Modelling and Networked Control of Water Distribution Networks

January 18, 2022

CA733

Control and Automation, Group 733

Aalborg University

Denmark





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Infinite-Horizon Linear-Quadrati Regulator

Tracking LQR and Integral Action

Velocity-Form LQR

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- Critical societal infrastructure, responsible for the provision of water to both domestic and industrial consumers.
- Network pressure must be controlled.
 - ► Underpressure → insufficient service pressure → dissatisfied end users.
 - Overpressure \rightarrow component failure \rightarrow repair costs, supply intermittency, etc.
- Network pressure tied to level in Elevated Water Reservoir(s) (EWR) → level control is key!
- Other key components are pumps, valves, and pipes.

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General optimal control problem

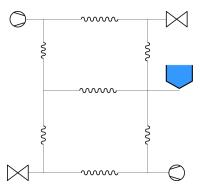
Linear-Quadratic Regulator Tracking LQR and

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We analyse a small-scale test WDN with the following layout:



Larger WDNs are typically not amenable to first-principles modelling.

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General optimal control problem

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The basic structure of an optimal control problem is sketched in the language of calculus of variations.

$$\dot{x} = f(x, u, t) \tag{1}$$

$$x(t_0) = x_0, x \in \mathbb{R}^n, \ u \in U \in \mathbb{R}^m$$
 (2)

where $t \in \mathbb{R}$ is the time and x, u are functions of t, with U the set of admissible controls.

Cost functional:

$$J = \mathcal{M}(x(T)) + \int_{0}^{T} \mathcal{L}(x(t), u(t)) dt$$
 (3)



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The LQR timeinvariant case with no terminal cost. System Dynamics:

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{4}$$

Cost functional:

$$J = \int_{t_0}^{\infty} \left(x^{T}(t) Q x(t) + u^{T}(t) R u(t) \right) dt$$
 (5)

State feedback control law:

$$u^*(t) = -R^{-1}B^T P x^*(t) (6)$$

P is time-invariant and fullfills the algebraic Riccati equation:

$$PA + A^T P + Q - PBR^{-1}B^T P = 0 (7)$$

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Let $\hat{x} = x - x_r$ and $\hat{u} = u - u_r$. Shifted coordinate system cost functional framed as an output tracking problem:

$$J = \int_{t_0}^{\infty} \left((C\hat{x})^T Q_y (C\hat{x}) + \hat{u}^T R \hat{u} \right) dt = \int_{t_0}^{\infty} \left(\hat{y}^T Q_y \hat{y} + \hat{u}^T R \hat{u} \right) dt$$
(8)

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States are extended with integral state x_i

$$u = -\bar{K}\bar{x} \tag{9}$$

$$\dot{\bar{x}} = \bar{A}\bar{x} + \bar{B}u + B_r r \tag{10}$$

$$y = \bar{C}\bar{x} \tag{11}$$

$$\bar{A} = \begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix}, \ \bar{B} = \begin{bmatrix} B \\ 0 \end{bmatrix}, \ B_r = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \ \bar{C} = \begin{bmatrix} C & 0 \end{bmatrix}$$
 (12)

$$\bar{K} = -\begin{bmatrix} K & -K_i \end{bmatrix} \tag{13}$$

 K_i can be an awkward weight to choose.



Velocity-Form LQR

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Deviation variables:

$$\Delta x_k = x_k - x_{k-1}, \ \Delta y_k = y_k - r_k, \ \Delta u_k = u_k - u_{k-1}$$
 (14)

Extended vectors and matrices:

$$\tilde{\zeta}_k = \begin{bmatrix} \Delta X_k \\ \Delta y_k \end{bmatrix}, \ \tilde{u}_k = \Delta u_k,$$

$$\tilde{A} = \begin{bmatrix} A & 0 \\ CA & I \end{bmatrix}, \ \tilde{B} = \begin{bmatrix} B \\ CB \end{bmatrix}, \ \tilde{C} = \begin{bmatrix} 0 & I \end{bmatrix}$$

Cost functional:

$$J = \sum_{k}^{\infty} \left(\tilde{\zeta}^{T} Q \tilde{\zeta} + \tilde{u}^{T} R \tilde{u} \right) \tag{16}$$

(15)

Origin regulation! If $\tilde{\zeta} \to 0 \Rightarrow \Delta y \to 0 \Rightarrow y \to r$.

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If linearising around equilibrium point x_e and corresponding operating point u_{op}

$$\Delta \tilde{x} = x - x_e, \ \Delta \tilde{u} = u - u_{op} \tag{17}$$

VF-LQR will penalize deviations from the point $\{x_e, u_{op}, r\}$! Incremental feedback law:

$$\Delta u^*(k) = -\Delta K(k) \Delta x(k) \tag{18}$$

Control input applied at time k is:

$$u^{*}(k) = \sum_{i=1}^{k} \Delta u^{*}(i)$$
 (19)

with

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$$\tilde{K} = (\tilde{B}^T P \tilde{B} - R)^{-1} (\tilde{B}^T P \tilde{A})$$
 (20)

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Standard LQR does not accommodate exogenous inputs (such as the the model of the consumer demand flows) but can be modified to do so:

$$u(k) = \sum_{i=1}^{k} \Delta u^*(i) - B^{\dagger} \mathcal{B} \delta(i)$$
 (21)

where B^{\dagger} is the Moore-Penrose pseudoinverse of B and \mathcal{B} is the disturbance input matrix.

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Open for questions!

