Pair Judge RM: Perform Best-of-N Sampling with Knockout Tournament

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Code: https://github.com/THU-KEG/PairJudgeRM/

Model: https://huggingface.co/THU-KEG/PairJudgeRM

Dataset: https://huggingface.co/datasets/THU-KEG/PairJudge-432K

Abstract

Best-of-N (BoN) sampling, a common strategy for test-time scaling of Large Language Models (LLMs), relies on reward models to select the best candidate solution from multiple generations. However, traditional reward models often assign arbitrary and inconsistent scores, limiting their effectiveness. To address this, we propose a Pairwise Judge Reward Model (PairJudge RM) combined with a knockout tournament for BoN sampling. Instead of assigning absolute scores, given one math problem, PairJudge RM judges two candidate solutions' correctness with chain-of-thought reasoning simultaneously. This approach eliminates the need for scoring and enables crossvalidation of solutions through parallel judgment. In the knockout tournament, PairJudge RM conducts pairwise Judgment between candidate solutions and eliminates the incorrect ones iteratively. We construct PAIRJUDGE-432K, a large-scale dataset of 432K pairwise judgments derived from NumiaMath and annotated using gemini-1.5-flash, and train the PairJudge RM via supervised fine-tuning. Experiments on MATH-500 and the Olympiad Bench demonstrate significant improvements over baseline reward models. And a 40% to 60% relative improvement is achieved on the top 50% challenging problems.

1 Introduction

Recently, test-time scaling has garnered significant attention from the research community, as it draw a blueprint for the next stage of scaling of Large Language Models (LLMs) (Snell et al., 2024; Wu et al., 2024; OpenAI, 2024). One of the most common practice to achieve test-time scaling is to use reward models (RMs) to perform the Best-of-N (BoN) Sampling at test time (Wang et al., 2023; Lightman et al., 2023; Wang et al., 2024b; Zhang et al., 2024b; Yuan et al., 2024b): the LLM generates N candidate solutions for a given problem, and

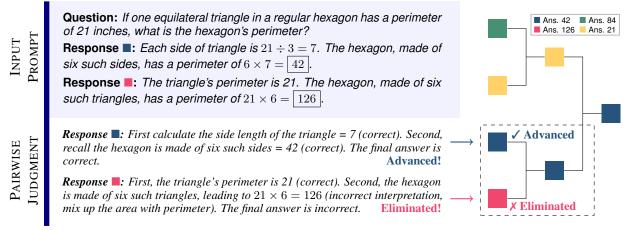
a learned reward model, scoring each candidate solution, selects the best one as the final output. The effectiveness of this strategy hinges on how accurate the score assigned by the reward model is to the candidate solutions.

However, assigning accurate and consistent scores is inherently challenging, even for human experts (Jonsson and Svingby, 2007; Abdul Gafoor and Jisha, 2014). An experiment conducted in NeurIPS 2021 shows that for different human experts guided by the same rubric, the scores assigned to the same candidate paper can vary significantly (Beygelzimer et al., 2021). This limitation is particularly pronounced in reward models, which are typically trained to assign relative scores rather than absolute, meaningful scores (Lambert et al., 2024; Liu et al., 2024). As a result, the scores assigned by reward models are often arbitrary and inconsistent, hindering the performance of BoN sampling (Liu et al., 2024; Kim et al., 2024).

To address this limitation, we propose a Pairwise Judge Reward Model (PairJudge RM) combined with a knockout tournament for BoN sampling. Instead of assigning absolute scores, PairJudge RM judges two candidate solutions' correctness simultaneously for a given reasoning problem.

In this setting, our approach eliminates the need for arbitrary scoring and enables cross-validation of solutions through pairwise judgment. To perform BoN sampling, we organize candidate solutions into a knockout tournament, where each pairwise judgment acts as a match. Rounds of matches are played until only one candidate remains, which is selected as the final output. Figure 1 presents the knockout tournament process with our PairJudge RM. Specifically, inlining with existing work (Lightman et al., 2023; Wang et al., 2023; Snell et al., 2024; Wu et al., 2024), math reasoning tasks are used as the testbed to evaluate the BoN sampling performance.

We construct PAIRJUDGE-432K, a large-scale



a) Input Prompt and Pairwise Judgment in PairJudge RM.

b) Knockout with PairJudge RM.

Figure 1: An example of the knockout tournament using the PairJudge RM. Panel (a) shows a pairwise judgment in the knockout tournament (Panel b) between response and . The PairJudge RM takes a question and responses as input, then judges their correctness pairwise. In the judgment, the PairJudge RM correctly identifies the response as correct and response as incorrect, leading to the elimination of the . The pairwise judgment process continues iteratively in the knockout tournament until only one response remains. For this question, the response is selected as the best candidate solution and Answer 42 is the final answer through the knockout.

dataset of 432K pairwise judgments derived from NumiaMath (LI et al., 2024) and annotated using gemini-1.5-flash. Using this dataset, we train the PairJudge RM via supervised fine-tuning. Experiments on MATH-500 and the Olympiad Bench demonstrate that PairJudge RM significantly outperforms traditional discriminative reward models. On the top 50% challenging problems in MATH-500, PairJudge RM achieves a 40% to 60% relative improvement over the baseline. Furthermore, our method outperforms the recently proposed Critic Model (Gao et al., 2024a; McAleese et al., 2024; Zhang et al., 2024b) under the same computational budget.

In summary, our contributions are as follows:

- We propose a Pairwise Judge Reward Model (PairJudge RM) combined with a knockout tournament for BoN sampling. This approach avoids the limitations of arbitrary scoring in traditional reward models and enables cross-validation of candidate solutions.
- We release PAIRJUDGE-432K, a large-scale dataset for training PairJudge RMs containing 432K annotated pairwise judgments, along with its construction pipeline.
- Experiments on MATH-500 and the Olympiad Bench demonstrate significant improvements compared to baselines in BoN sampling. Specifically, on the top 50% difficult problems, PairJudge RM achieves a 40% to 60%

relative improvement over baseline models.

2 Preliminaries

Best-of-N Sampling in Math Reasoning Given a math problem $x \in \mathcal{X}$ and the N candidate solutions $\{y_1, y_2, \ldots, y_N\}$ sampled from a Large Language Model (LLM), the BoN Sampling aims to select the best candidate solution y^* from the N candidate solutions based on an external selection mechanism. Typically, there are two types of reward models (RMs) serving as the external selection mechanism: the Outcome Reward Model and the Process Reward Model.

Outcome Reward Model Given a math problem x and a candidate solution y, the Outcome Reward Model assigns a numerical score s(y) to the candidate solution y. The Outcome Reward Model selects the candidate solution with the highest score as the final output:

$$y^* = \arg \max_{y \in \{y_1, y_2, \dots, y_N\}} s(y).$$
 (1)

The Outcome Reward Model is typically trained on a preference dataset \mathcal{D} , consisting of pairs (x, y_c, y_r) , where y_c is the chosen response and y_r is the rejected response. The model is trained to assign a higher reward to y_c than to y_r , minimizing the following objective:

$$\mathcal{L} = -\mathbb{E}\left[\log \sigma(R_{\psi}(x, y_c) - R_{\psi}(x, y_r))\right], \quad (2)$$

where $\sigma(\cdot)$ is the sigmoid function and ψ is the parameters of the Reward Model. \mathcal{L} is the loss function for preference learning indicating the probability of the chosen response y_c being preferred over the rejected response y_r . This objective ensures that the reward model learns to identify responses that align better with human preferences.

Process Reward Model Given a math problem x and a corresponding candidate solution y, the Process Reward Model first requires to split the candidate solution y into a sequence of reasoning steps $\{a_1, a_2, \ldots, a_M\}$. The Process Reward Model assigns a numerical score $s(a_i)$ to each reasoning step a_i . The score of the entire candidate solution y is the mean of the scores of all reasoning steps:

$$s(y) = \frac{1}{M} \sum_{i=1}^{M} s(a_i).$$
 (3)

The Process Reward Model selects the candidate solution with the highest score as the final output with the same mechanism as the Outcome Reward Model in Equation 1.

The Process Reward Model is typically trained on a dataset with process labels \mathcal{D}_{proc} , where each solution y to a problem x, the dataset contains a series of process labels $\{l_1, l_2, \ldots, l_M\}$, where $l_i \in \{0, 1\}$ indicates whether the reasoning step a_i is correct or incorrect. Then the Process Reward Model is trained to predict the correctness of each reasoning step a_i .

3 PairJudge RM and Knockout

In this section, we introduce the PairJudge RM and the knockout tournament, which are the core components of our proposed method for performing BoN Sampling at test time.

3.1 Pairwise Judge Reward Model

Definition Given a math problem x and two candidate solutions y_1 and y_2 , the PairJudge RM is designed to simultaneously check the correctness of the two candidate solutions. Specifically, the PairJudge RM is trained to judge the correctness of the two candidate solutions, denoted as c_1 and c_2 , respectively.

$$c_1, c_2 = \text{PairJudge}(x, y_1, y_2),$$
 (4)

where $c_1, c_2 \in \{0, 1\}$ indicate whether the candidate solutions y_1 and y_2 are correct or incorrect.

Implementation Inspired by the Generative Reward Model (GenRM) (Zhang et al., 2024b) and LLM-as-a-Judge (Zheng et al., 2023), we implement the PairJudge RM as a generative model. Specifically, given a math problem x and two candidate solutions y_1 and y_2 , the PairJudge RM first generates a reasoning text using chain-ofthought (Wei et al., 2022) to verify the correctness of the two candidate solutions. Based on the reasoning text, the PairJudge RM then predicts the correctness of the two candidate solutions by directly generating the correctness labels c_1 and c_2 . The detailed prompt for performing pairwise verification with chain-of-thought is provided in Table 4 in the Appendix.

3.2 Knockout Tournament

To perform BoN Sampling with the PairJudge RM, we introduce a knockout tournament to select the best candidate solution.

Algo 1: Knockout for Best-of-N Sampling

Input: Math problem x,

N candidate solutions

 $\mathcal{Y} = \{y_1, y_2, \dots, y_N\},\$

Pairwise Judge Reward Model: PairJudge

Output: Best candidate solution y_{best}

Step 1: Group candidates into teams

Partition \mathcal{Y} into k teams, where members of a team share the same final answer.

Step 2: Initialize the knockout pool

Add all N candidates to the initial pool \mathcal{P} .

Step 3: Perform the knockout rounds while $|\mathcal{P}| > 1$ do

Pair each candidate y_i with an unpaired y_j from a different team.

Remove y_i and y_j from \mathcal{P} .

foreach $pair(y_i, y_j)$ do

Compute correctness scores c_i , c_j using PairJudge (x, y_i, y_j) .

if $c_i > c_j$ then

 y_i advances.

else if $c_i > c_i$ then

 y_j advances.

else if c_i, c_j both correct then

Randomly select one to advance.

else

☐ Both incorrect and eliminated.

Add advancing candidates back to \mathcal{P} .

Step 4: Return the best solution

Output the last remaining y in \mathcal{P} as y_{best} .

Specifically, we first group the N candidate solutions into k teams, where candidates that share the same answer are placed in the same team. Then, we pair up the candidate solutions from each team to compete with candidate solutions from other teams. In each match, only the candidate solution that receives the correct label from the PairJudge RM advances to the next round. If both candidate solutions receive the correct label, one is randomly selected to advance. This process continues until only one candidate solution remains or early termination occurs when all candidate solutions are from the same team.

The detailed procedure of the knockout tournament is shown in Algorithm 1.

4 PAIRJUDGE-432K dataset collection

To train the PairJudge RM, we collect a large-scale dataset named PAIRJUDGE-432K, which contains 432K annotated pairwise judgments derived from NumiaMath (LI et al., 2024) with gemini-1.5-flash. In the following, we describe the detailed procedure of collecting the PAIRJUDGE-432K dataset.

4.1 Dataset Format

Since the PairJudge RM is designed as a generative model to judge the correctness of candidate solutions, the training dataset has the same format as the one for Supervised Fine-Tuning, consisting of prompt-completion pairs. Specifically, each prompt is constructed by filling the template shown in Table 4 with a math problem x and two candidate solutions y_1 and y_2 . The completion is a chain-of-thought reasoning text that judges the correctness of the two solutions and provides the correctness labels c_1 and c_2 .

4.2 Math Problem Collection

We first collect math problems from the Numia-Math dataset (LI et al., 2024), which contains 860K problems ranging from high school math exercises and international mathematics olympiad competition problems. Because these data are primarily collected from online exam paper PDFs and mathematics discussion forums, we remove low-quality problems with messy formatting, OCR errors, or missing answers. We also remove multiple-choice (MCQ) and True/False questions to avoid random guessing in candidate solutions. Following community conventions, we remove proof problems as well, due to the difficulty of verifying candidate

Dataset	Original Count	Filtered Count		
AMC/AIME	4,070	289		
AoPS Forum	30,192	9,017		
Chinese K-12	276,554	63,779		
GSM8K	7,342	6,539		
Math	7,477	5,988		
Olympiads	150,563	52,766		
ORCA Math	153,314	149,550		
Synthetic AMC	62,108	94		
Synthetic Math	167,874	136,921		
Total	859,494	425,943		

Table 1: Statistics of the datasets before and after filtering. AMC-related datasets shrink significantly because most AMC problems are multiple-choice.

solutions. The detailed filtering criteria are listed in Table 5 of the Appendix.

4.3 Candidate Solution Generation

For each math problem x, we generate k=24 candidate solutions $\{y_1, y_2, \ldots, y_k\}$ using Llama-3.1-8B-instruct (AI, 2025). We employ the same four-shot in-context examples for all problems as the prompt. The candidate solutions are decoded with a temperature of 1.0 and a Top-P value of 0.5 to balance diversity and quality.

4.4 Pairwise Judgment Annotation

We use gemini-1.5-flash to annotate the PairJudge RM training data on the NumiaMath dataset. To align the generated training data distribution with the solution-judgment distribution in the knockout tournament, we conduct a knockout tournament for each math problem x and its candidate solutions $\{y_1, y_2, \dots, y_k\}$ to select the best solution y_{best} . During the knockout tournament, we record all pairwise judgments among candidate solutions and retain only those records that correctly judge solution correctness for the PairJudge RM. Specifically, due to cost considerations, we only run the knockout tournament for questions whose candidate solutions are not all correct or all incorrect. As a result, we conducted 343K tournaments and recorded 2.2M comparisons. Among these, 1.3M correctly evaluated both candidate solutions and were used as raw training data for the Pair-Judge RM. Finally, we filtered out samples where the response did not strictly follow the instructions in Table 4, ending up with 432K training samples.

5 Experiments

In this section, we demonstrate the effectiveness of the PairJudge RM and the knockout tournament

Туре	Reward Model	Llama-3.1-8B-Inst		Qwen-2.5-7B-Inst		Llama-3.1-70B-Inst			Avg.		
Турс		@16	@32	@64	@16	@32	@64	@16	@32	@64	Avg.
			M	ATH-50	0						
ORM	ArmoRM-Llama3-8B	51.6	49.2	49.8	<u>77.6</u>	77.4	76.4	64.8	64.8	65.8	64.2
	SkyworkRM-Llama3.1-8B	51.4	51.0	51.0	<u>77.6</u>	76.4	<u>78.0</u>	66.4	66.6	67.4	65.1
	EurusRM-7B	55.2	53.4	53.4	76.6	77.0	77.4	68.0	66.6	67.6	66.1
	Pair-Preference-Llama3-8B	48.0	47.6	49.0	76.0	77.4	75.6	64.0	63.4	60.2	62.4
	Math-Shepherd-7B	49.5	50.1	49.2	74.7	75.3	75.9	63.5	62.8	63.6	62.7
PRM	RLHFlow-8B-Mistral-Data	51.0	51.0	50.2	75.4	76.2	76.6	64.0	63.0	64.8	63.6
I KIVI	RLHFlow-8B-DS-Data	55.2	57.0	56.2	75.8	76.0	76.2	66.2	66.4	65.4	66.0
	RLHFlow-8B-LLaMA-Data	55.5	56.8	56.0	76.0	76.3	76.5	66.7	67.0	66.0	66.3
	Majority Voting	<u>57.0</u>	<u>58.8</u>	<u>58.8</u>	77.4	<u>77.6</u>	<u>78.0</u>	70.2	72.8	<u>73.6</u>	69.4
	PairJudge RM & Knockout	61.0	64.6	65.6	80.2	79.8	80.4	72.2	75.6	77.4	73.0
			Olym	piad Be	ench						
	ArmoRM-Llama3-8B	16.1	15.9	16.7	39.3	40.1	40.4	29.2	29.8	30.1	28.7
ORM	SkyworkRM-Llama3.1-8B	19.9	20.0	18.7	39.9	40.0	<u>41.0</u>	29.8	30.4	29.8	29.4
OKW	EurusRM-7B	<u>20.4</u>	19.6	20.1	37.9	39.4	39.1	30.1	30.7	32.4	30.0
	Pair-Preference-Llama3-8B	17.7	19.1	17.2	39.4	38.9	38.1	26.5	27.4	25.5	27.8
	Math-Shepherd-7B	15.2	13.7	13.1	34.8	34.5	35.1	25.3	26.0	24.1	24.6
PRM	RLHFlow-8B-Mistral-Data	16.4	14.5	14.5	36.1	35.9	36.3	26.7	27.1	25.2	25.9
PKW	RLHFlow-8B-DS-Data	18.5	19.6	19.3	35.4	34.8	34.2	28.9	29.5	30.1	27.8
	RLHFlow-8B-LLaMA-Data	18.7	20.0	19.7	35.8	35.2	34.7	29.1	29.4	30.3	28.1
	Majority Voting	20.3	22.4	23.3	40.0	40.7	39.9	<u>35.6</u>	<u>35.9</u>	<u>36.7</u>	32.8
<u> </u>	PairJudge RM & Knockout	22.7	24.9	25.5	41.9	<u>40.2</u>	41.2	33.9	36.7	37.8	33.9

Table 2: Different reward models' best-of-N sampling performance on MATH-500 and Olympiad Bench with three different LLMs: Llama-3.1-8B-Inst, Qwen-2.5-7B-Inst, and Llama-3.1-70B-Inst. The results are reported in terms of accuracy. The pass@1 accuracy of these three LLMs are 42.0, 73.6, and 59.2 on MATH-500, and 12.3, 35.7, and 25.9 on Olympiad Bench, respectively. @16, @32, and @64 denote the accuracy with Best-of-16, Best-of-32, and Best-of-64 sampling, respectively. The best results are in bold, and the second-best results are underlined.

in performing BoN Sampling at test time. We first introduce the experimental setup, including the dataset, evaluation metrics, and baselines. Then, we present the experimental results and analysis.

5.1 Experimental Setup

Dataset We evaluate BoN Sampling on MATH-500 (Hendrycks et al., 2021) and Olympiad Bench (He et al., 2024) to coverage from the high-school-level math problems to the olympiad-level math problems. To study the generalizability of our PairJudge RM, we test it with three LLMs that have different capabilities and come from different model families: Llama-3.1-8B-Instruct (AI@Meta, 2024), Llama-3.1-70B-Instruct (AI@Meta, 2024), and Qwen2.5-7B-Instruct (Qwen Team, 2024).

Training Details We use Qwen2.5-7B-Instruct as the base model and perform supervised fine-tuning on our PAIRJUDGE-432K dataset to obtain the

PairJudge RM. We set the learning rate to 1×10^{-5} with the Adam optimizer and a batch size of 128. The model is trained for 8 epochs.

Baselines We compare our PairJudge RM with both outcome and process reward model, which is trained to assign a score to each candidate solution and then select the candidate solution with the highest score as the final output. For the Outcome Reward Model, we use EurusRM-7B (Yuan et al., 2024a), SkyworkRM-Llama3.1-8B (Liu and Zeng, 2024), and ArmoRM-Llama3-8B (Gao et al., 2024b) as representatives of state-of-the-art outcome reward models. In Outcome Reward Model, we also include a Pair-Preference-Llama3-8B as the representative of the pairwise preference model (Dong et al., 2024; Zhao et al., 2023; Ye et al., 2024; Jiang et al., 2023; Lee et al., 2024). These models also take the candi-

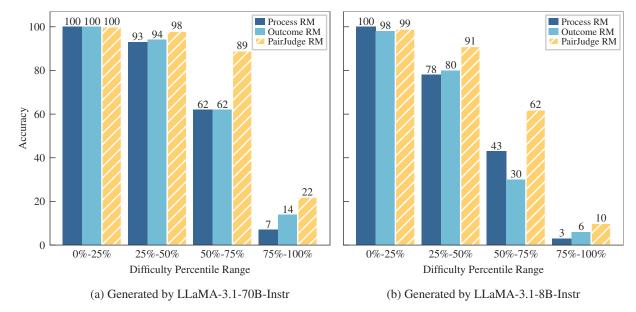


Figure 2: Comparison of Process RM, Outcome RM, and PairJudge RM across difficulty percentiles in MATH-500. Candidate solutions are generated by (a) LLaMA-3.1-70B-Instr and (b) LLaMA-3.1-8B-Instr. Process RM and Outcome RM refer to EurusRM-7B and RLHFlow-8B-DS-Data, respectively. As shown, PairJudge RM consistently outperforms both, except on the easiest problems. Notably, for the hardest 50% problems, PairJudge RM achieves a 40%–60% relative improvement.

date solution pairs as input, but they are trained as same as the outcome reward model to assign scores to the candidate solutions, instead of judging the correctness of the candidate solutions via chain-of-thought reasoning. For the Process Reward Model, we leverage three off-the-shelf opensource models: Math-Shepherd (Wang et al., 2023), RLHFlow-8B-Mistral-Data, and RLHFlow-8B-Deepseek-Data (Dong et al., 2024). For fair comparison, we also reimplement the Math-Shepherd model with MCTS data generated by Llama-3.1-8B-Instruct, denoted as RLHFlow-8B-LLaMA-Data. We select the candidate solution with the highest reward-model score as the final output of BoN Sampling. Moreover, we include a majorityvoting baseline, which selects the candidate solution that receives the most votes from the N candidate solutions as the final output.

5.2 Results

The experimental results are summarized in Table 2. Our proposed method, PairJudge RM, consistently outperforms baseline models, including majority voting, across all datasets and generation models. Notably, PairJudge RM achieves an average improvement of 6.7% on MATH-500 and 3.9% on Olympiad Bench compared to the strongest baseline model (excluding majority voting). Interestingly, majority voting outperforms the baseline re-

ward model on MATH-500, suggesting that existing reward models may lack robustness in scoring candidate solutions. These findings align with previous research (Liu et al., 2024; Kim et al., 2024), which highlights the limitations of baseline reward models in reliably assessing solution correctness.

5.3 Difficulty Analysis

To further investigate scenarios in which the Pair-Judge RM outperforms the baseline reward model, we analyze the performance of the Pair-Judge RM and the baseline reward model on math problems with different levels of difficulty. We define the difficulty of a math problem as the fraction of incorrect answers among the candidate solutions:

Difficulty =
$$\frac{\text{\#incorrect answers}}{\text{\#candidate solutions}}$$
. (5)

Specifically, we calculate this difficulty when the number of candidate solutions is n=64. We then divide the math problems into four percentile groups based on their difficulty level and evaluate the performance of the PairJudge RM and baseline models on each percentile in the MATH-500 dataset. Figure 2 shows the results. Except for the easiest problems, the PairJudge RM consistently outperforms the baseline models across all difficulty levels. On the challenging problems (Difficulty > 0.5), the PairJudge RM achieves a relative

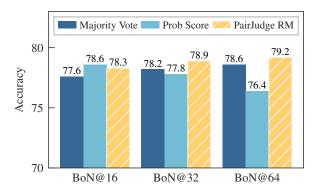


Figure 3: Comparison of the Critic Model with Majority vote/ Prob Score bewteen PairJudge RM on BoN sampling. Accuracy is reported in percentage.

improvement of 40% to 60% over the baseline models. These findings indicate that the PairJudge RM has strong potential to enhance BoN Sampling on challenging math problems.

6 Comparison with Critic Model

Critic Model (Gao et al., 2024a; McAleese et al., 2024), also known as LLM-as-a-Judge (Zheng et al., 2023; Bai et al., 2023), uses one LLM to critique the response of another LLM to a given prompt. In contrast to ordinary RMs, the output of the Critic Model is a critique in the form of chain-of-thought reasoning rather than a numerical score. This setting is similar to that of the PairJudge RM, as both methods generate a chainof-thought reasoning text to evaluate the response. The key difference is that PairJudge RM performs pairwise judgment, while Critic Model uses pointwise judgment. Recently, Critic Model has been applied in the math and code reasoning domains to verify candidate solutions and assign numerical scores (McAleese et al., 2024; Gao et al., 2024a). In this section, we compare the effectiveness of PairJudge RM and Critic Model in correctness verification and Best-of-N Sampling at the test time.

6.1 Comparison on Correctness Verification

We compare the performance of the PairJudge RM and the Critic Model on correctness verification. Specifically, given one question and two candidate solutions, both models are tasked with judging the correctness of these solutions. For a fair comparison, we train the PairJudge RM and the Critic Model using the same computational budget and training data. In particular, we use the same questions from the MATH-500 training set (Hendrycks et al., 2021) and candidate solutions generated by Llama-3.1-8B-Instruct (AI@Meta, 2024) for both

Model	MATH	Olympiad	Avg.
Critic Model	67.7	56.9	62.3
PairJudge RM	70.4	64.2	67.3

Table 3: Comparison of the PairJudge RM and LLM-as-a-Judge on the MATH-500 and Olympiad datasets on correctness verification task. Candidates are generated by Qwen-2.5-7B-Instruct. Accuracy is reported.

models. Since each training example for the Pair-Judge RM contains two candidate solutions, the training data for the Critic Model is twice as large. All other training details follow Section 5.1.

After training, we evaluate both models on the MATH-500 and Olympiad datasets. We sample 8,000 candidate solutions from each dataset to form the test set for the Critic Model. To avoid bias, these candidate solutions are generated by Qwen-2.5-7B-Instruct (Qwen Team, 2024) using the test split. We then pair each solution with another solution that produces a different answer for the same question, yielding 4,000 pairs for evaluating the PairJudge RM.

As shown in Table 3, the PairJudge RM outperforms the Critic Model on both the MATH-500 and Olympiad datasets. This result suggests that pairwise judgment is more effective than pointwise judgment when judging correctness. Notably, on the more challenging Olympiad dataset, the PairJudge RM achieves a larger improvement, highlighting its potential for difficult math problems.

6.2 Comparison on Best-of-N Sampling

As described in Section 6.1, the primary difference between the PairJudge RM and the Critic Model is the judgment process. The PairJudge RM performs pairwise judgment, while the Critic Model performs pointwise judgment. This difference makes it challenging to apply the Critic Model for BoN Sampling at test time. If the Critic Model verifies two candidate solutions with different answers as correct, it remains unclear which one is superior. In contrast, the PairJudge RM directly compares correctness to select the better solution.

To enable BoN Sampling with the Critic Model, two approaches are typically adopted.

Combine with Majority Voting One option is to combine the Critic Model with majority voting. The Critic Model first judges the correctness of each candidate and removes those marked as incorrect. Majority voting is then applied to the remaining solutions to determine the final output.

Problem: A regular hexagon can be divided into six equilateral triangles. If the perimeter of one of the triangles is 21 inches, what is the perimeter, in inches, of the regular hexagon?

Solution 1: Since the perimeter of the triangle is 21, each side of the triangle has length $\frac{21}{3} = 7$. The hexagon is made up of six of these triangles, so the perimeter of the hexagon is $6 \cdot 7 = 42$. So the final answer is $\boxed{42}$.

Critic Model Probabilistic Score: 0.0006 | Pair Judge RM Correctness Judgment: True

Solution 2: Since the perimeter of one of the triangles is 21 inches, the side length of the triangle is $\frac{21}{3} = 7$ inches. Since there are six equilateral triangles, the perimeter of the hexagon is six times the side length of one triangle, which is $6 \cdot 7 = 42$ inches. So the final answer is $\boxed{42}$.

Critic Model Probabilistic Score: 0.9999 Pair Judge RM Correctness Judgment: True

Figure 4: Comparison of the Critic Model with Probabilistic Score and PairJudge RM in evaluating two correct solutions to a math problem. The Critic Model assigns drastically different probabilistic scores (0.0006 vs. 0.9999), highlighting its inconsistency, while PairJudge RM consistently identifies both as correct.

Use Probabilistic Score Another approach is to use the probabilistic score assigned by the Critic Model to each candidate solution. In this method, the Critic Model is prompted to generate a to-ken—either "correct" or "incorrect"—within its reasoning text to indicate correctness. Zhang et al. (2024b) suggest that the probability of generating the token "correct" can serve as the score for each candidate solution. The candidate with the highest score is then selected as the final output.

To prevent data leakage, we use the MATH-500 test split and candidate solutions generated by Qwen-2.5-7B-Instruct (Qwen Team, 2024) to evaluate both models on BoN Sampling. For a fair comparison, we reuse the Critic Model and Pair-Judge RM trained in Section 6.1 for this evaluation.

As shown in Figure 3, the PairJudge RM consistently outperforms the Critic Model on the MATH-500 dataset. This result demonstrates that, under the same training budget, the PairJudge RM is more effective at BoN Sampling than the Critic Model. Additionally, the Critic Model using a probabilistic score underperforms compared to majority voting, likely due to its tendency to assign highly polarized scores. As illustrated in Figure 4, even similar candidate solutions receive substantially different probabilistic scores. This observation suggests that the probabilistic scoring method faces robustness and stability issues similar to those observed in reward models (Liu et al., 2024; Kim et al., 2024).

7 Related Work

7.1 Test Time Scaling and Best-of-N Sampling

Test-time scaling, introduced with o1(OpenAI, 2024), improves model performance during inference by dedicating more computation (Snell et al., 2024; Wu et al., 2024) Approaches include Monte Carlo Tree Search (Zhang et al., 2024a; Gao

et al., 2024b) and long-chain-of-thought (Min et al., 2024; Qwen Team, Alibaba, 2023) Best-of-N Sampling is one such approach, generating *N* candidate solutions and selecting the best via a reward model (Wang et al., 2023; Lightman et al., 2023; Wang et al., 2024b; Zhang et al., 2024b) Performing BoN sampling with tournament-style selection was first introduced for instruction-following tasks (Lee et al., 2024) Our knockout method differs by grouping candidates with identical answers to avoid unnecessary comparisons and using generative judgment via CoT reasoning (Wei et al., 2022) rather than discriminative scoring.

7.2 Reward Models and Critic Models

Reward models (RMs) assign numerical scores to LLM outputs for feedback during training and inference (Lambert et al., 2024; Liu et al., 2024; Wang et al., 2024a; Lightman et al., 2023). They can operate in pointwise or pairwise manners (Yuan et al., 2024a; Jiang et al., 2023). Critic models evaluate response quality, especially in reasoning tasks like math and code (Gao et al., 2024a; McAleese et al., 2024; Zheng et al., 2023; Li et al., 2024). Unlike RMs, Critic Models function offering textual feedback instead of numerical score. PairJudge RM differs from them by simultaneously judging the correctness of two responses using chain-of-thought reasoning instead of pointwise evaluation like Critic Models and numerical scoring like RMs.

8 Conclusion

We propose the Pairwise Judge Reward Model (PairJudge RM) with a knockout tournament for BoN Sampling. PairJudge RM uses CoT reasoning to simultaneously evaluate two candidate solutions, eliminating arbitrary scoring and enabling cross-validation. The knockout tournament iteratively eliminates incorrect solutions through pair-

wise judgments. We create a 432K pairwise judgment dataset to train PairJudge RM. Experiments show PairJudge RM significantly outperforms baseline RMs in BoN Sampling on benchmarks.

Limitation

The main limitation of the proposed method lies on the inference time. To conduct the BoN Sampling with the PairJudge RM, serval rounds of pairwise verification are required to select the best candidate solution. This process is time-consuming and may not be suitable for latency-sensitive applications. However, the proposed method can be potentially accelerated by parallel computing or other optimization techniques to reduce the inference time. For example, the multiple pairwise verification can easily be parallelized to multiple GPUs to speed up the inference process since they are independent of each other. Moreover, with popularization of the inference-time scaling, it is a common practice to increase the computational resources to improve the performance of the model in solving complex reasoning tasks like math problems (Snell et al., 2024; Wu et al., 2024).

Ethical Considerations

In this work, all the data and models are acquired from public datasets and pre-trained models, and no human subjects are involved in the experiments. Considering the potential hallucination and bias in the pre-trained models, it is worth nothing that the user should be cautious when applying the proposed method to real-world applications such as use PairJudge RM to check human student's homework in the educational system.

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A Dataset Information

In this section, we provide detailed information about the datasets used in this work. The datasets employed for evaluating mathematical reasoning tasks come from various sources, each contributing unique characteristics for comprehensive benchmarking.

MATH-500 is a collection of problems designed to test mathematical reasoning capabilities. It covers a wide range of mathematical concepts and problem types, from basic algebra to more advanced topics. Specifically, MATH-500 is newer, IID version of MATH (Hendrycks et al., 2021), which is a widely used benchmark for avoide the data leakage issue. Size: 500 problems. Source: https://github.com/openai/prm800k/tree/main/prm800k/math_splits. License: MIT License.

Olympiad Bench is derived from a collection of problems from various international mathematical olympiads. It includes a broad range of challenging problems, covering topics like number theory, combinatorics, geometry, and algebra. This dataset is particularly useful for testing a model's ability to handle competition-level mathematical reasoning tasks. Size: 8,476 problems Source: https://arxiv.org/abs/2402.14008. License: MIT License

NuminaMath-CoT is a dataset that includes 860k math problems, where each solution is formatted in a Chain of Thought (CoT) manner. The sources of the dataset range from Chinese high school math exercises to US and international mathematics olympiad competition problems. The data were primarily collected from online exam paper PDFs and mathematics discussion forums. The processing steps include (a) OCR from the original PDFs, (b) segmentation into problem-solution pairs, (c) Translation into English, (d) realignment to produce a CoT reasoning format, and (e) final answer formatting. Size: 860k problems. Source: https://huggingface.co/ datasets/AI-MO/NuminaMath-CoT. Li**cense:** Apache License 2.0.

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Computational Resources

This section provides detailed information about the computational resources used in our experiments. The training of Qwen-2.5-7B-Instr on the PAIRJUDGE-432K was conducted on an 8-GPU H100 server, with an estimated training duration of approximately 24 hours. The construction of the PAIRJUDGE-432K relies on the gemini-1.5-flash API server through Google Cloud. The associated API costs for this project amounted to approximately 2,000 USD.

C Future Work

Application in Reinforcement Learning

In this work, we mainly focus on how to perform the BoN Sampling at test time with the PairJudge RM. This experiment setting follows the existing work (Wang et al., 2023; Lightman et al., 2023; Wang et al., 2024b; Zhang et al., 2024b) and helps us to compare the performance of the PairJudge RM with baseline models and verify the effectiveness of the proposed method. However, the Pair-Judge RM can also be applied at the Reinforcement Learning (RL) training stage to improve the performance of the model in solving complex reasoning tasks like math problems. To apply in the training stage, the PairJudge RM need to assign a numerical score to the candidate solutions just like the discriminative reward model (Lambert et al., 2024; Liu et al., 2024). Such a numerical score could acquired by the winning rate of the candidate solutions in the knockout tournament, which can be used as the reward signal to guide the training of the model. In the future, we plan to explore the application of the PairJudge RM in the RL training stage to improve the performance of the model in solving complex reasoning tasks like math prob-

Alternative Tournament Strategies

In this work, we introduce the knockout tournament to select the best candidate solution, where the candidate solutions are viewed as players in the tournament and each pairwise comparison is viewed as a match between two players. The main reason for choosing the knockout tournament is that it is one of the most naive tournament design that could select the best candidate under time complexity O(N), where N is the number of candidate solutions. It worth noting that there are tons of alternative tournament strategies that could be used to select the best candidate solution, such as the round-robin tournament, the Swiss-system tournament, and the double-elimination tournament (Devriesere et al., 2024) Such alternative tournament strategies could be potentially used to improve the

performance of the PairJudge RM in selecting the best candidate solution, and we plan to explore the application of the alternative tournament strategies in the future work.

D Potential Improvement

Due to the computational limitation and resource constraints, there are several potential improvements that could be made to further improve the performance of the PairJudge RM.

- Bigger Model Capacity: Due to the computational limitation, although we presents a promising and scalable dataset contruction in Section 4, the PairJudge RM is trained with the Qwen-2.5-7B-Instr, which is a relatively small model compared to the state-of-the-art models like the Qwen-2.5-70B-Instr, LLaMA-3.1-70B-Instr and QwQ-32B. According to the Chinchilla Law (Hoffmann et al., 2022), under the same training data and training time, a model with larger capacity can achieve better performance than a model with smaller capacity.
- More Data Scaling Dimension: The Pair-Judge RM is trained with the PAIRJUDGE-432K dataset, which contains 343K training data for the PairJudge RM. Now there are two directions to further improve the performance of the PairJudge RM: 1) use more models rather than only LLama-3.1-8B-Instruct to generate the candidate solutions, and 2) Use more models rather than only gemini-1.5-flash to annotate the training data. This two directions could potentially magnitudes the size of the training data and improve the performance of the PairJudge RM.
- Long-Cot Base Model: The PairJudge RM is trained with the Qwen-2.5-7B-Instruct. Considering the recent success of the Long-Cot models such as the QwQ-32B (Qwen Team, Alibaba, 2023) in reasoning task, it is worth exploring the application of the PairJudge RM with the Long-Cot models to further improve the performance of the PairJudge RM.

E Artifacts in Our Research

Our work is built upon several key artifacts that have played a crucial role in enabling the development and evaluation of the proposed Pairwise Judge Reward Model (PairJudge RM).

PyTorch (Paszke et al., 2019) is a widely used

deep learning framework that provides flexible and efficient tools for building neural networks, making it an essential artifact for modern NLP research. Its dynamic computational graph and GPU acceleration have been pivotal in enabling rapid prototyping and experimentation, especially in the context of transformer-based models and reinforcement learning.

The **Hugging Face Transformers** library (Wolf et al., 2020) is another critical tool that has revolutionized NLP. It provides an extensive collection of pre-trained models and tools for working with transformer architectures, making it easier for researchers and practitioners to fine-tune models on domain-specific tasks. The library's user-friendly API, combined with its comprehensive model hub, has democratized access to state-of-the-art models such as BERT, GPT, T5, and more.

DeepSpeed (Rasley et al., 2020) is a library developed by Microsoft that aims to optimize largescale model training. It introduces techniques such as mixed-precision training and model parallelism to significantly reduce memory usage and speed up training, making it an essential tool for training large transformer models. DeepSpeed's support for efficient distributed training has enabled researchers to scale up their models while reducing computational costs.

The **Llama** model (AI@Meta, 2024) is a family of large language models developed by Meta AI. Trained on approximately 15 trillion tokens, Llama models are available in sizes ranging from 8 billion to 405 billion parameters. They have demonstrated superior performance across various NLP benchmarks, making them a valuable resource for tasks such as text generation, translation, and summarization.

The **Qwen** model (Qwen Team, 2024) is a series of large language models developed by Alibaba Cloud. The Qwen series includes models with varying parameter counts ranging from 0.5B to 72B. These models have shown competitive performance across diverse benchmarks, including language understanding, generation, multilingual proficiency, coding, mathematics, and reasoning. The Qwen series has been instrumental in advancing the capabilities of LLMs in various applications.

All these artifacts have played a crucial role in enabling our research and have significantly contributed to our research.

F Experiments Replication

To facilitate the replication of our experiments, we plan to release the codebase, model checkpoints, and dataset used in this work. The codebase will be made available on GitHub, along with detailed instructions on how to reproduce the experiments. As for the model checkpoints and dataset, we plan to provide in Huggingface. All the experiments will be conducted on three times and the average results will be reported to minimize the randomness in the experiments.

G Prompt Templates

Here we provide the prompt templates for the Pair-Judge RM in Table 4, which is used to guide the Pair-Judge RM to judge the correctness of two candidate solutions to a given math problem.

H Math Problem Filtering Criteria

Here we provide the filtering criteria applied to the dataset in Table 5, which is used to remove low-quality, proof-based, or multiple-choice problems.

Table 4: Prompt Template for PairJudge RM, the {question}, {response_a}, and {response_b} are placeholders for the math question, response A, and response B, respectively.

Task Objective:

Evaluate the correctness of two responses (Response A and Response B) to a given math question. Perform a step-by-step verification of each response's accuracy. After completing the step-by-step checks, provide a final correctness judgment for each response.

Steps to Follow:

0. Extract Answers from both Responses:

- Read and both responses to identify the final answers provided.
- If the responses provide different answers, make sure there are is no possible way that both responses can be correct. It must be the case that one response is correct and the other is incorrect or both are incorrect.

1. Step-by-Step Verification of Correctness:

- For each response (Response A and Response B):

Carefully examine each step of the solution provided. Check the following:

- Mathematical accuracy: Ensure all calculations, algebraic simplifications, and mathematical operations are correct.
- Logical consistency: Verify that each step follows logically from the previous one and that the reasoning is sound.
- Completeness: Make sure that all necessary steps are included to fully solve the problem and reach the final answer.

While performing this step-by-step evaluation, refer to the **Additional Tips** section for helpful techniques to validate each response's accuracy. **Attention:** When checking the correctness of a single step, you should never first conclude the correctness of this step (for example, *"This step is incorrect because..."* is strictly forbidden). You should neutrally check this step, provide evidence about its correctness, and then finally draw a conclusion about the correctness of this step. In other words, you should first employ the techniques in **Additional Tips** to check the correctness of this step, and then draw a conclusion about the correctness of this step.

2. Final Conclusion:

- After completing the step-by-step verification for each response, sum up the information you have now, then finally determine whether each response's answer is **correct** or **incorrect**.
- Provide the final judgment for each response, the output should in-closed with the following tags:
- If Response A's answer is correct:
- <resp_a_judge>Correct</resp_a_judge>
- If Response A's answer is incorrect:
- <resp_a_judge>Incorrect</resp_a_judge>
- If Response B's answer is correct:
- <resp_b_judge>Correct</resp_b_judge>
- If Response B's answer is incorrect:
- <resp_b_judge>Incorrect</resp_b_judge>
- **Note:** The responses A and response B can be either correct or incorrect, or both correct, or both incorrect. You should provide the final judgment for each response. There is no guarantee that at least one response is correct or incorrect.

Additional Tips:

- Key Validation Techniques (to apply during Step 1):
- Re-derive Key Parts of the Solution: Independently calculate or derive crucial steps of the solution to verify their correctness.
- Verify Calculations: Double-check all mathematical operations (e.g., addition, multiplication, division) to confirm accuracy.
- Compare Responses: If needed, compare similar steps between Response A's and Response B's answers to identify discrepancies or inconsistencies.
- The final output format should be as follows:

Final Judgment:

Response A: <resp_a_judge>Correct/Incorrect</resp_a_judge>
Response B: <resp_b_judge>Correct/Incorrect</resp_b_judge>

Question: <question> {question} </question>

Response A: <response_a> {response_a} </response_a>
Response B: <response_b> {response_b> </response_b>

Table 5: Filtering criteria applied to the dataset to remove low-quality, proof-based, or multiple-choice problems.

Filter Type	Criteria
Bad Quality Problems	Problems with messy formatting, OCR errors, or empty ground truth (gt).
Equations in Ground Truth	gt contains "=" (indicating it might be an equation rather than a clear ground true).
Multiple Questions	Problems with patterns indicating multiple sub-questions (MULTI_QUESTION).
Yes/No Questions	Solutions with patterns indicating yes/no, true/false (YESNO_QUESTIONS).
Text Answers	Ground truth containing patterns indicating textual answers (TEXT_ANSWER).
Proof Problems	Problems with patterns indicating proof problems (PROVE_PATTERN).
Multiple Choice Questions	Problems with patterns indicating multiple-choice questions (MCQ_OPTIONS).