Part I

INTRODUCTION TO NETWORK ANALYSIS

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UCDAVIS VETERINARY MEDICINE
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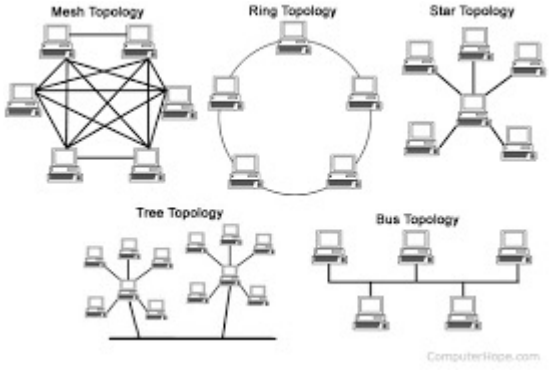
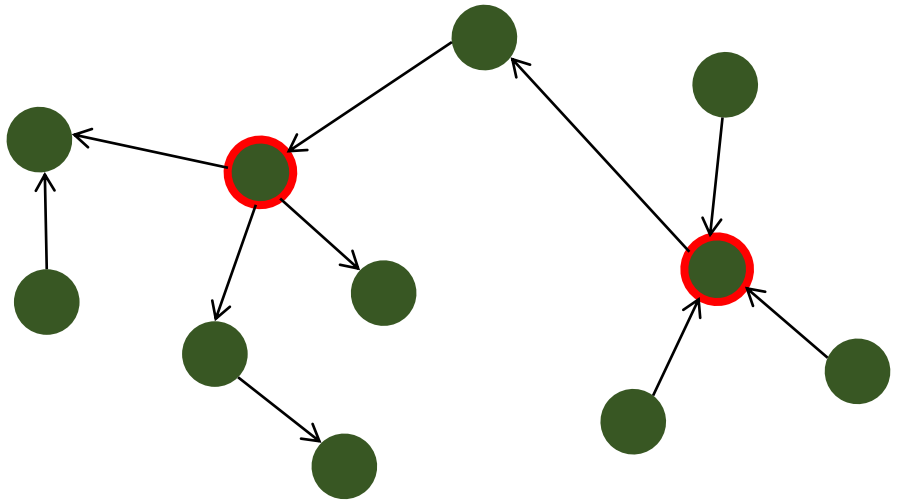
I.A - What is a network?



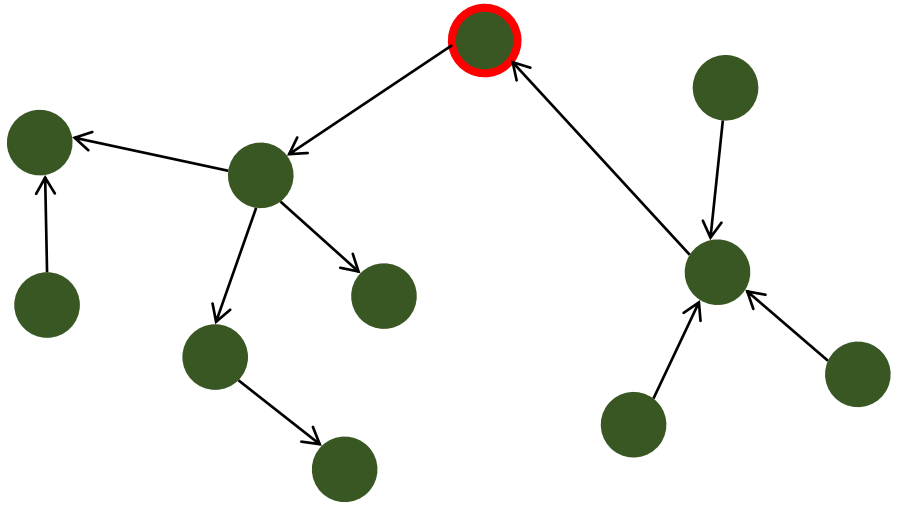
Why represent events in a network?

To describe contact dynamics

Identify individuals that are very active

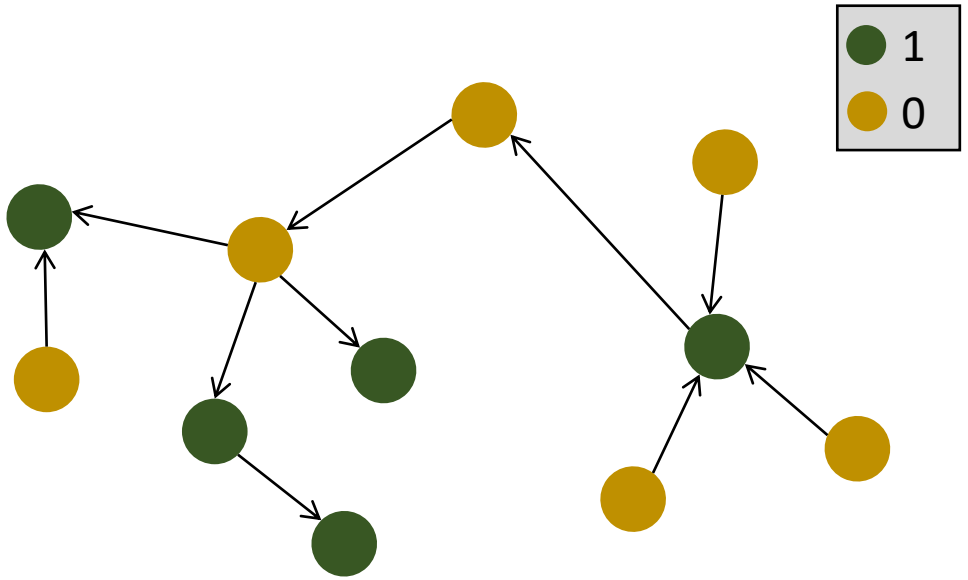


Identify individuals that are intermediate



Why represent events in a network?

Model contact dynamics:



- Inference: Associations between attributes and activities in a network
- Prediction: Are there reproducible patterns that we can predict?

Applications in Preventative Veterinary Medicine

- Surveillance, Prevention and Control
  - Identify **strategic** nodes for **risk-based targeted** intervention
    - **Surveillance**: Diagnostic testing, road checks

Pig sampling - Serbia

FAO African Swine Fever: Detection and diagnosis - 2017

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Road blocks and CO2 culling chamber - Lithuania

FAO African Swine Fever: Detection and diagnosis - 2017

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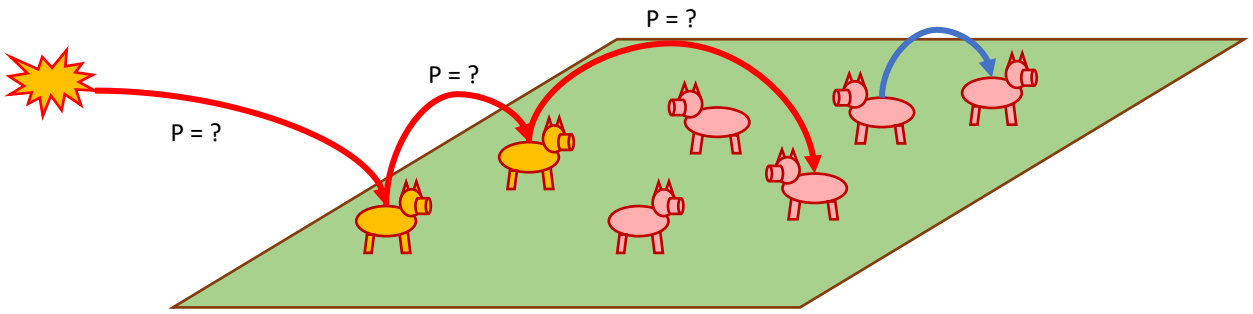
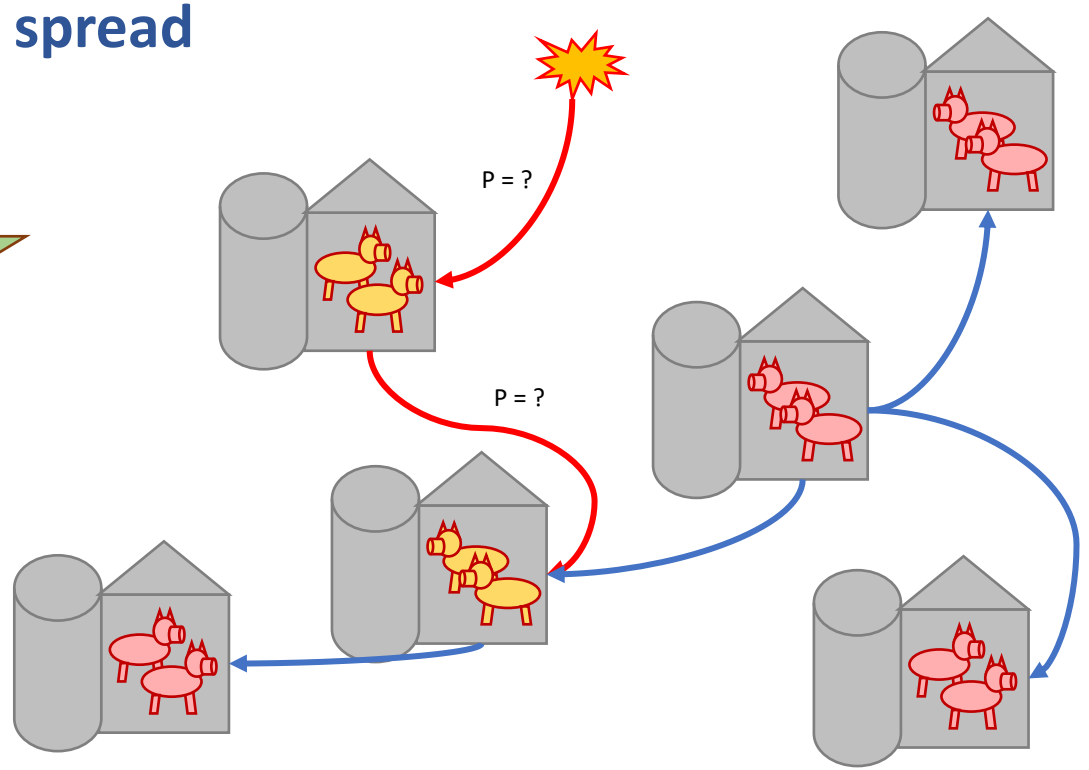
  - Modelling of disease **introduction** and **spread**





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- **Risk Factor** Analysis

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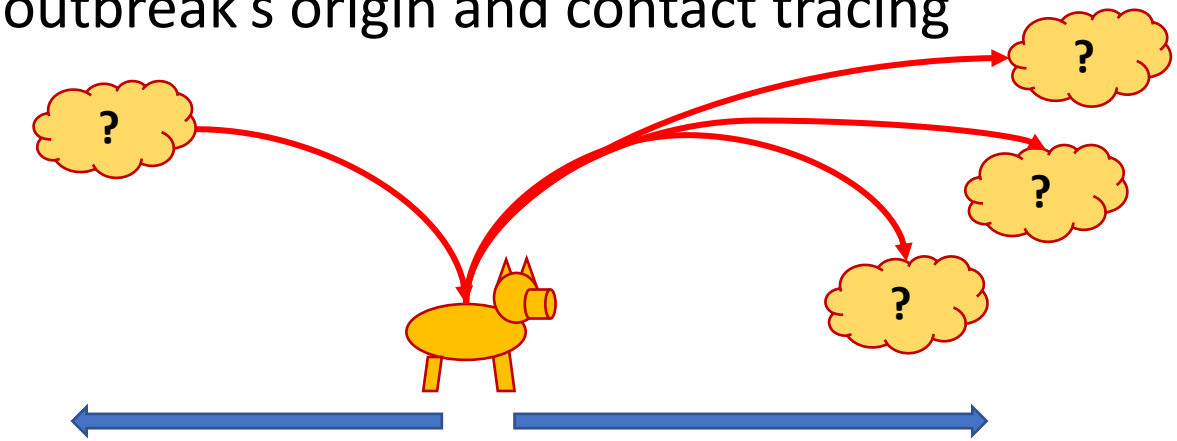
  - Modelling of disease **introduction** and **spread**
- **Risk Factor** Analysis
- **Outbreak** investigation
  - Traceability of the outbreak's origin and contact tracing



Applications in Preventative Veterinary Medicine

- Surveillance, Prevention and Control
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  - Traceability of the outbreak's origin

Source: San Diego Zoo

- Other uses in the animal world
  - Behavioral studies in social animals: i.e. contact patterns in a herd of cattle, primate behavioral studies

Applications in Preventative Veterinary Medicine

- Surveillance, Prevention and Control
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- Other uses in the animal world
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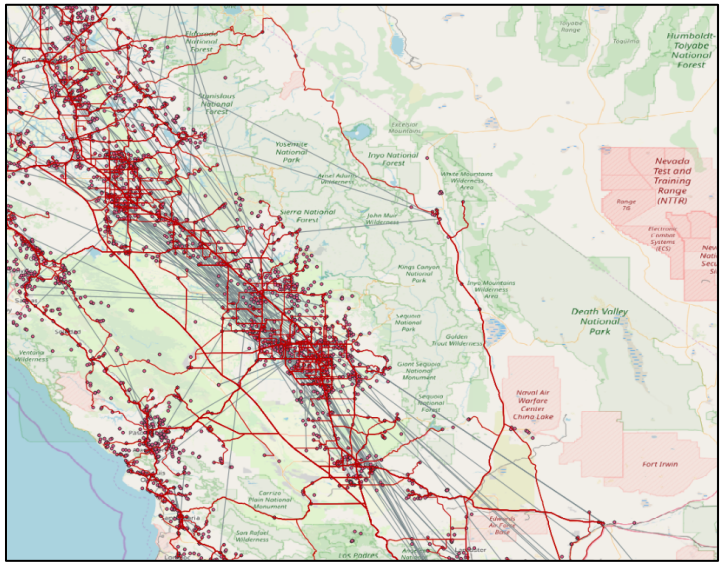
Graph theory

What is a graph? (in the context of network analysis)

"Mathematical repesentation of a network"



$$G = (V, E)$$

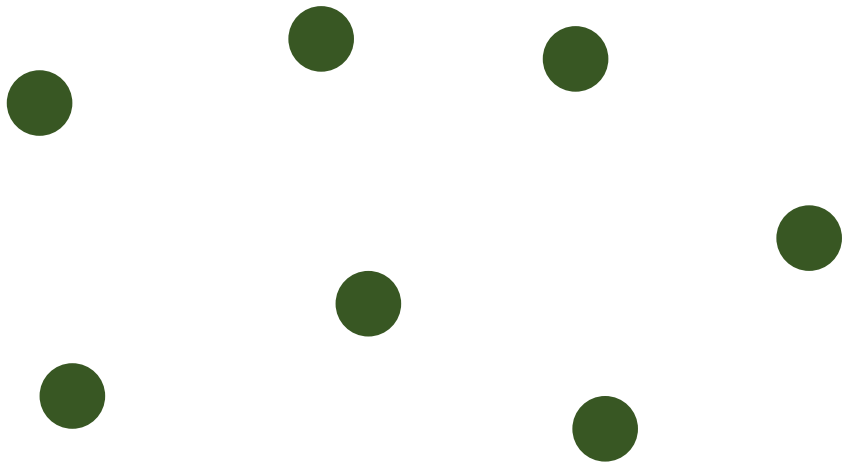
Where:

V = Vertice

E = Edge

I.B - Elements of a network

Nodes (vertices)

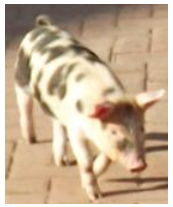


Agents or individuals forming a network

$$V = \{1, 2, 3, \dots, i\}$$

I.B - Elements of a network

Nodes (vertices)



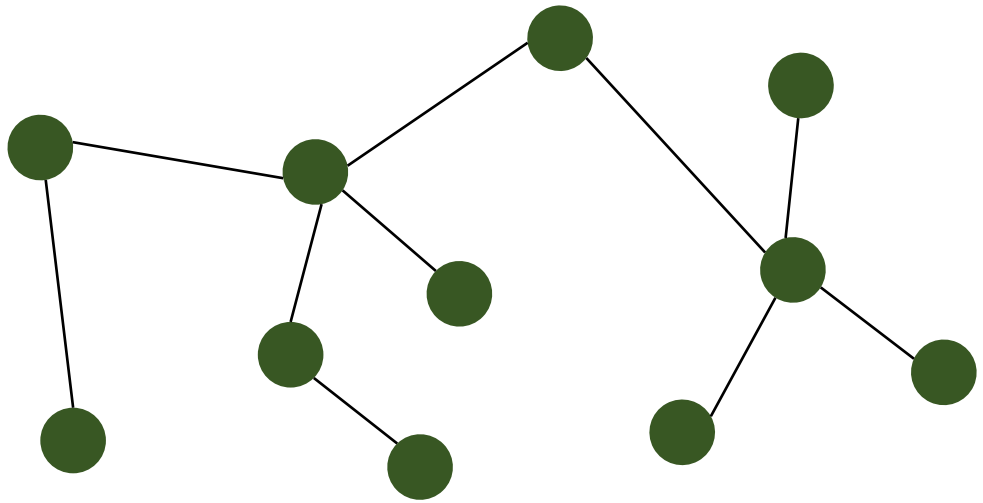


Agents or individuals forming a network:
farms, animals, humans, markets

$$V = \{1, 2, 3, \dots, i\}$$

I.B - Elements of a network

Edges (links, contacts)



Connection between a pair of nodes (dyad)

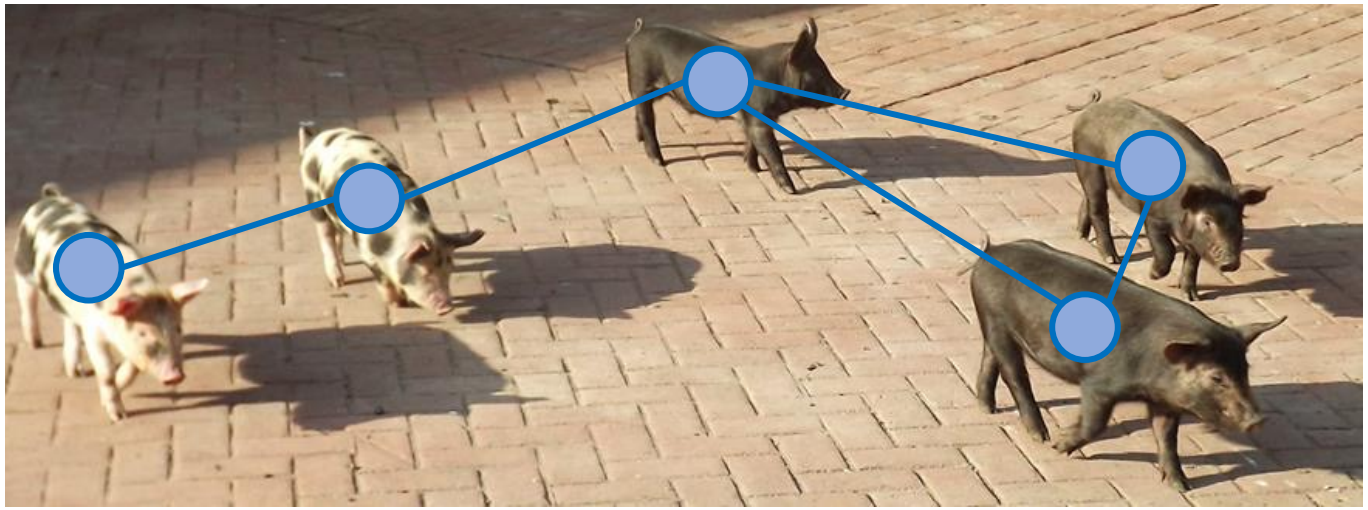
$$E = \{(1, 2), (1, 3), \dots , (i, j)\}$$

In a network, connected nodes are considered as **neighbours**

Each of the connected nodes belongs to a **neighborhood**

I.B - Elements of a network

Edges (links, contacts)



Connection between a pair of nodes (dyad):
Animal shipments, human movements, social contacts

$$E = \{(1, 2), (1, 3), \ldots, (i, j)\}$$

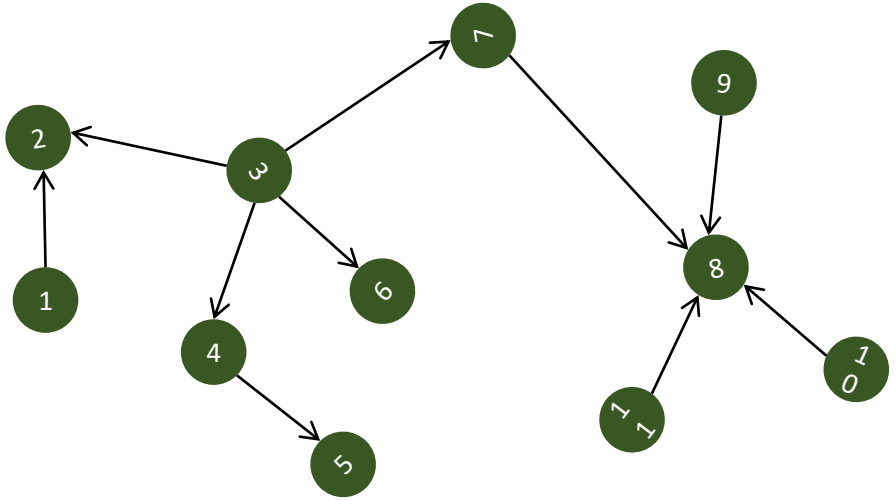
In a network, connected nodes are considered as **neighbours**

Each of the connected nodes belongs to a **neighborhood**

I.B - Elements of a network

Representation or Directionality

$E = \{(1 \rightarrow 2), (3 \rightarrow 2), \ldots, (i \rightarrow j), \ldots, (11 \rightarrow 8)\}$



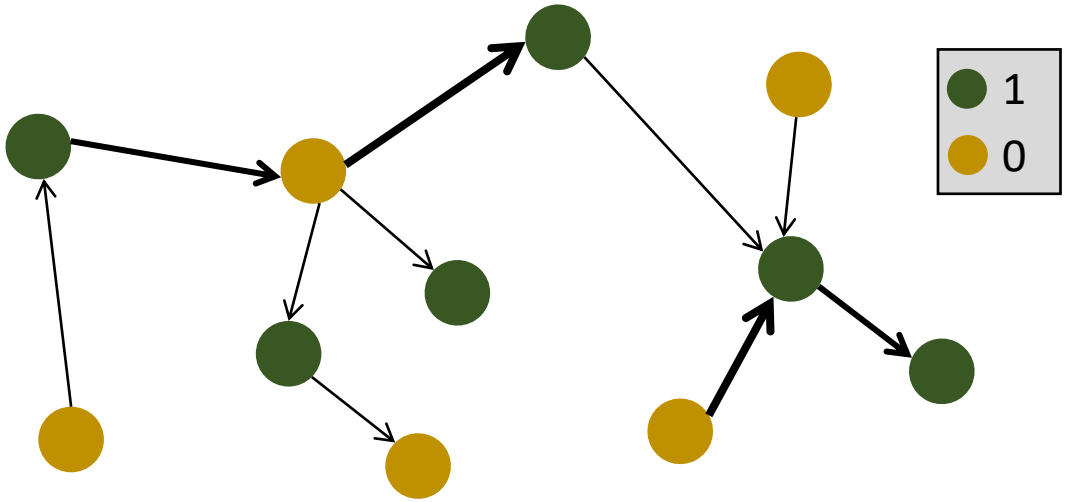
	1	2	3	….	J	…	11
1	0	1	0	…	0	…	0
2	0	0	0	…	0	…	0
3	0	1	0	…	0	…	0
….	…	…	…	…	…	…	…
I	0	0	0	…	1	…	0
…	…	…	…	…	…	…	…
11	0	0	0	…	0	…	0

I.B - Elements of a network

Atributes

Nodes: farm size, type…

Edges: movement size, cause…



$V = \{0, 1, 0, \dots, i\}$

$E = \{(1), (3), \dots, (x_i)\}$

I.C.a - Data sources

Define **nodes**:

- What is the unit of analysis (e.g. farm, animal, etc)

Define **edges**:

- Frequency of contacts
- Duration of contacts

I.C.a - Data structure

Nodes dataset			
ID	**Farm**	**Farm size**	**Farm type**
1	Swine and company	800	fattening farm
2	Les Cochonets	1200	sow farm
3	The farmhouse	50	small-scale farm
4	The Boar	20	reproductive males
5	Ham & sausage	65	slaughterhouse
6	The trading post	0	market

Edges dataset				
ID_origin	**ID_destination**	**Shipment size**	**Date**	**Reason**
2	6	10	02-25-2018	Sale of adult sows
2	1	15	03-12-2018	Piglets for fattening
4	1	1	10-15-2017	Insemination
1	5	100	06-30-2018	Slaughter
6	3	2	07-10-2018	Sale of adult sows
2	1	100	02-15-2018	Piglets for fattening

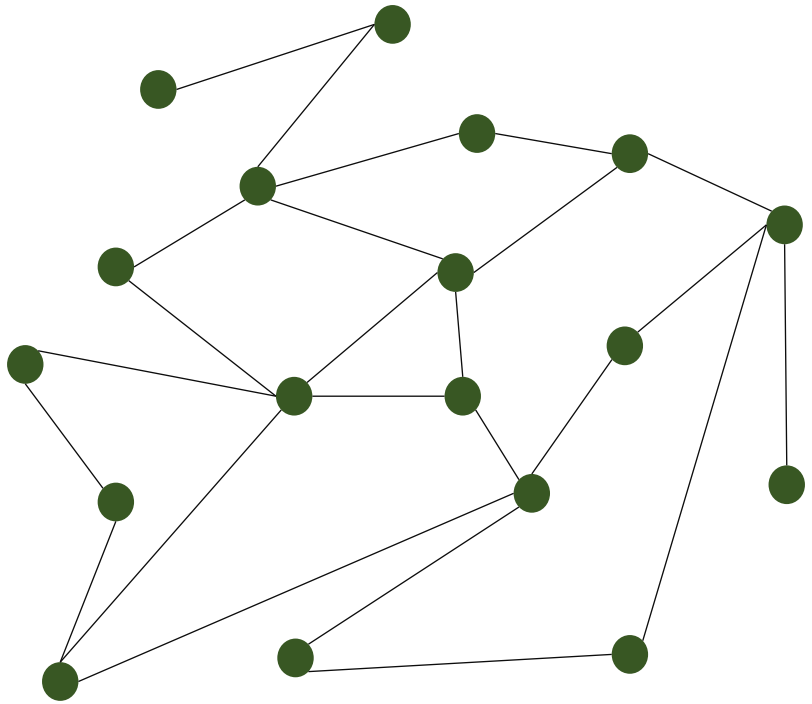
Measurement of contacts

- **Retrospective**
  - National movement records
  - Farm registry and population census
  - Surveys
- **Prospective**
  - Visual observation
  - Tracking with GPS or PIT (Passive Integrated Transporter) systems

I.C.a - Data sources

Passive Surveillance

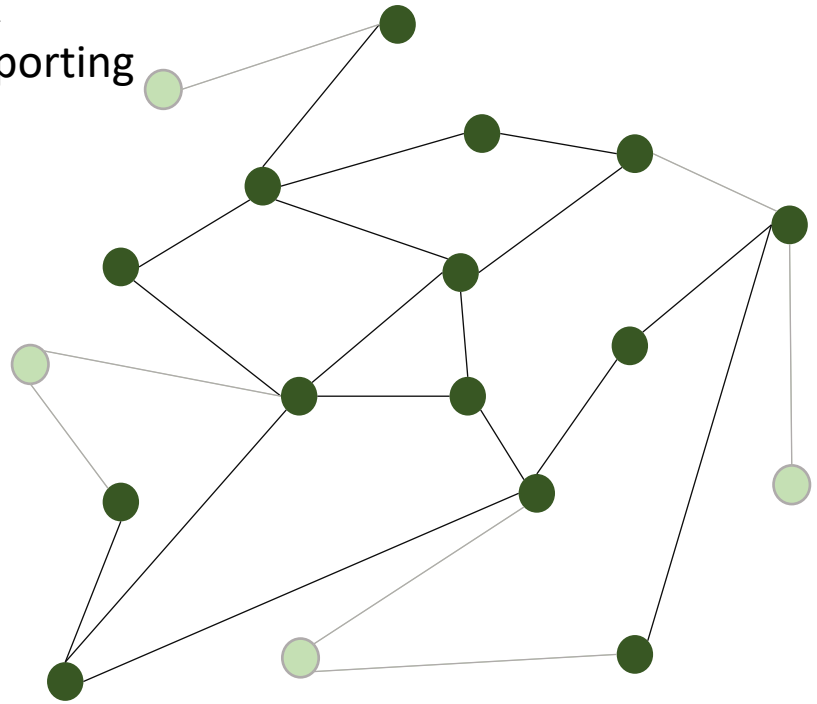
- Mandatory movement registries
- Population census
- GPS data (entire population)
  - ➢ Complete network



I.C.a - Data sources

Passive Surveillance

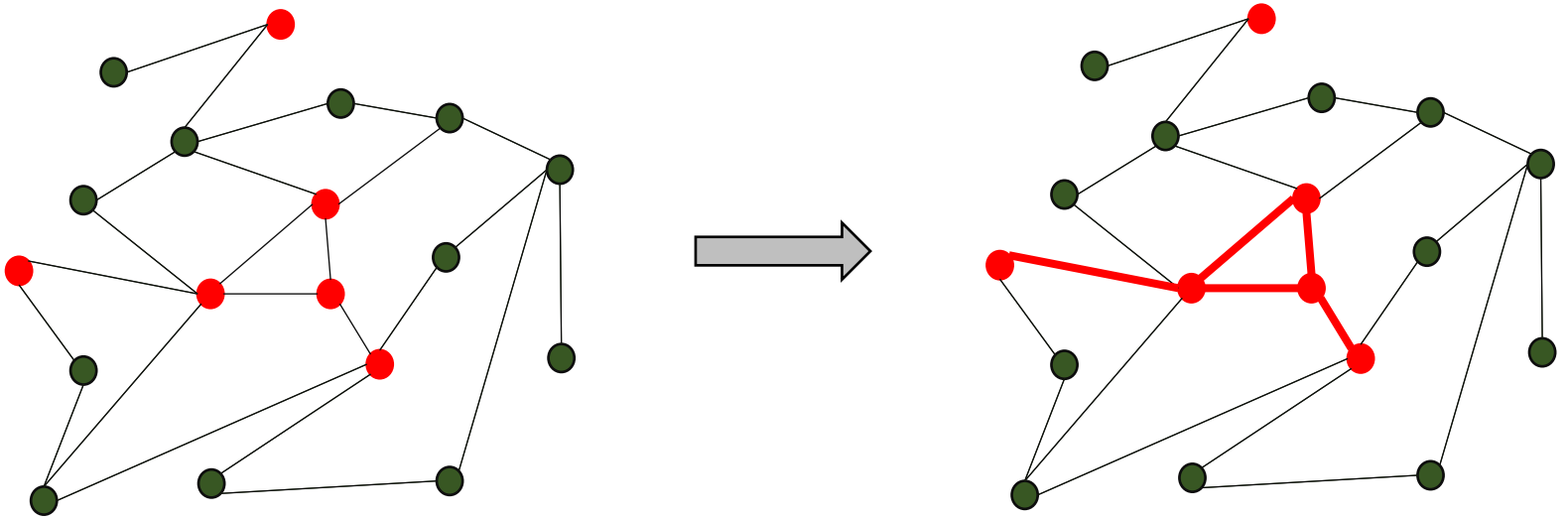
- Mandatory movement registries
- Population census
- GPS data (entire population)
  - Complete network
  - Impact of underreporting



I.C.a - Data sources

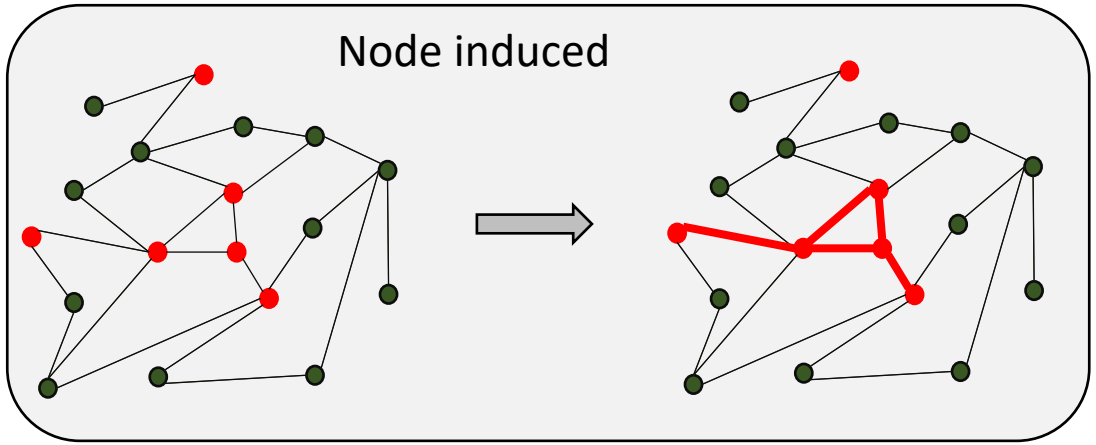
Active surveillance

- Surveys
- Observation of a sample of the population
  - ➢ Subgraph sampling and incomplete network

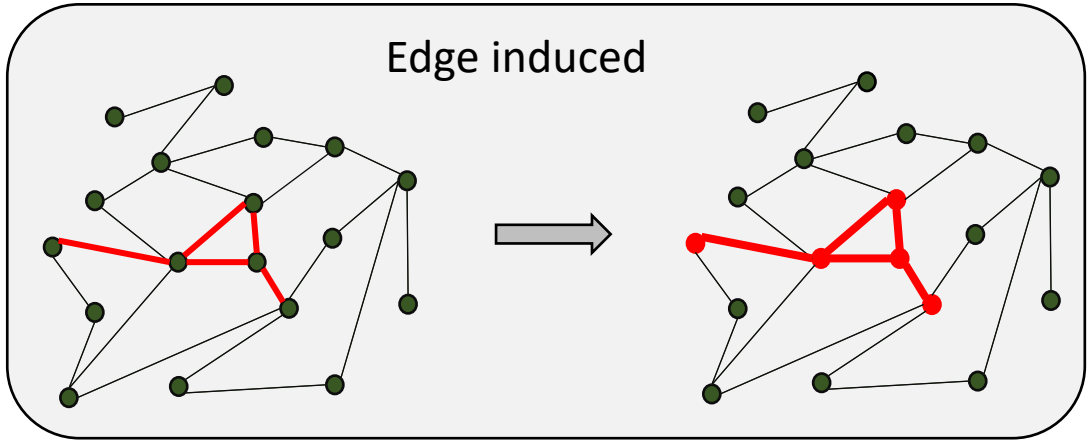


I.C.b - Sampling method

Random Sampling Methods



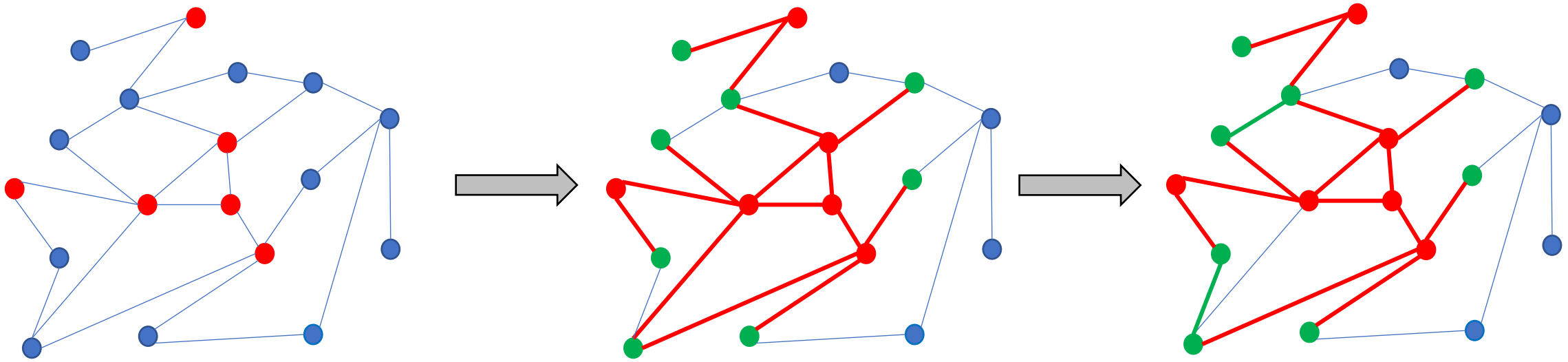
- Sample a group of nodes
- Identify all contacts between these nodes



- Sample a group of edges
- Identify all nodes connected to these edges

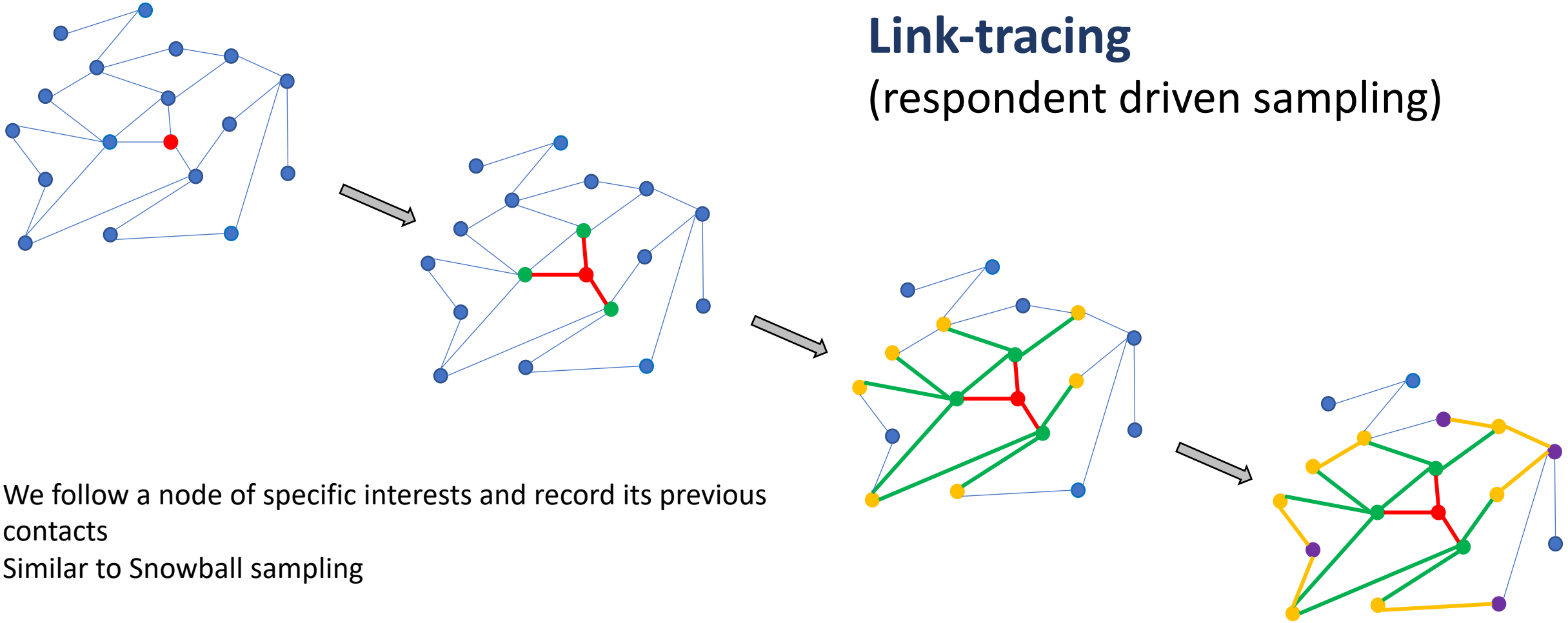
I.C.b - Sampling method

Egocentric Sampling



- Sample a group 1 of nodes
- Identify all contacts linked members of group 1
- Identify all nodes (group 2) directly connected to group 1
- Identify all contacts between members of group 2

I.C.b - Sampling method



Link-tracing
(respondent driven sampling)

We follow a node of specific interests and record its previous contacts
Similar to Snowball sampling

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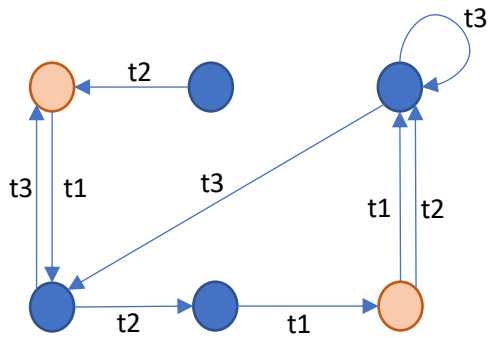
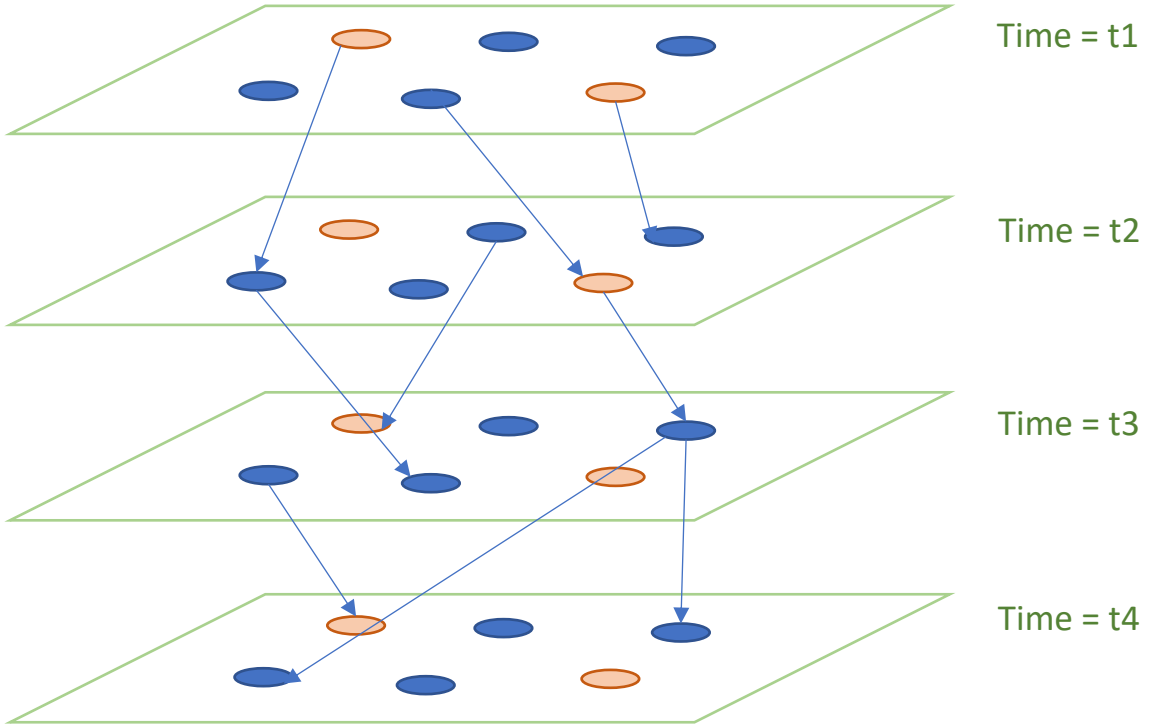
Static Network Analysis

- Network Type

Static Network



Dynamic Network

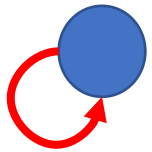


Why use a static network?

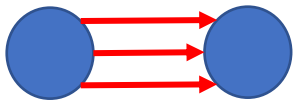
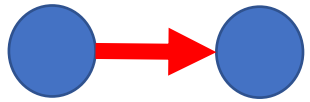
- Simplicity of use
  - If data resolution isn't high enough.
  - If there are technological (computational) limitations.
- Simplicity of interpretation
  - More generalizable for predictions.
  - Are we interested in an "average population" network?
  - Are the dynamics repeatable patterns?

II.A - Network simplification

- Loops
  - Are they important?



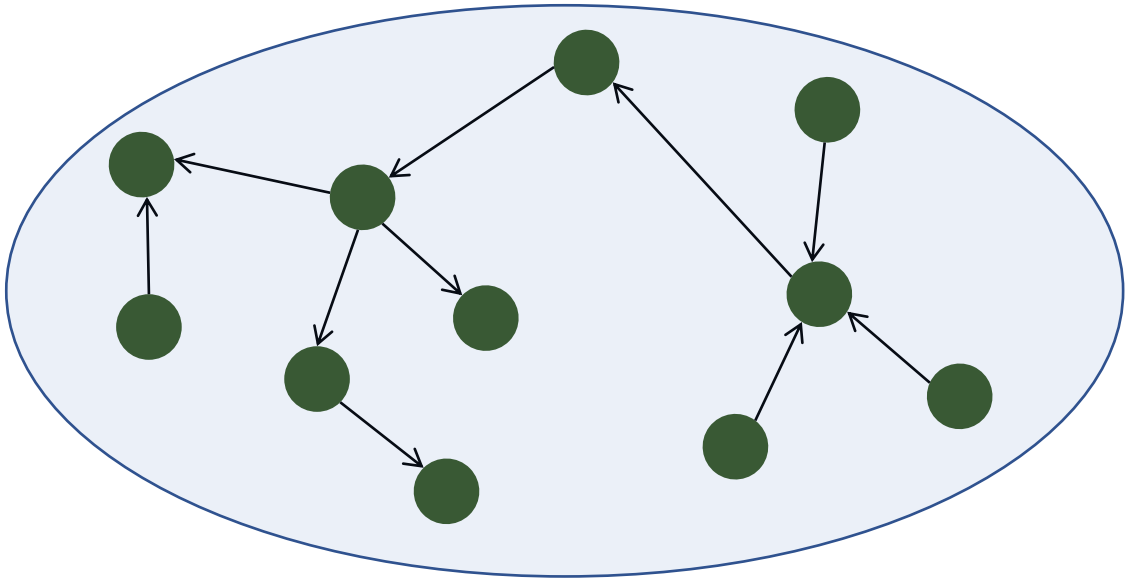
- Multiple Edges
  - Meaning? Social/Commercial relationship? Useful for modeling?

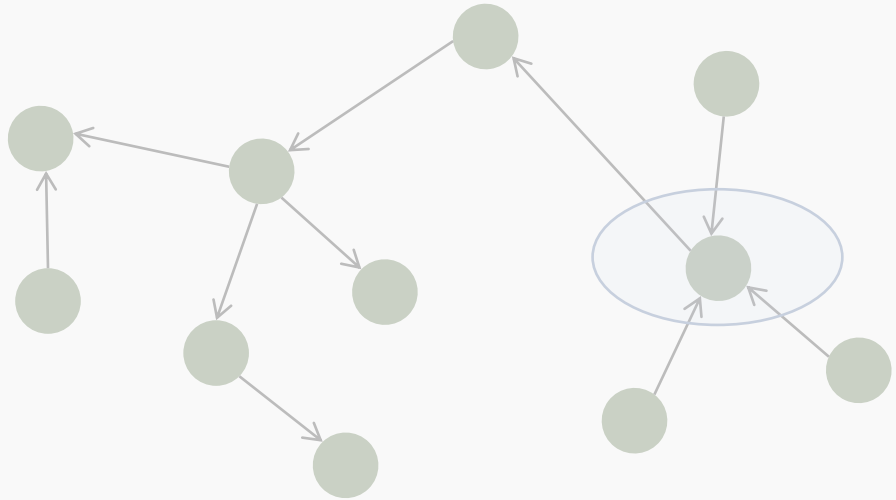

- Directed or undirected
  - Is it relevant to disease transmission in a given context: direct animal contact vs shipment


II.B - Network properties

General properties



Local properties



II.B.a - General properties

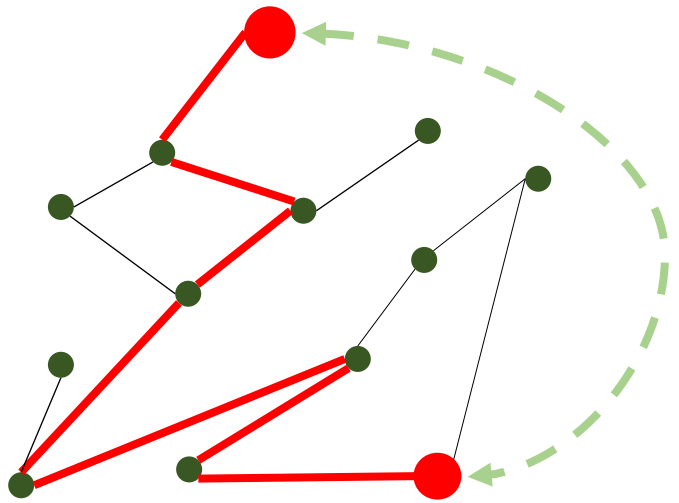
- Size
- Diameter
- Average path length
- Density
- Fragmentation
- Clustering Coefficient

II.B.a - General properties

Size: Number of nodes in the network (1 to ∞)

Diameter: The shortest path between the most distant nodes (1 to ∞)

Average path length:
Average length of shortest paths between every pair of nodes (1 to ∞)



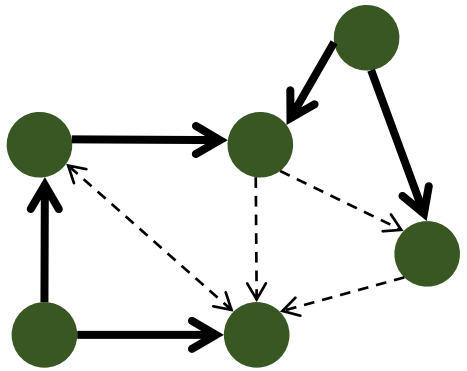
II.B.a - General properties

Density: Proportion of observed edges in network to all theoretical edges that could exist (0 to 1).

$$D = \frac{E}{N(N-1)} \qquad D = \frac{2E}{N(N-1)}$$

$D \approx 1$

F $\approx$ 0

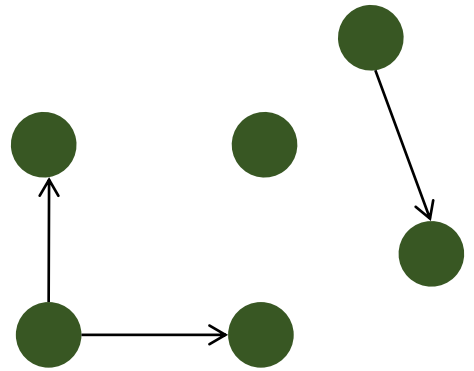


- **Fragmentation**: Proportion of pairs of nodes that are not connected to each other (0 to 1). High fragmentation means more isolated nodes

$$F = 1 - \frac{2\sum_i \sum_{j<1} r_{ij}}{N(N-1)}$$

$D \approx 0$

F $\approx$ 1



II.B.a - General properties

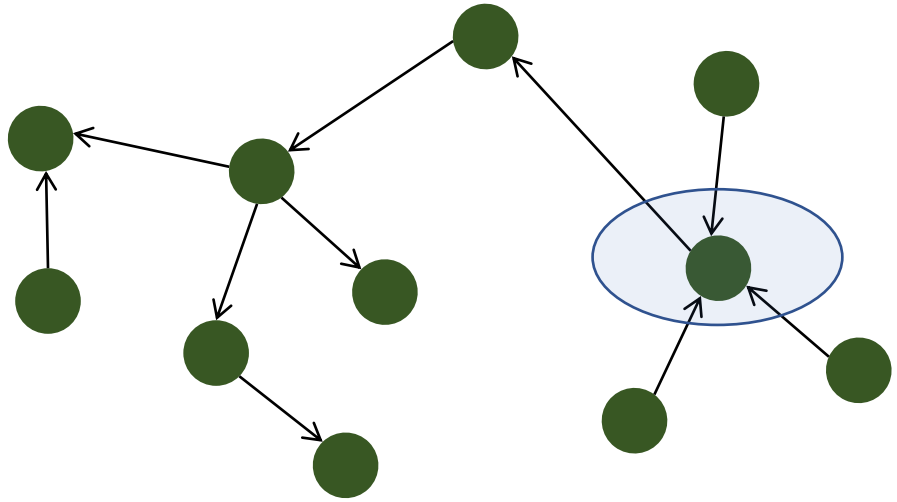
- **Clustering coefficient**: The probability of individual nodes being connected
- Values of 1 indicates that all nodes are directly connected to all others. Value of 0 indicates absence of contacts.

$$CC = \frac{1}{N}\sum_{i=1}^{N}\frac{e_{jz}}{k_i(k_i - 1)}$$

II.B - Network properties

General properties

Local properties



II.B.b - Local properties

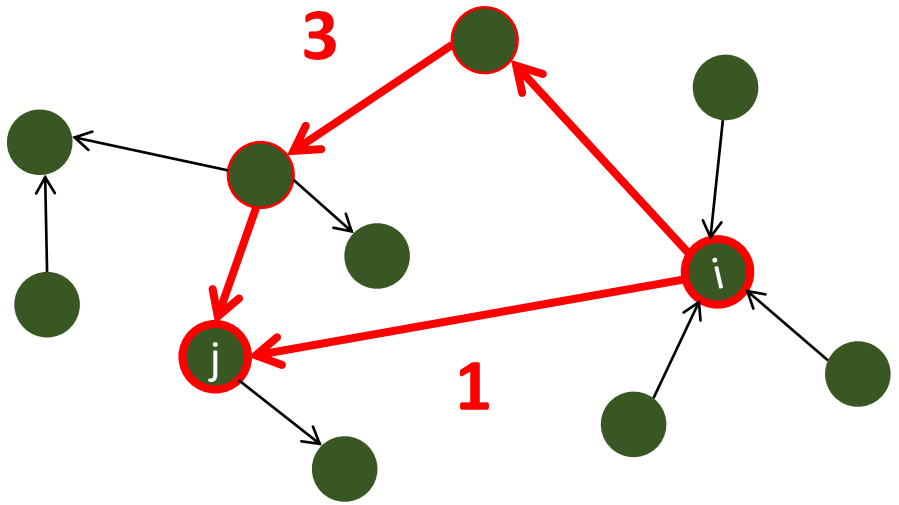
Connection measures:

- Path length
- Geodesic distance
- Farness

II.B.b - Local properties

Path length Number of steps to get from node *i* to node *j*

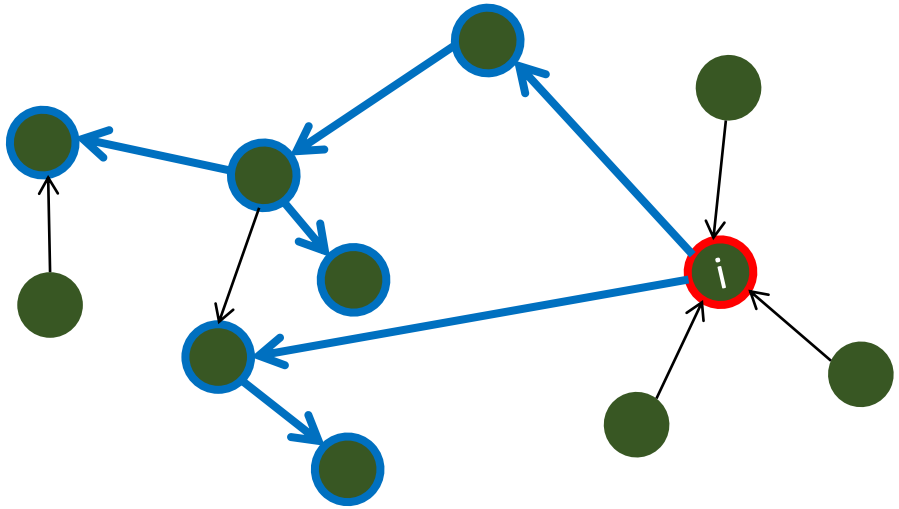
Geodesic distance The shortest path length



II.B.b - Local properties

Farness Sum of the shortest paths (in or out)

outF = 1+2+3+3+1+2 = 12



II.B.b - Local properties

Centrality measures:

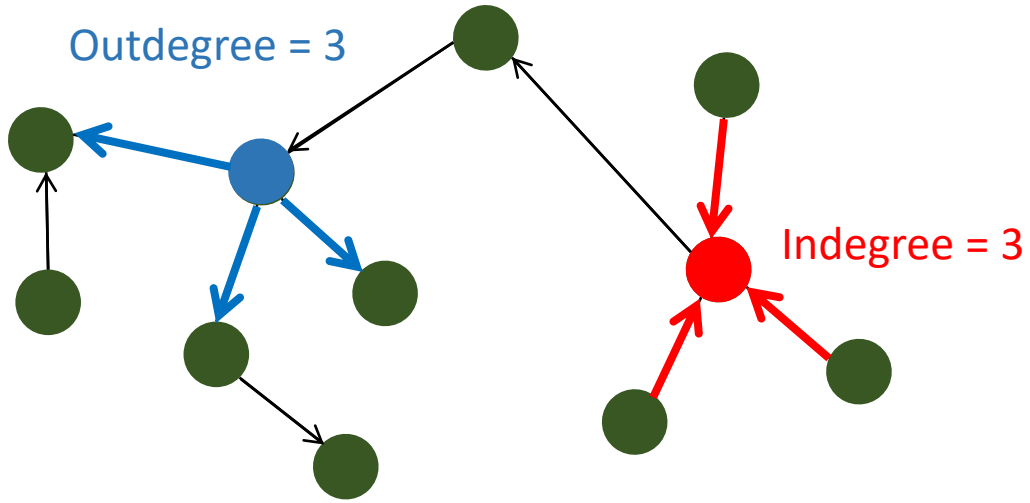
- Degree
- Betwenness
- Closeness

II.B.b - Local properties

Degree (Dc_i), Number of edges for each node.

Outdegree Do_i: Number of edges originating from node *i*.

Indegree Di_i : Number of contacts arriving at node *i*.



Relative degree centrality

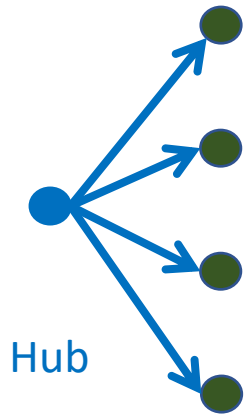
$$RDc_i = \frac{Dc_i}{N-1}$$

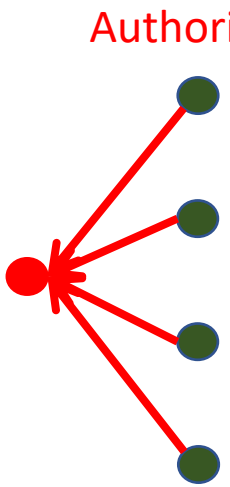
$$RDi_i = \frac{Di_i}{N-1}$$

$$RDo_i = \frac{Do_i}{N-1}$$

II.B.b - Local properties

Hubs and Authorities are nodes with high outedgree and high indegree.





Hub score

$$a_i = \sum_{(j,i)\in E} h_j$$

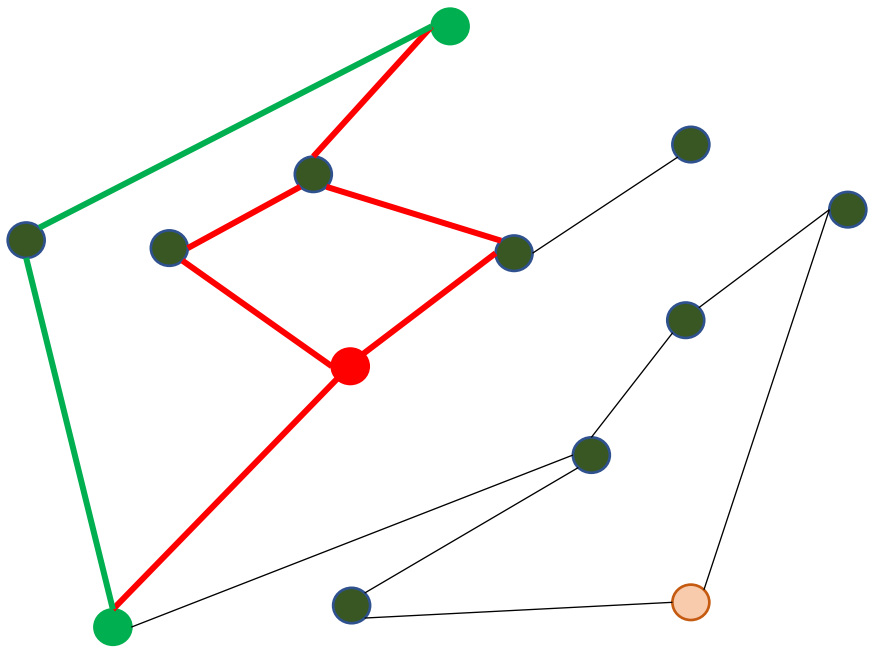
Authority score

$$h_i = \sum_{(i,j)\in E} a_j$$

II.B.b - Local properties

Betwenness (Bc_i) Estimate of the probability that the shortest path between any pair of nodes passes through node *i.*

$$Bc_i = \sum \frac{\#SP\ through\ i}{\#SP}$$



Relative betweenness centrality

For non-directed networks

$$RBc_i = \frac{Bc_i}{(N-1)\,(N-2)/2}$$

For directed networks

$$RBc_i = \frac{Bc_i}{(N-1)(N-2)}$$

II.B.b - Local properties

Closeness (Cc_i) Inverse of sum shortest paths between node *i* and all other nodes. Describes how close node *i* is to all other nodes.

Non-directed networks:

$$Cc_i = \frac{1}{\sum_{j=1}^{N} d_i(n_j, n_i)}$$

Relative closeness centrality

$$RCc_i = (N-1) \times Cc_i$$

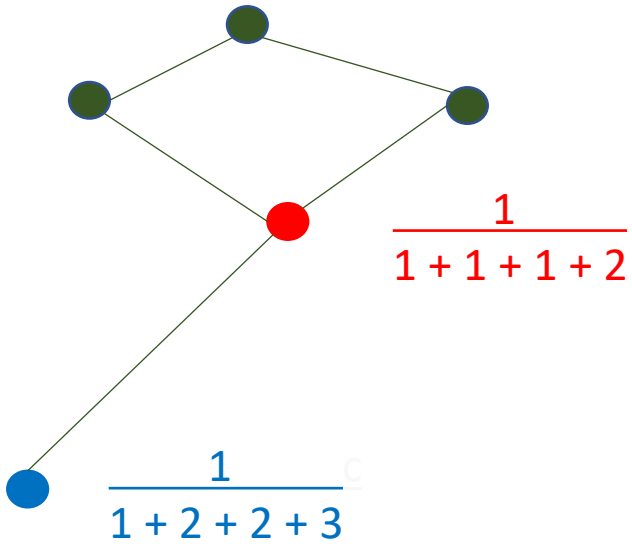
Directed networks:

In-closeness

$$Ci_i = \frac{1}{\sum_{j=1}^{N} d_i(n_j, n_i)}$$

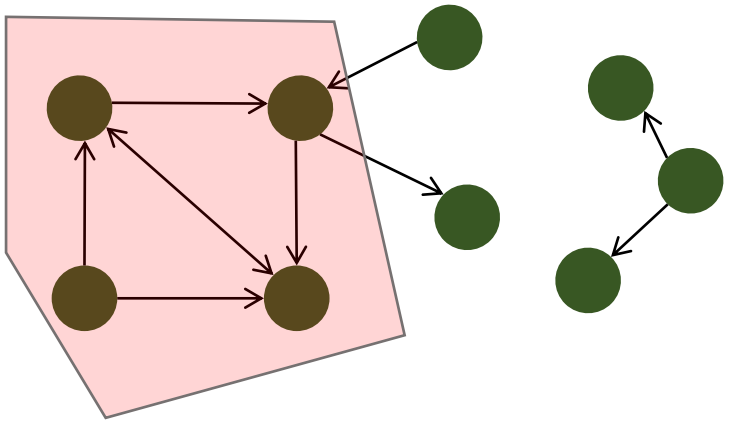
Out-closeness

$$Co_i = \frac{1}{\sum_{j=1}^{N} d_o(n_j, n_i)}$$



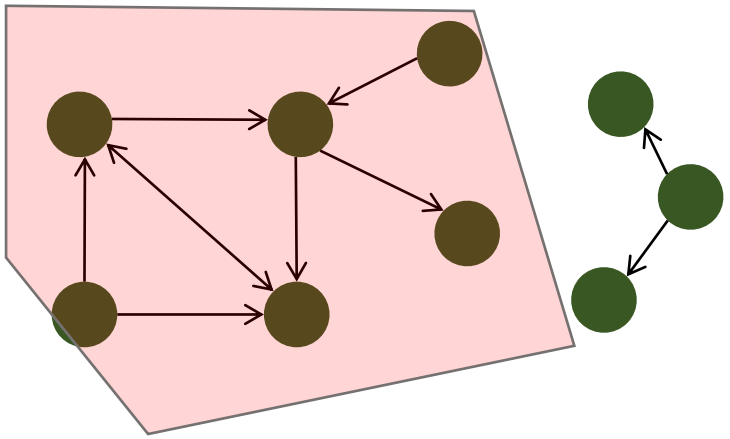
II.C – Components and communities

Strong Component



A subset of the network in which any pair of nodes *i* and *j* can can directly reach each other

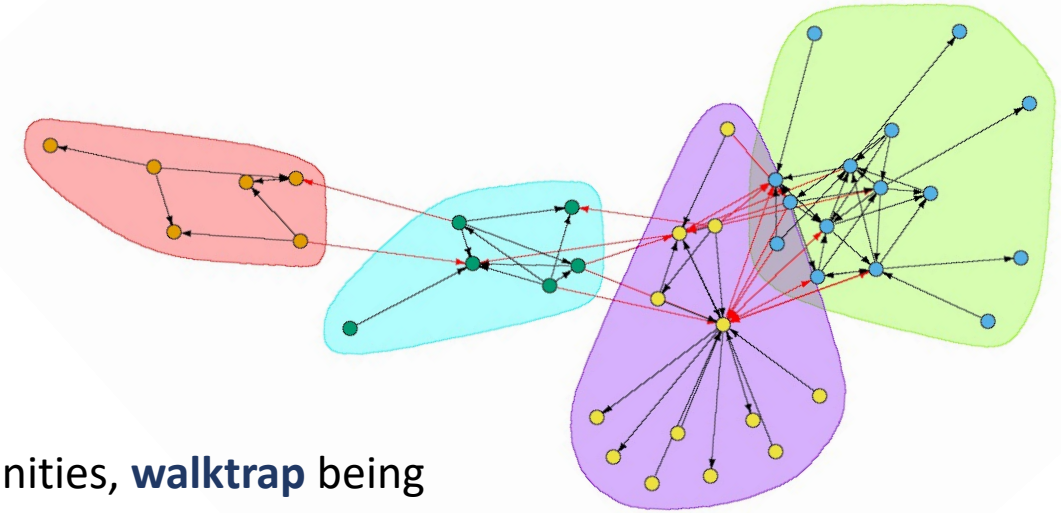
Weak Component



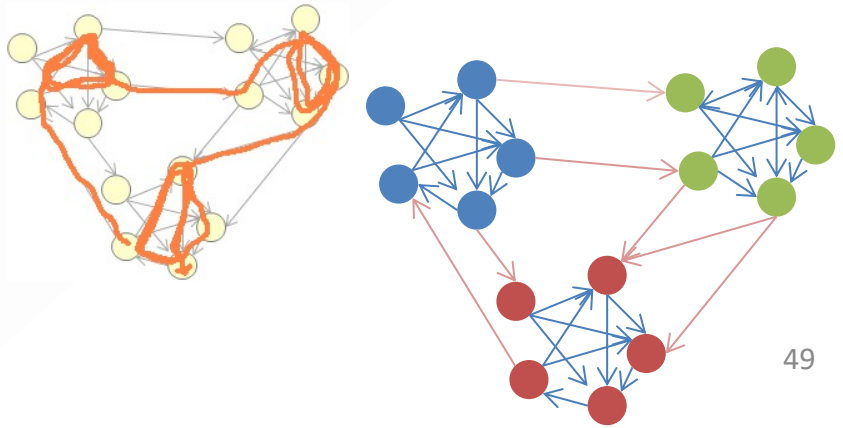
A subset of the network in which any pair of nodes *i* and *j* can can indirectly reach each other

II.C - Components and communities

Communities: Many intra-community contacts and few inter-community contacts

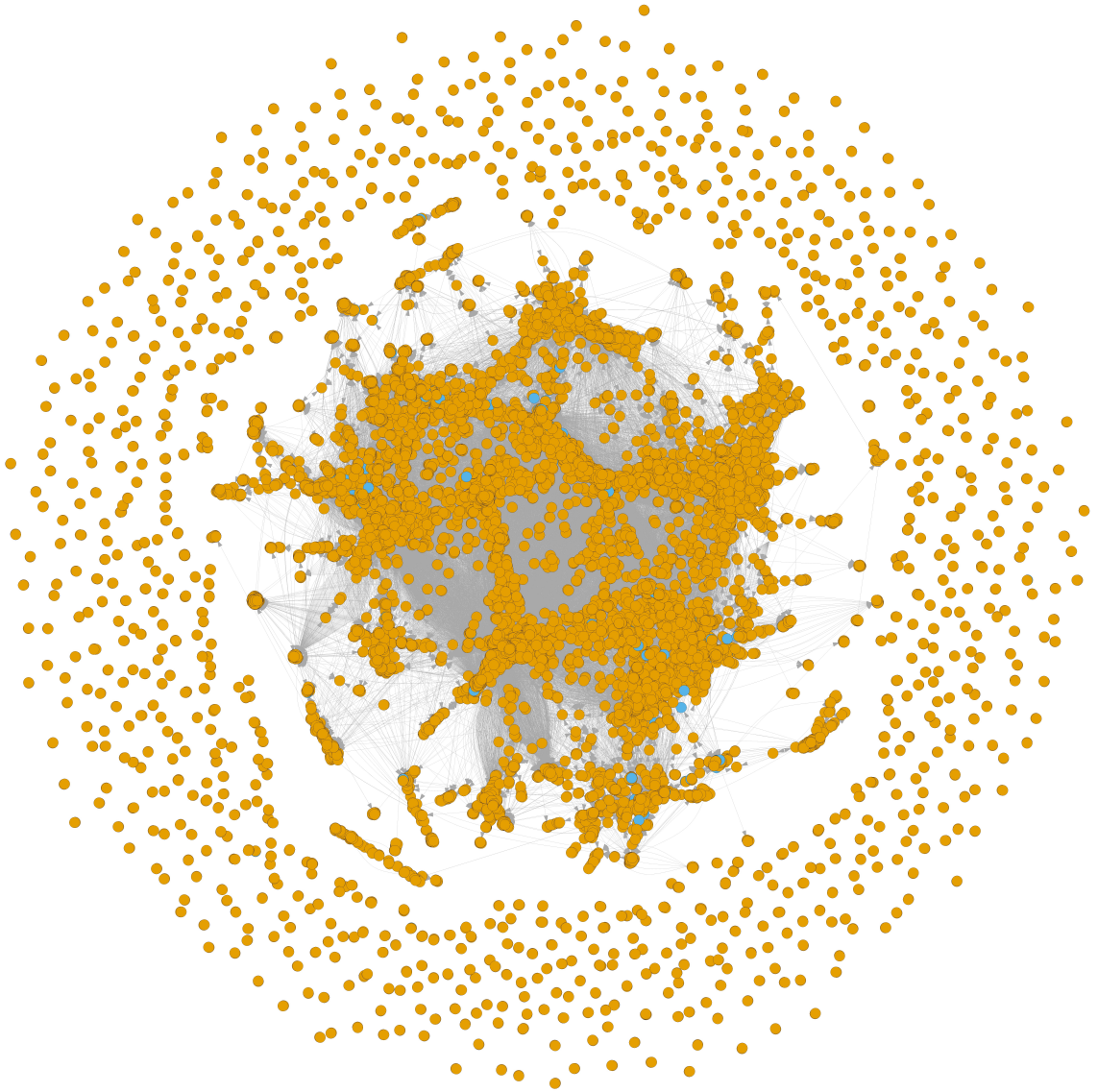


There are many algorithms to detect communities, **walktrap** being one of the most popular.
I generates random paths between nodes to detect zones where it circulates more frequently within a group of nodes.



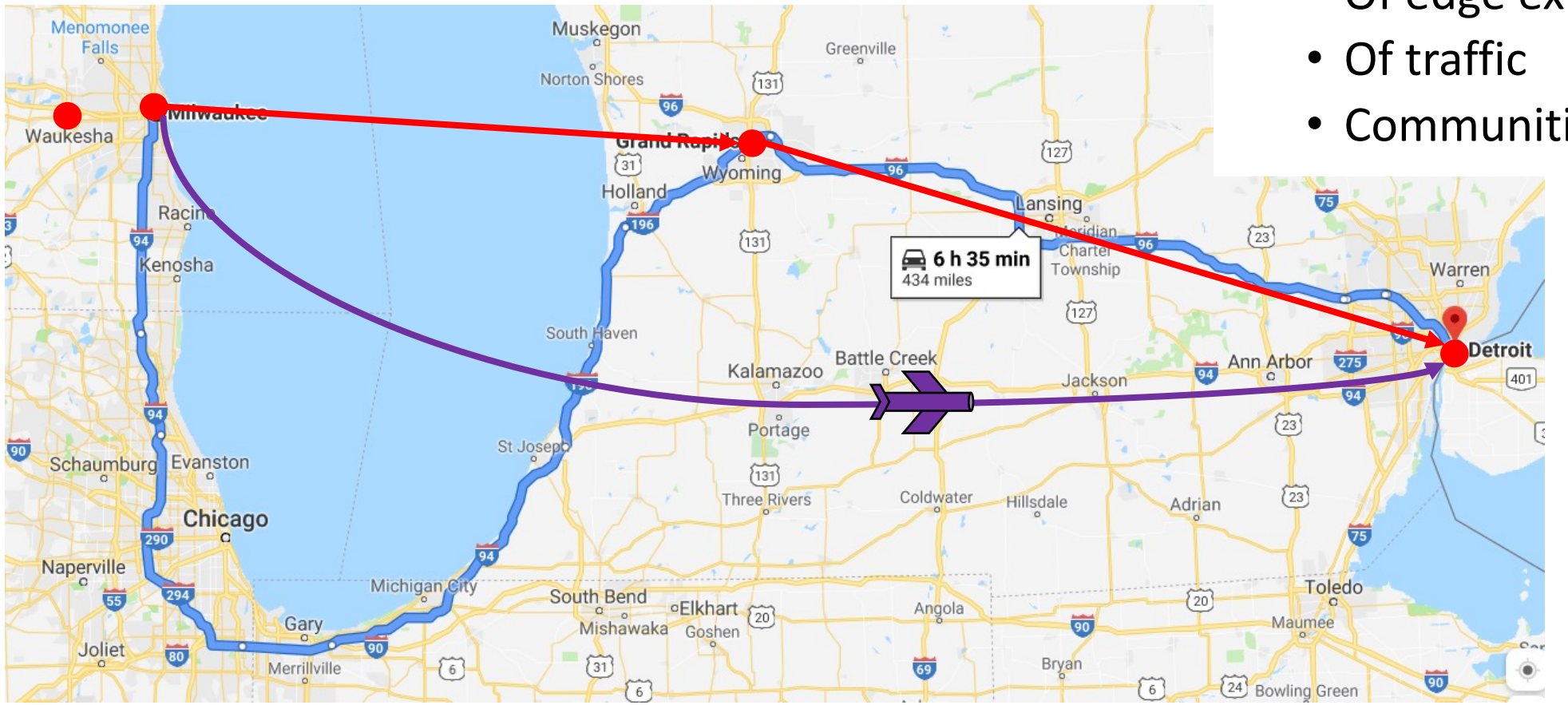
Limitations of static network analysis and graph

- Network size and legibility
- No visible timeline in a static network
- Doesn’t take into account spatial characteristics
  - Location based on network algorithm



Spatial considerations

- Mean distance
  - Euclidian vs real distance?
  - In km or travel time?



- Spatial clustering
  - Of nodes
  - Of node attributes
  - Of edge extremities
  - Of traffic
  - Communities

Questions?

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