Formulating Existing Force Structure Optimization

Table of Contents

1 Introduction

A formulation for a MIP goal program to formalize force structure optimization.

2 Sets

$SRCS$
$phases$

3 Parameters

- `weight(phase)` The priority weighting by phase, summing to unity.
- `excess(SRC,phase,n)` The excess fill by src, in a phase, if supply level n is chosen.
- `demandmet(SRC,phase,n)` The demand met by src, in a phase, if supply level n is chosen.
- `strength(SRC,n)` The total strength associated with src, is supply level n is chosen.
- `lower(SRC)` The lower bound for supply level n for src.
- `upper(SRC)` The upper bound for supply level n for src.
- `bounds(SRC)` $\{lower(SRC)...upper(SRC)\}$ The integer range of supply levels for src.
- `targetstrength` The targeted end strength for our goal program.

4 Variables

We define a few free variables.

`selected` is a binary variable determining if a level of supply for an SRC was chosen:

$$selected(SRC, n) \in \{0, 1\}$$
$$\forall SRC \in SRCS$$
$$\forall n \in bounds(SRC)$$

Since we have end-strength goals, we introduce necessary variables to implement absolute value relations and constraints:

$posdev \in R_+$ the "chosen" positive deviation for end strength
$negdev \in R_+$ the "chosen" negative deviation for end strength

$strdev \in R$ surrogate for the relating positive and negative deviation

5 Constraints

5.1 Fills

∀ SRC ∈ SRCS

$$(1) \qquad wfill(SRC) = \sum_{\substack{n \in bounds(SRC) \\ \forall phase \in phases}} weight(phase) * demandmet(SRC, phase, n) * selected(SRC, n)$$

$$(2) \qquad wexcess(SRC) = \sum_{\substack{n \in bounds(SRC) \\ \forall phase \in phases}} weight(phase) * excess(SRC, phase, n) * selected(SRC, n)$$

$$(3) \qquad \sum_{n \in bounds(SRC)} selected(SRC, n) = 1$$

5.2 Total Weighted Fill

$$(4) \qquad totalfill = \sum_{SRC \in SRCS} wfill(SRC)$$

5.3 Total Excess Fill

$$(5) \qquad totalexcess = \sum_{SRC \in SRCS} wexcess(SRC)$$

5.4 End Strength

$$(6) \qquad endstrength = \sum_{\substack{\forall SRC \in SRCS \\ \forall n \in bounds(SRC)}} strength(SRC, n) * selected(SRC, n)$$

5.4.1 Deviation

End strength is a goal, which may deviate absolutely. We provide a fill deviation variable that captures deviation from the expectation.

Strength deviation is the measure of observed strength less the target. It may be positive or negative.

$$(7) \qquad strengthdev = posdev - negdev$$

Absolute strength deviation is the sum of our two positive variables, this represents $|strengthdev|$, which will contribute to the objective.

$$absstrengthdev = posdev + negdev \tag{8}$$

Codify our endstrength goal.

$$endstrength + strengthdev = target \tag{9}$$

5.5 Objectives

In the legacy example, we have 2 objectives that are maximized hierarchically.

We want to select a force structure where

- we minimize deviation from the force structure target
- maximize (or preserve) weighted fill
- maximize (or preserve) weighted excess

$$\begin{aligned} &\min absstrengthdev \\ &\max totalfill \\ &\max totalexcess \end{aligned} \tag{10}$$

We could formulate this as a composite weighted objective function using the Big M method or similar (weights may need adjusting):

$$\max z = -10000 * absstrengthdev + 1000 * totalfill + totalexcess \tag{11}$$

6 Additional Goals

6.1 Deviation From Original Supply

We can also codify a desire to retain the original balance of structure as much as possible by choosing to deviate as little as possible from some baseline for each SRC.

6.2 Additional Parameters

$baseline(SRC) \in bounds(SRC)$

6.3 Additional Variables

$posSRCdev(SRC) \in$ R_+ "chosen" positive deviation in SRC structure
$negSRCdev(SRC) \in$ R_+ "chosen" negative deviation in SRC structure
$SRCdev(SRC) \in$ R surrogate for relating positive and negative structure deviation

6.4 Additional Constraints

$$SRCdev(SRC) = posSRCdev(SRC) - negSRCdev(SRC) \tag{12}$$

$$absSRCdev(SRC) = posSRCdev(SRC) + negSRCdev(SRC) \tag{13}$$

Codify our structure deviation goal:

$\forall \mathrm{SRC} \in \mathrm{SRCs}$

$$\sum_{n \in \mathrm{bounds(SRC)}} \mathrm{selected(SRC, n)} + \mathrm{SRCdev} = \mathrm{baseline(SRC)} \tag{14}$$

$$\mathrm{totalSRCdev} = \sum_{\mathrm{SRCinSRCS}} \mathrm{absSRCdev(SRC)} \tag{15}$$

7 Revised Objective

$$\begin{aligned} &\min \mathrm{absstrengthdev} \\ &\max \mathrm{totalfill} \\ &\max \mathrm{totalexcess} \\ &\min \mathrm{totalSRCdev} \end{aligned} \tag{16}$$

We could formulate this as a composite weighted objective function using the Big M method or similar (weights may need adjusting):

$$\max z = -10000 * \mathrm{absstrengthdev} + 1000 * \mathrm{totalfill} + \mathrm{totalexcess} - 0.01 * \mathrm{totalSRCdev} \tag{17}$$

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