Modelling and Networked Control of Water Distribution Networks

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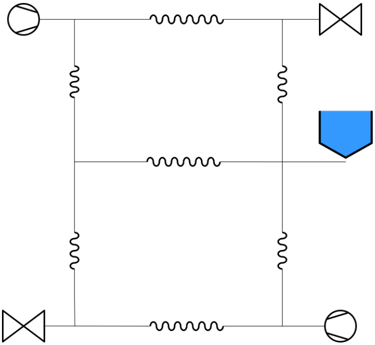
Agenda

- Critical societal infrastructure, responsible for the provision of water to both domestic and industrial consumers.
- Network pressure must be controlled.
  - Underpressure $\rightarrow$ insufficient service pressure $\rightarrow$ dissatisfied end users.
  - Overpressure $\rightarrow$ component failure $\rightarrow$ repair costs, supply intermittency, etc.
- Network pressure tied to level in Elevated Water Reservoir(s) (EWR) $\rightarrow$ level control is key!
- Other key components are pumps, valves, and pipes.

We analyse a small-scale test WDN with the following layout:



Larger WDNs are typically not amenable to first-principles modelling.

General optimal control problem

The basic structure of an optimal control problem is sketched in the language of calculus of variations.

$$\dot{x} = f(x, u, t) \tag{1}$$

$$x(t_0) = x_0,\ x \in \mathbb{R}^n,\ \ u \in U \in \mathbb{R}^m \tag{2}$$

where $t \in \mathbb{R}$ is the time and x, u are functions of t, with U the set of admissible controls.

Cost functional:

$$J = \mathcal{M}(x(T)) + \int_0^T \mathcal{L}(x(t), u(t))dt \tag{3}$$

Infinite-Horizon Linear-Quadratic Regulator

The LQR timeinvariant case with no terminal cost.
System Dynamics:

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{4}$$

Cost functional:

$$J = \int_{t_0}^{\infty} \left(x^T(t)Qx(t) + u^T(t)Ru(t) \right) dt \tag{5}$$

State feedback control law:

$$u^*(t) = -R^{-1}B^T P x^*(t) \tag{6}$$

P is time-invariant and fullfills the *algebraic Riccati equation*:

$$PA + A^T P + Q - PBR^{-1}B^T P = 0 \tag{7}$$

Let $\hat{x} = x - x_r$ and $\hat{u} = u - u_r$. Shifted coordinate system cost functional framed as an output tracking problem:

$$J = \int_{t_0}^{\infty} \left((C\hat{x})^T Q_y (C\hat{x}) + \hat{u}^T R \hat{u}\right) dt = \int_{t_0}^{\infty} \left(\hat{y}^T Q_y \hat{y} + \hat{u}^T R \hat{u}\right) dt \tag{8}$$

States are extended with integral state x_i

$$u = -\bar{K}\bar{x} \tag{9}$$

$$\dot{\bar{x}} = \bar{A}\bar{x} + \bar{B}u + B_r r \tag{10}$$

$$y = \bar{C}\bar{x} \tag{11}$$

$$\bar{A} = \begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix},\ \bar{B} = \begin{bmatrix} B \\ 0 \end{bmatrix},\ B_r = \begin{bmatrix} 0 \\ 1 \end{bmatrix},\ \bar{C} = \begin{bmatrix} C & 0 \end{bmatrix} \tag{12}$$

$$\bar{K} = -\begin{bmatrix} K & -K_i \end{bmatrix} \tag{13}$$

K_i can be an awkward weight to choose.

Deviation variables:

$$\Delta x_k = x_k - x_{k-1},\ \Delta y_k = y_k - r_k,\ \Delta u_k = u_k - u_{k-1} \tag{14}$$

Extended vectors and matrices:

$$\begin{aligned} \tilde{\zeta}_k &= \begin{bmatrix} \Delta x_k \\ \Delta y_k \end{bmatrix},\ \tilde{u}_k = \Delta u_k, \\ \tilde{A} &= \begin{bmatrix} A & 0 \\ CA & I \end{bmatrix},\ \tilde{B} = \begin{bmatrix} B \\ CB \end{bmatrix},\ \tilde{C} = \begin{bmatrix} 0 & I \end{bmatrix} \end{aligned} \tag{15}$$

Cost functional:

$$J = \sum_{k_0}^{\infty} \left(\tilde{\zeta}^T Q \tilde{\zeta} + \tilde{u}^T R \tilde{u} \right) \tag{16}$$

Origin regulation! If $\tilde{\zeta} \to 0 \Rightarrow \Delta y \to 0 \Rightarrow y \to r$.

If linearising around equilibrium point x_e and corresponding operating point u_{op}

$$\Delta\tilde{x} = x - x_e, \;\; \Delta\tilde{u} = u - u_{op} \tag{17}$$

VF-LQR will penalize deviations from the point $\{x_e, u_{op}, r\}$!

Incremental feedback law:

$$\Delta u^*(k) = -\Delta K(k)\Delta x(k) \tag{18}$$

Control input applied at time k is:

$$u^*(k) = \sum_{i=1}^{k} \Delta u^*(i) \tag{19}$$

with

$$\tilde{K} = (\tilde{B}^T P \tilde{B} - R)^{-1}(\tilde{B}^T P \tilde{A}) \tag{20}$$

Exogenous input disturbance-accommodating LQR

Standard LQR does not accommodate exogenous inputs (such as the the model of the consumer demand flows) but can be modified to do so:

$$u(k) = \sum_{i=1}^{k} \Delta u^*(i) - B^\dagger \mathcal{B} \delta(i) \tag{21}$$

where $B^\dagger$ is the Moore-Penrose pseudoinverse of B and $\mathcal{B}$ is the disturbance input matrix.

References

Open for questions!

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