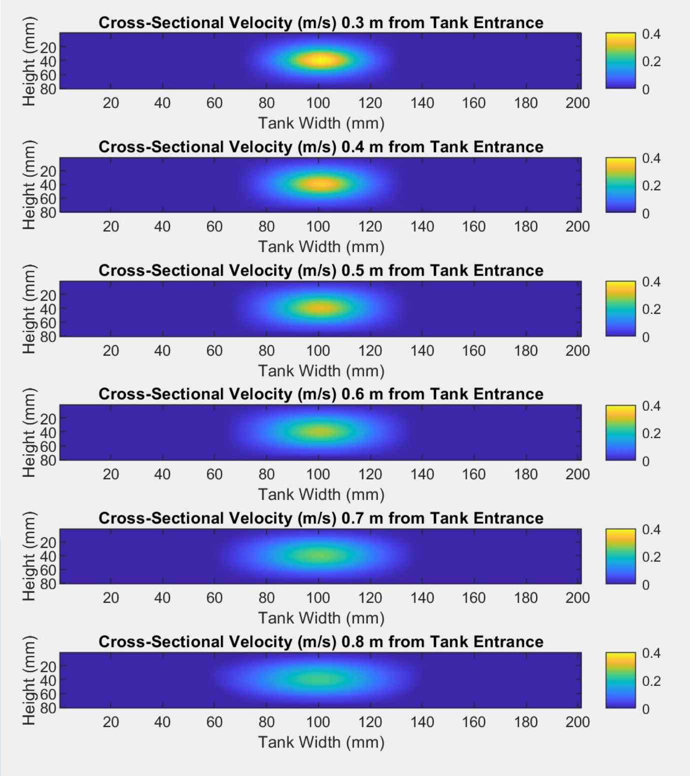


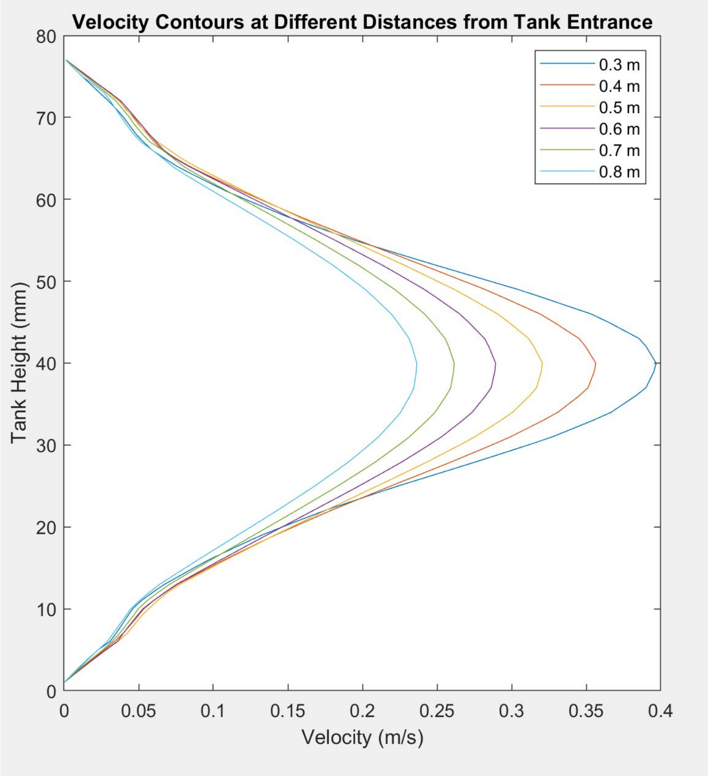


Appendix B - CFD/MATLAB/Experimental Comparison

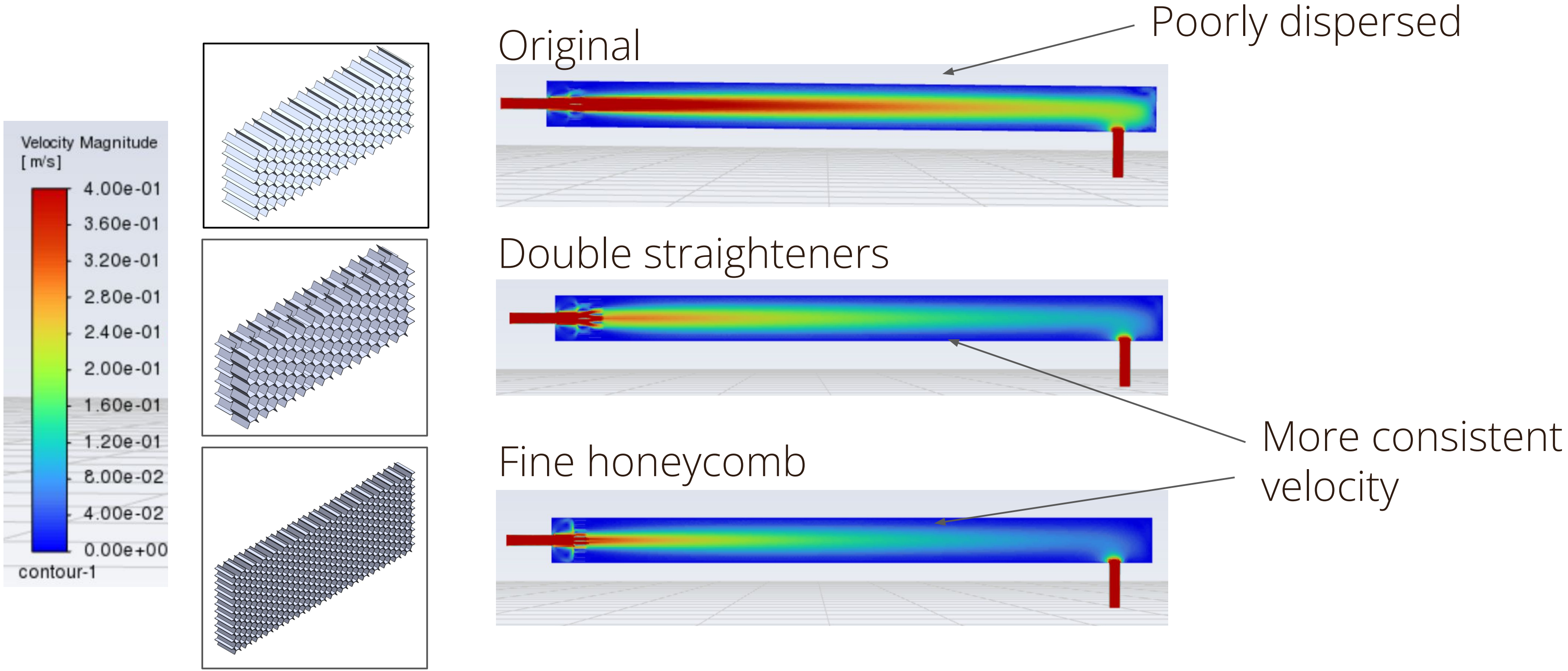
	ANSYS	**MATLAB** ***Duct***	**Actual System**
Max tank velocity	0.89 in/s *Friction?*	0.94 in/s	0.98 in/s
Average tank velocity	-	0.47 in/s	-
Mass flow rate in pipe	0.22 kg/s	0.22 kg/s	-
Volumetric flow rate of pump *(Theoretical MAX)*	-	176.0317 GPH	- (1250 GPH)
Reynolds Number	-	45	-
Entrance length	11.81 in *Flow straighteners at beginning*	11.10 in	Appears to match up with theory, fairly developed flow near 12 in
Pressure drop across entire tank	1.45e-04 psi *Roughly 1 Pascal*	2.55e-04 psi	-
Pump delivery pressure (*Theoretical MAX)*	-	1.124 psi	- (6.77 psi)

Appendix C - Original Flow Straightener





Appendix C - Iterations of Flow Straightener



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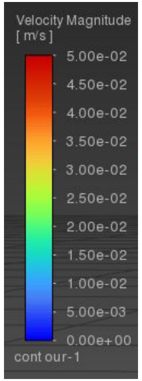
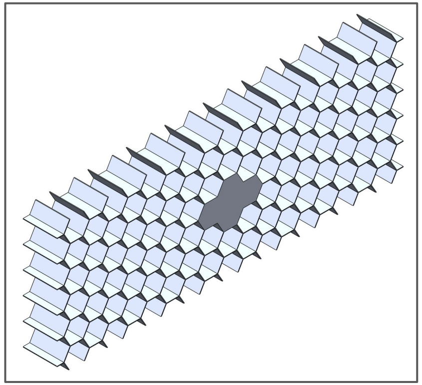
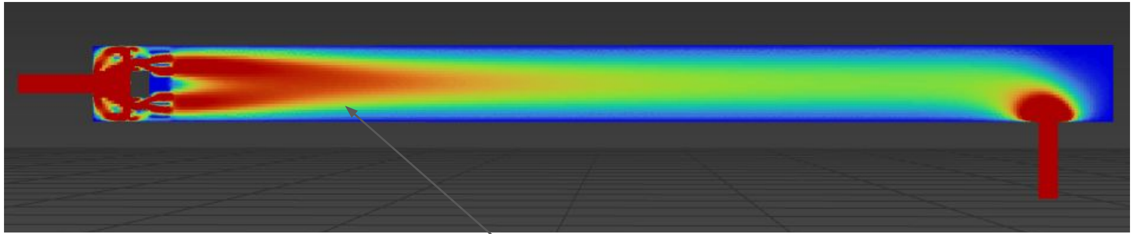
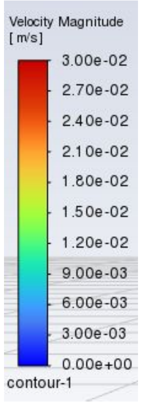


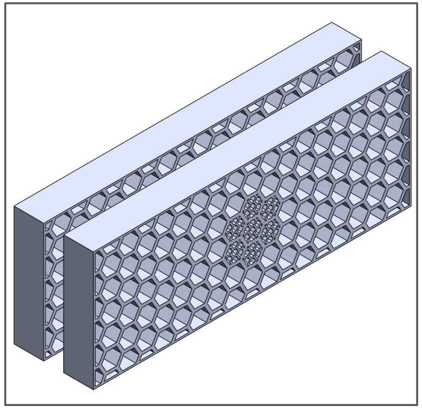


Blockage

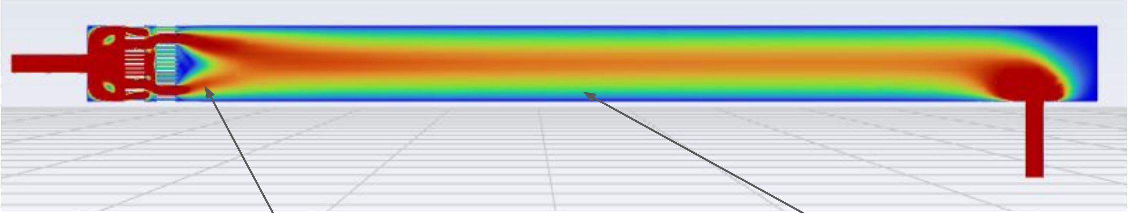


Better dispersion





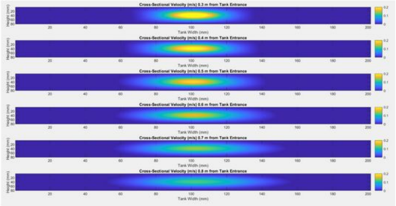
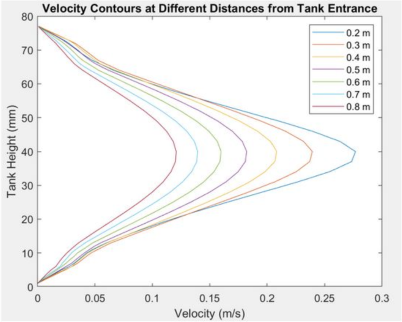
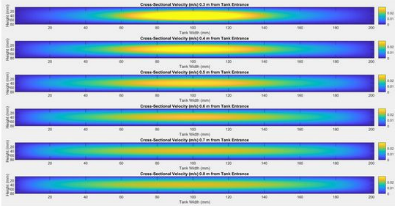
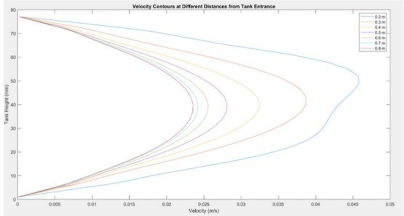
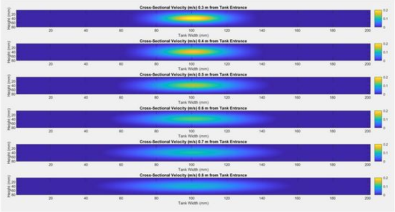
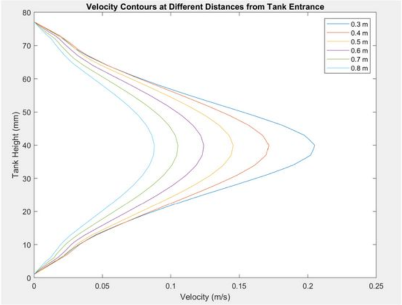
Final Iteration



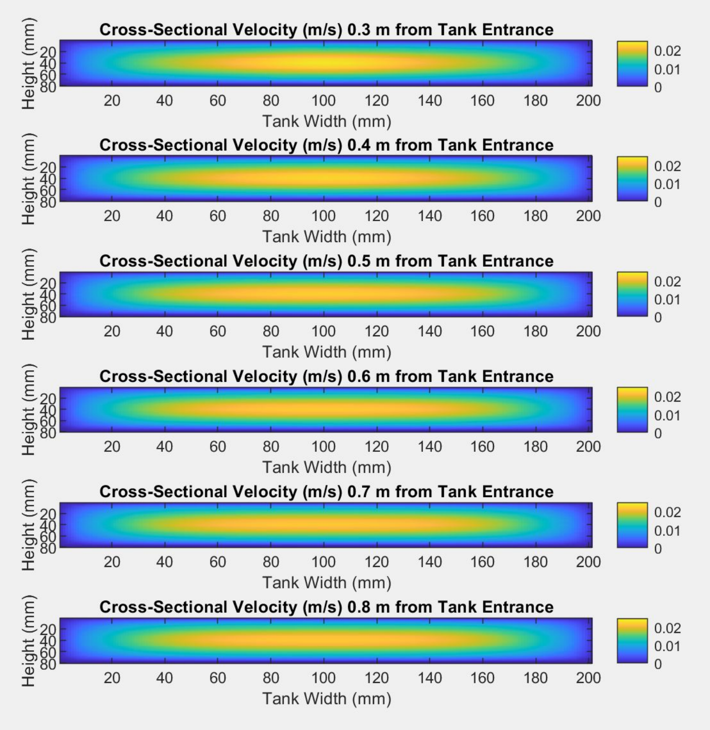
Better dispersion

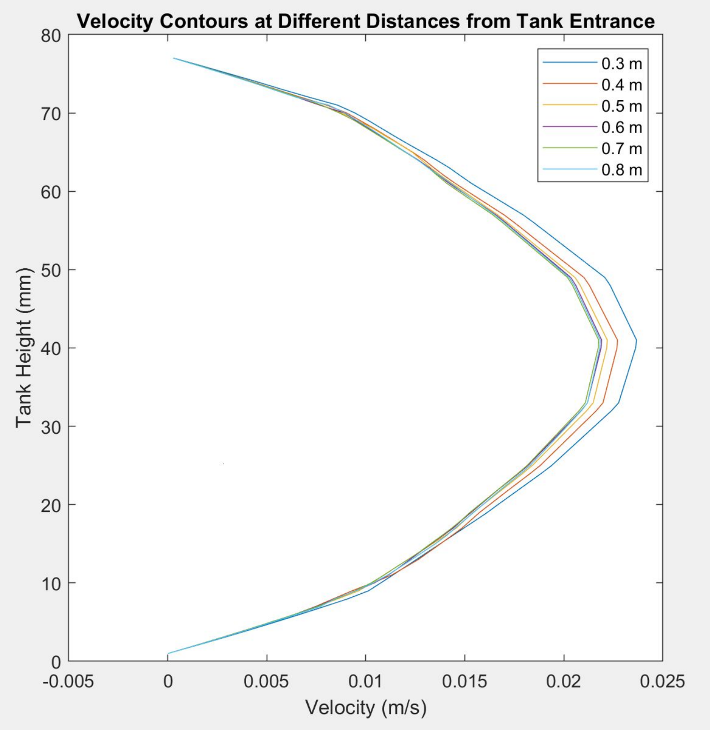
Consistent velocity

Appendix C - Iterations of Flow Straightener

Iteration 2 (Double)		
Iteration 3 (Obstacle)		
Iteration 4 (Fine Honeycomb)		

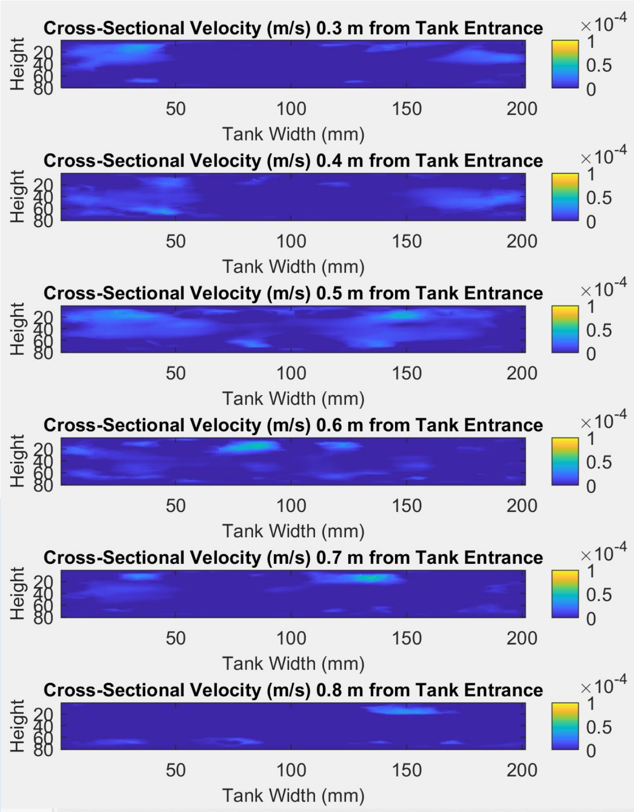
Appendix C - Final Flow Straightener



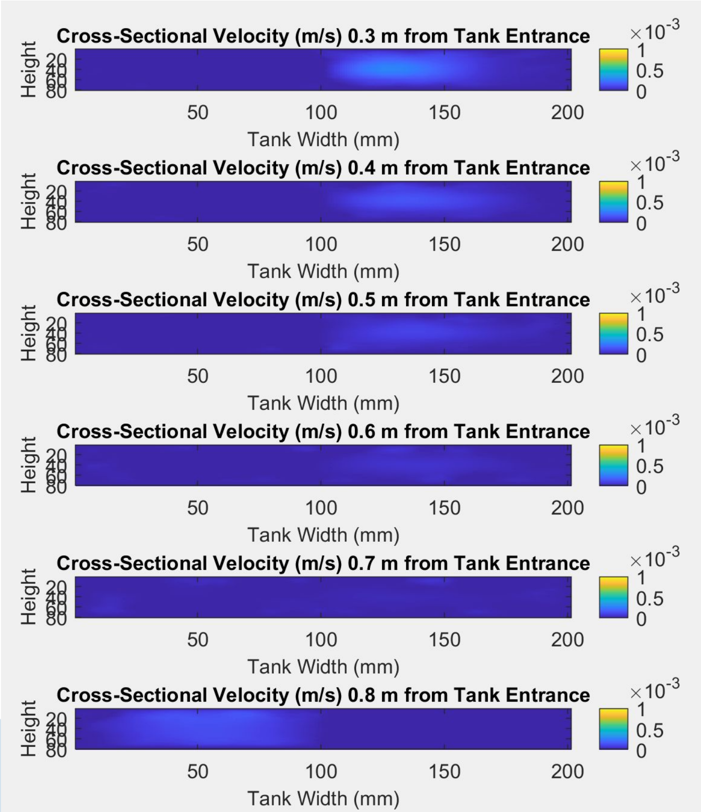


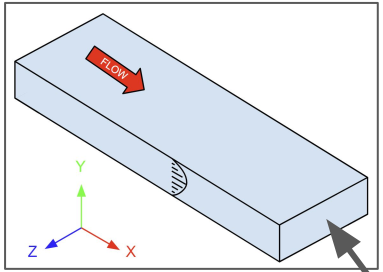
Appendix D - ANSYS Output (v,w = 0)

v = 0



w = 0

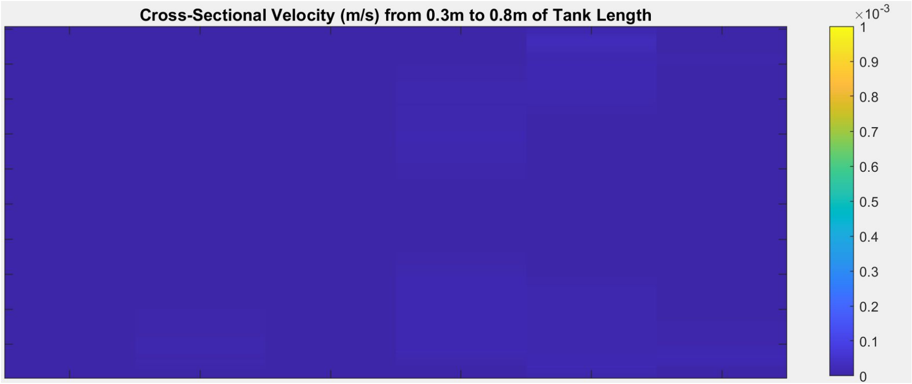




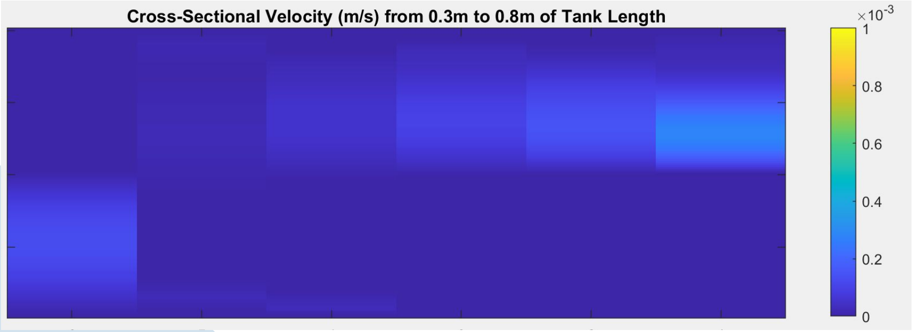
This cross-section

Appendix D - ANSYS Output (v,w = 0)

v = 0



w = 0



This cross-section