# Signcryption with Proxy Re-encryption

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#### Abstract

Confidentiality and authenticity are two of the most fundamental problems in cryptography. Many applications require both confidentiality and authenticity, and hence an efficient way to get both together was very desirable. In 1997, Zheng proposed the notion of "signcryption", a single primitive which provides both confidentiality and authenticity in a way that's more efficient than signing and encrypting separately. Proxy re-encryption is a primitive that allows a semi-trusted entity called the "proxy" to convert ciphertexts addressed to a "delegator" to those that can be decrypted by a "delegatee", by using some special information given by the delegator, called the "rekey". In this work, we propose the notion of signcryption with proxy re-encryption (SCPRE), and motivate the same. We define security models for SCPRE, and also propose a concrete unidirectional, non-interactive identity-based SCPRE construction. We also provide complete proofs of security for the scheme in the security models defined. We finally provide directions for further research in this area.

#### 1 Introduction

Cryptography has found various applications in email, e-commerce, secure web protocols, etc. Most of these applications demand both confidentiality and authenticity to be provided. So an efficient way of providing both together was desired. It was achieved in 1997, when Zheng[4] proposed a primitive called signcryption, which provides confidentiality and authenticity in a way that's more efficient than combining any encryption and signature schemes. Later several signcryption schemes have been published. The first

identity-based signcryption scheme was by Malone-Lee[5] in 2002. Later in 2003, Boyen[6] proposed the first provably secure id-based scheme. In 2005, a more efficient scheme was provided by Chen and Malone-Lee[7]. There have also been other signcryption schemes, and some are variants of signcryption, like those in [8].

Proxy re-encryption is a concept introduced by Blaze et al[9] in 1998, that allows a semi-trusted entity called the "proxy" to convert ciphertexts addressed to an entity B called the "delegator", to another entity C called the "delegatee", while maintaining that the proxy cannot learn anything about the underlying plaintext, and C cannot learn anything about the underlying plaintext without co-operation from the proxy. B does this delegation by providing a special piece of information, called the "rekey", to the proxy. Proxy re-encryption has found various applications like secure email forwarding, etc. But the scheme by Blaze et al [9] was inherently bidirectional - allowed only two-way delegations. Also collusion between proxy and the delegator or delegatee compromised the other entity's secret key. These problems were first fully addressed by Ateniese et al [11]. Later, several proxy re-encryption schemes were proposed, including the first CCA-secure scheme by Canetti et al [12], the first identity-based scheme by Matsuo [13], and the first CCA-secure id-based scheme by Green and Ateniese [14], and the first RCCA-secure and collusion resistant scheme secure in the standard model by Libert and Vergnaud [16].

## 1.1 Signcryption with Proxy Re-encryption (SCPRE)

We introduce the notion of signcryption with proxy re-encryption (SCPRE). Thus an SCPRE scheme provides confidentiality, authenticity, and proxy re-encryption capabilities in a very efficient way. Such a primitive can have several applications, including:

- Authentic email forwarding: Email is an ideal candidate for applying signcryption. A natural application of SCPRE here is to allow sign-crypted messages (emails) to be redirected to a delegated person when the actual receiver is unavailable.
- Secure and authentic distributed storage: One of the well-known applications of proxy re-encryption[11] can be extended using SCPRE, when the authenticity of the content stored is desirable.

## 1.2 Contributions of this work

Our contributions presented in this work, can be summarized as:

- Introduction of the notion of signcryption with proxy re-encryption (SCPRE)
- Definition of security models for the confidentiality and non-repudiation of SCPRE, which are slightly weaker than a natural adaptation of the chosen ciphertext attack (CCA2) and the chosen message attack (CMA) models, respectively.
- A concrete construction of an efficient unidirectional, non-interactive, identity-based SCPRE are provided. The scheme is non-transitive and key optimal.
- The confidentiality and non-repudiation of the scheme are formally proved, in the security models defined.

## 2 The Scheme

#### 2.1 Definition of SCPRE

An SCPRE scheme consists of the following algorithms:

- 1. Setup: The algorithm accepts a security parameter l, and outputs a master secret key s.
- 2. Extract: The algorithm accepts an identity  $ID_u$  and outputs the secret key  $S_u$
- 3. Extract-rekey: It accepts two  $ID_1$  and  $ID_2$  and outputs the rekey from  $ID_1$  to  $ID_2$ .
- 4. Signcrypt: It accepts message m, and two identities  $ID_1$  and  $ID_2$  and outputs the signcryption on m from  $ID_1$  to  $ID_2$ .
- 5. De-signcrypt: It accepts a signcryption  $\phi$  and an identity  $ID_r$  and outputs the de-signcryption of  $\phi$  by  $ID_r$ .
- 6. Re-encrypt: It accepts a signcryption  $\phi$ , and an identity  $ID_d$  and outputs the re-encrypted (second level) signcryption  $\phi'$  of  $\phi$  to  $ID_d$ .

7. De-re-encrypt: It accepts a second-level signcryption  $\phi'$  and  $ID_d$  and outputs the de-signcryption of  $\phi'$  by  $ID_d$ .

#### 2.2The Scheme

Our SCPRE scheme is derived from the identity-based signcryption scheme proposed by Chen et al[7]. Our scheme is presented below.

## Setup

Let l be the security parameter of the system. Let  $\mathbb{G}_1$  and  $\mathbb{G}_2$  be two prime ordered groups of order  $q = \Theta(2^l)$ , where  $\mathbb{G}_1$  is represented additively, and  $\mathbb{G}_2$  is represented multiplicatively. Let P be a generator of  $\mathbb{G}_1$ . Let  $e: \mathbb{G}_1 \times \mathbb{G}_1 \to \mathbb{G}_2$  be a bilinear pairing. We assume that the BCDH assumption holds in  $\langle e, \mathbb{G}_1, \mathbb{G}_2 \rangle$ .

We use four hash functions  $H_0, H_1, H_2$ , and  $H_3$  in our scheme, where

 $H_0: \{0,1\}^* \to \mathbb{G}_1,$ 

 $H_1: \mathbb{G}_1 \times \{0,1\}^n \to \mathbb{Z}_q^*,$   $H_2: \mathbb{G}_2 \to \{0,1\}^{n+t}$ 

 $H_3: \mathbb{G}_1 \times \{0,1\}^* \to \mathbb{G}_1.$ 

Here n is the number of bits in the message, and t is the number of bits used to represent an element in  $\mathbb{G}_1$ .

The private key generator (PKG) chooses the master secret key  $s \in_R \mathbb{Z}_q^*$ and sets the master public key  $P_{pub} = sP$ .

The public parameters published are param =  $< e, \mathbb{G}_1, \mathbb{G}_2, n, q, P, P_{vub}, H_0, H_1, H_2 >$ 

Each user u has his identity  $ID_u$  as his public key. He gets two secret keys  $S_u$  and  $S_{u||delegatee}$  by providing  $ID_u$  and  $ID_u||$  "delegatee" to the following Extract algorithm, respectively.

## $\mathsf{Extract}(ID_u)$

The PKG computes the secret key as  $S_u = s \cdot H_0(ID_u)$ , where  $H_0(ID_u)$  is generally denoted as  $Q_u$ 

#### $Signcrypt(m, S_A, ID_B)$

To encrypt a message m to B, user A does the following steps.

- 1. Choose  $r \in_R \mathbb{Z}_q^*$
- 2. Compute  $X = rQ_A$  and  $h = H_1(X||m)$
- 3. Compute the signature  $Z = (r+h)S_A$
- 4. Choose  $k \in_R \mathbb{G}_2$
- 5. Compute  $w = e(S_A, Q_B)^r$  and set  $\lambda = w \cdot k$
- 6.  $y = H_2(k) \oplus (m||Z)$
- 7. The signcryption is  $\phi = \langle X, y, \lambda, ID_A \rangle$ .

 $\mathsf{De}\text{-}\mathsf{signcrypt}(\phi=< X, y, \lambda, ID_A>, S_B\ )$ 

The receiver B, upon receiving the signcryption  $\phi$ , does the following.

- 1.  $w = e(X, S_B)$
- 2. Compute  $k = \lambda \cdot w^{-1}$
- 3. Recover  $m||Z = y \oplus H_2(k)$
- 4.  $h_1 = H_1(X||m)$
- 5. If  $e(Z, P) = e(P_{pub}, X + h_1Q_A)$  then  $< m, (X, Z), ID_A >$  is output as the message and signature. Otherwise,  $\perp$  is output.

Rekey-Extract $(S_B, ID_C)$ 

B sends  $rk_{B\to C} = \langle -S_B + H_3(e(S_B, Q_{C||delegatee})) \rangle$  to proxy.

Re-encrypt(  $\phi = \langle X, y, \lambda, ID_A \rangle$ ,  $rk_{B \to C}, ID_B, ID_C \rangle$ )

Proxy computes re-encrypted signcryption  $\phi' = \langle X, y, \lambda \cdot e(X, rk_{B \to C}), ID_A, ID_B \rangle$  and sends  $\phi'$  to C.

De-re-encrypt( $\phi' = \langle X, y, \lambda', ID_A, ID_B \rangle, S_{C||delegatee}$ )

On receipt of a level 2 signcryption, C decrypts using the following algorithm:

- 1.  $w = e(X, H_3(e(Q_B, S_{C||delegatee})))$
- 2. Compute  $k = \lambda' \cdot w^{-1}$
- 3. Recover  $m||Z = y \oplus H_2(k)|$
- 4.  $h_1 = H_1(X||m)$
- 5. If  $e(Z, P) = e(P_{pub}, X + h_1Q_A)$  then output  $\langle m, (X, Z), ID_A \rangle$ ; else output  $\perp$ .

#### 2.3 Correctness

We now show the correctness of our scheme. Let  $\langle X, y, \lambda', ID_A, ID_B \rangle$  be a second-level signcryption addressed to C.

$$y \oplus H_2(\lambda' \cdot e(X, H_3(e(Q_B, S_{C||delegatee})))^{-1}) = y \oplus H_2(k \cdot e(X, H_3(e(S_B, Q_{C||delegatee}))) \cdot e(X, H_3(e(S_B, Q_{C||delegatee})))^{-1})$$

$$= y \oplus H_2(k)$$

$$= m||Z$$

This shows that the signature  $\langle m, (X, Z), ID_A \rangle$  is recovered correctly, which implies the correctness of the scheme (due to correctness of the underlying signature [3]).

# 3 Security Models

We define the security models for confidentiality and non-repudiation. It can be seen that for most applications unforgeability of signcryptions is not necessary and non-repudiation would suffice.

## 3.1 Confidentiality

The security model we define is slightly weaker than the natural adaptation of the adaptive chosen ciphertext attack model, called the IND-SCPRE-CCA2. It is a given-ID attack model.

At the start of the game, the adversary is given a challenge identity  $ID_{B^*}$ , which is the identity of the user whose confidentiality the adversary is challenged to break.

#### Phase 1:

In this phase, the adversary is given access to the following oracles:

- Random oracles  $H_0, H_1, H_2, H_3$
- Extract (ID): returns the secret key of the identity ID
- Extract-rekey  $(ID_1, ID_2)$ : returns the rekey from  $ID_1$  to  $ID_2$
- Signcrypt $(m,ID_1,ID_2)$ : returns a signcryption  $\phi$ , on message m from  $ID_1$  to  $ID_2$
- De-signcrypt  $(\phi, ID_2)$ : if  $\phi$  is a valid (first level) signcryption addressed to identity  $ID_2$ , it returns the corresponding message and signature  $\langle m, (X, Z), ID_1 \rangle$ ; else it returns  $\bot$ , indicating rejection.
- Re-encrypt $(\phi, ID_1, ID_2)$ : returns the signcryption  $\phi'$ , which is the reencryption of the signcryption  $\phi$  from  $ID_1$  to  $ID_2$
- De-re-encrypt( $\phi'$ ,  $ID_2$ ): if  $\phi'$  is a valid second level signcryption addressed to  $ID_2$ , it returns the corresponding message and signature m, (X, Z),  $ID_1 >$ ; else it returns  $\bot$ , indicating rejection.

The adversary can access the above oracles with the following restrictions:

- It is not allowed to query  $Extract(ID_{B^*})$
- It is not allowed to query both Extract-rekey $(ID_{B^*}, ID_3)$  and Extract $(ID_3)$  | "delegatee"), for any  $ID_3$
- It is not allowed to query both Re-encrypt( $\phi'$ ,  $ID_{B^*}$ ,  $ID_3$ ) and Extract( $ID_3$  ||"delegatee"), for any  $\phi'$ ,  $ID_3$

## Challenge Phase

At the end of phase 1, the adversary gives two messages  $m_0$  and  $m_1$  and a sender identity  $ID_A$  to the challenger. The challenger now chooses a random bit  $b \in_R \{0,1\}$  and outputs the challenge signcryption as  $\phi^* = \text{Signcrypt}(m_b, S_A, ID_{B^*})$ .

### Phase 2:

As in phase 1, the adversary can make polynomial number of queries to the oracles with the following additional restrictions:

- It is not allowed to query De-signcrypt( $\phi^*$ ,  $ID_{B^*}$ )
- It is not allowed to query De-re-encrypt(Re-encrypt( $\phi^*$ ,  $ID_{B^*}$ ,  $ID_3$ ),  $ID_3$ ) for any  $ID_3$
- It is not allowed to query  $rk_{B^*\to C} = \text{Extract-rekey } (ID_{B^*}, ID_C)$  and De-re-encrypt $(\phi^{*'}, ID_C)$ , where  $\phi^{*'} = \text{Re} \text{encrypt}(\phi^*, rk_{B^*\to C}, ID_{B^*}, ID_C)$ .

Finally, the adversary outputs its guess b'. The adversary wins the game iff b = b'. We say that the system is IND-SCPRE-CCA2 secure if  $Pr[b = b'] \ge \frac{1}{2} + \epsilon$  where  $\epsilon$  is negligible in the security parameter l. The value  $\epsilon$  is called the advantage of the adversary. So, the system is IND-SCPRE-CCA2 secure if any PPT adversary has only negligible advantage in this game.

## 3.2 Non-repudiation

The security model we define is slightly weaker than the natural adaptation of the adaptive chosen message attack model (CMA), called the EUF-SCPRE-CMA. It is a given-ID attack model.

At the start of the game, the adversary is given a challenge identity  $ID_{A*}$ . The adversary can query the oracles specified in the confidentiality game polynomial number of times with the following restrictions:

- It is not allowed to query  $Extract(ID_{A^*})$
- It cannot query both Extract-rekey  $(ID_{A^*}, ID_3)$  and Extract $(ID_3||$ "delegatee"), for any  $ID_3$ .

• It cannot query both Re-encrypt( $\phi'$ ,  $ID_{A^*}$ ,  $ID_3$ ) and Extract( $ID_3$ ||"delegatee") for any  $\phi'$ ,  $ID_3$ 

Finally, the adversary produces a signature  $\langle m, (X, Z), ID_{A^*} \rangle$  such that Signcrypt $(m, ID_{A^*}, ID_3)$  was never queried, for any  $ID_3$ . It succeeds if the signature is valid. So, the system is EUF-SCPRE-CMA secure if the probability that any PPT adversary succeeds in the above game is negligible.

## 4 Security Results

The following are the security results for our SCPRE scheme:

## 4.1 Confidentiality

**Theorem 1** If there exists a probabilistic polynomial time adversary that wins in the IND-SCPRE-CCA2 game with advantage  $\epsilon$ , then there exists a probabilistic polynomial time machine that solves the BCDH problem with probability

$$\geq \epsilon \cdot \frac{1}{q_2} \left( 1 - \left( q_s \left( \frac{q_s + q_1}{q} \right) + \sum_{i=1}^3 \alpha_i(l) \right) \right)$$

where  $q_1$ ,  $q_2$  and  $q_s$  are the number of queries made by the adversary to  $H_1$ ,  $H_2$  and Signcrypt oracles, respectively, and  $\alpha_i(l)$  are negligible, for all i.

#### 4.2 Non-repudiation

**Theorem 2** If there exists a probabilistic polynomial time adversary that wins in the EUF-SCPRE-CMA game with probability  $\epsilon$ , then there exists a probabilistic polynomial time machine that breaks Chen et al's scheme's non-repudiation with probability

$$\geq \epsilon \cdot \left(1 - \left(q_s\left(\frac{q_s + q_1}{q}\right) + \beta_1(l) + \sum_{i=2}^{3} \alpha_i(l)\right)\right)$$

where  $q_1$ ,  $q_2$  and  $q_s$  are the number of queries made by the adversary to  $H_1$ ,  $H_2$  and Signcrypt oracles, respectively, and  $\beta_1(l)$  and  $\alpha_i(l)$  are negligible, for all i.

## 5 Proof of Confidentiality (IND-SCPRE-CCA2)

#### 5.1 Security as a signcryption scheme

We now show a PPT algorithm (simulator) that simulates an environment indistinguishable from the real world (as in the model) to an adversary which has any non-negligible advantage in winning the IND-CCA2 game (specified in [7]) of signcryption scheme, can solve a BCDH problem with non-negligible probability.

#### 5.1.1 Simulation

#### Setup

The simulator chooses a bilinear pairing  $e : \mathbb{G}_1 \times \mathbb{G}_1 \to \mathbb{G}_2$  where  $(\mathbb{G}_1, +)$  and  $(\mathbb{G}_2, .)$  are groups of the same prime order q. We assume that the Bilinear Computational Diffie Hellman (BCDH) assumption holds on  $\langle e, \mathbb{G}_1, \mathbb{G}_2 \rangle$ .

Initially the simulator is given an instance of BCDH problem on  $(e, \mathbb{G}_1, \mathbb{G}_2)$ , (P, aP, bP, cP) as the challenge.

The simulator then sets up the public parameters to the adversary as  $Param = < e, \mathbb{G}_1, \mathbb{G}_2, q, P, P_{pub} = aP, n, H_0, H_1, H_2 >$  where  $H_0, H_1, H_2$  are defined as random oracles. Here, n is the length of the message, in bits. It also chooses an identity,  $ID_{B^*}$ , and presents it to the adversary as the challenge identity.

#### Oracle simulations

In all simulations, any list L is assumed to be empty at the start of the simulation.

Initially the random oracles  $H_0$ ,  $H_1$  and  $H_2$  are simulated as shown below.

#### $H_0(ID)$ oracle:

If  $ID = ID_{B^*}$  then output bP.

Else, if there exists entry of the form  $\langle x, ID, Q, S \rangle$  in the list  $L_0$ , then output Q. Else, choose  $x \in_R \mathbb{Z}_q^*$ , set Q = xP and  $S = xP_{pub}$ . Make an entry  $\langle x, ID, Q, S \rangle$  into  $L_0$  list, and output Q.

#### $H_1(X||m)$ oracle:

If an entry of the form  $\langle (X||m), h_1 \rangle$  exists in  $L_1$ , then output  $h_1$ . Else, choose  $h_1 \in_R \mathbb{Z}_q^*$ , enter  $\langle (X||m), h_1 \rangle$  into  $L_1$ , and output  $h_1$ .

## $H_2(k)$ oracle:

If an entry of the form  $\langle k, h_2 \rangle$  already exists in  $L_2$ , then output  $h_2$ . Else, choose  $h_2 \in_R \{0,1\}^{n+t}$ , enter  $\langle k, h_2 \rangle$  into  $L_2$ , and output  $h_2$ .

The  $H_0$  oracle query of ID sets the secret key of ID. The simulation of the Extract oracle is given below.

### $\underline{\text{Extract}(ID) \text{ oracle:}}$

If an entry of the form  $\langle x, ID, Q, S \rangle$  exists in  $L_0$ , then output S. Else query  $H_0(ID)$  to set the secret key of ID; now, fetch the entry of the form  $\langle x, ID, Q, S \rangle$  from  $L_0$  list and output S

## Signcrypt $(m, ID_1, ID_2)$ oracle:

Case 1:  $ID_1 \neq ID_{B^*}$ 

- 1.  $S_1 = \text{Extract}(ID_1)$
- 2. Output Signcrypt $(m, S_1, ID_2)$

## Case 2: $ID_1 = ID_{B^*}$

In steps 1 to 4, a signature on m from  $ID_1$  is created, using the control the simulator has over the random oracles. In steps 5 to 9, the normal encryption routine is carried out as in the algorithm.

- 1. Choose  $r, h \in_R \mathbb{Z}_q^*$ .
- 2.  $Q_{B^*} = H_0(ID_{B^*})$
- 3. Compute  $X = rP hQ_{B^*}$ , and set  $Z = rP_{pub}$ .
- 4. Add the entry  $\langle (X||m), h \rangle$  to the  $L_1$  list.
- 5.  $S_2 = \text{Extract}(ID_2)$
- 6. Compute  $w = e(X, S_2)$
- 7. Choose  $k \in_R \mathbb{G}_2$ , and encrypt it as  $\lambda = w.k$

- 8. Now encrypt the message as  $y = H_2(k) \oplus (m||Z)$
- 9. Output  $(X, y, \lambda, ID_{B^*})$ .

De-signcrypt( $\phi = (X, y, \lambda, ID_1), ID_2$ ) oracle:

Case 1:  $ID_2 \neq ID_{B^*}$ 

- 1.  $S_1 = \operatorname{Extract}(ID_1)$
- 2. Output  $De signcrypt(\phi, S_2)$

Case 2:  $ID_2 = ID_{B^*}$ 

With very high probability, the adversary could have computed a valid sign-cryption only by querying the appropriate k to the  $H_2$  oracle. Thus, the simulator does the following.

For each entry  $\langle k, h_2 \rangle$  in  $L_2$  list do:

- 1. Compute  $(m||Z) = y \oplus h_2$
- 2. Compute  $Q_1 = H_0(ID_1)$ , and  $h_1 = H_1(X||m)$
- 3. Validate the sign by checking if  $e(Z, P) = e(P_{pub}, X + h_1Q_1)$ . If so, proceed to the next step. If not, go to next entry in  $L_2$ .
- 4. Validate the correctness of k by checking if  $\lambda = k \cdot e(Z h_1S_1, bP)$ . If so, output  $\langle m, (X, Z), ID_1 \rangle$  as the message and signature. If not, go to the next entry in  $L_2$ .
- 5. If no message-signature was returned till this step, output  $\perp$ .

## 5.1.2 Reduction

In phase 1, the adversary queries the oracles with the restrictions specified in the security model.

At the end of phase 1, the adversary outputs two messages,  $m_0, m_1$  and a sender identity  $ID_A$ . The simulator does the following: set  $X^* = cP$ , choose  $y^* \in_R \{0,1\}^{n+t}$  and  $\lambda^* \in_R \mathbb{G}_2$ , and finally output  $\langle X^*, y^*, \lambda^*, ID_A \rangle$  as the challenge to the adversary.

In phase 2, the adversary queries the oracles, again with the restrictions specified in the security model. At the end of phase 2, the adversary outputs its guess b' as mentioned in the game, which the simulator discards.

In subsection 5.1.3, we prove that its infeasible for any adversary to distinguish our oracle simulations from the real world (the model) with any non-negligible probability. Hence, any adversary that has advantage  $\epsilon$ , should do either of these: distinguish that the challenge ciphertext is invalid, or attempt to win the game by computing the bit, with  $\epsilon$  probability.

It can be seen that the challenger ciphertext is valid as long as  $H_2(k)$  where  $k = \lambda^*/e(X^*, S_{B^*})$  is set properly. Also,  $y^*$  can be a signcryption of  $m_0$  or  $m_1$  with equal probability, and so the adversary has to know the value of  $H_2(k)$  to get any information about  $m_b$ . Thus, in either case the adversary has to query  $H_2(k)$ ; but, then the simulator can solve the BCDH instance with probability  $1/q_2$ , by choosing a random element x from  $L_2$  list and outputting  $\lambda^*/x$ , thereby showing a contradiction.

#### 5.1.3 Analysis

We analyze the simulation of each oracle and show that the simulation is indistinguishable from real world, for any adversary.

### Random oracles' and Extract oracle's indistinguishability

It can be easily seen that the random oracles  $H_0, H_1, H_2$  and Extract oracle are simulated as in the real world.

#### Signcrypt oracle's indistinguishability

The signcrypt oracle produces valid signcryptions, except in Step 4 of Case 2 when an entry  $\langle (X||m), h \rangle$  already exists in the  $L_1$  list and therefore an error happens. The probability that this bad event happens in any of the  $q_s$  signcrypt queries is  $\leq q_s \left(\frac{q_s+q_1}{q}\right)$ , as X is a random element in  $\mathbb{G}_1$ .

#### De-signcrypt oracle's indistinguishability

It can be observed that the De-signcrypt oracle is simulated correctly in all cases, except for the case when the adversary has produced a valid signcryption without making the query  $H_2(k)$ , but the De-signcrypt oracle rejects it

and returns  $\perp$ .

But as  $H_2$  is a random oracle, and because only a negligible fraction of the signature space has valid signatures, the probability that the adversary can create a valid signcryption without querying  $H_2(k)$  is negligible (in l). Now, the probability that this bad event happens in at least one of the  $q_d$  de-signcrypt queries that the adversary makes, is also negligible, say,  $\alpha_1(l)$ .

Thus, the probability that the adversary does *not* distinguish any of the oracles' simulations from the real world is

$$\geq 1 - \left(q_s\left(\frac{q_s + q_1}{q}\right) + \alpha_1(l)\right)$$

which can be seen as  $1 - \nu(l)$ , where  $\nu(l)$  is negligible in l.

#### 5.2 Proof: Part 1

We prove that the security of our system as a signcryption scheme in the IND-CCA2 model under given-ID attacks by probabilistic polynomial time adversaries, is maintained even in the presence of Extract-rekey and De-reencrypt oracles. The security model is same as that of the IND-CCA2 model (of course, here the adversary is given the Extract-rekey and De-re-encrypt oracles also), except that in Phase 1 there is one additional restriction:

• It cannot query both Extract-rekey  $(ID_{B^*}, ID_3)$  and Extract $(ID_3||$ "delegatee"), for any  $ID_3$ 

and in Phase 2 there are two additional restrictions:

- It cannot query both Extract-rekey  $(ID_{B^*}, ID_3)$  and Extract $(ID_3||$ "delegatee"), for any  $ID_3$
- It cannot query  $rk_{B^*\to C}=$  Extract-rekey  $(ID_{B^*},ID_C)$  and De-re-enc-rypt $(\phi^{*'},ID_C)$ , where  $\phi^{*'}=$  Re encrypt $(\phi^*,rk_{B^*\to C},ID_{B^*},ID_C)$ , for any  $ID_C$

#### 5.2.1 Simulation and Reduction

The simulator interacts with the IND-CCA2 challenger of the signcryption scheme. We denote each oracle  $\mathcal{O}$  provided by the challenger as  $\mathcal{O}'$ .

#### Setup

The simulator is given the public parameters,  $Param = \langle e, \mathbb{G}_1, \mathbb{G}_2, q, P, P_{pub}, n, H'_0, H'_1, H'_2 \rangle$  where  $H'_0, H'_1, H'_2$  are defined as random oracles, by the challenger. Here, n is the length of the message, and  $e : \mathbb{G}_1 \times \mathbb{G}_1 \to \mathbb{G}_2$  where  $\mathbb{G}_1$  and  $\mathbb{G}_2$  are groups of same prime order q, is a bilinear pairing, where the BCDH assumption holds. The simulator also gets an identity,  $ID_{B^*}$ , as its challenge identity.

The simulator now sets up the public parameters for the adversary as  $Param = < e, \mathbb{G}_1, \mathbb{G}_2, q, P, P_{pub}, n, H_0, H_1, H_2, H_3 >$ , where  $H_3$  is a random oracle, and also gives the same  $ID_{B^*}$  as its challenge identity.

#### Oracle Simulations

Queries to the  $H_0(ID)$ ,  $H_1(X||m)$ ,  $H_2(k)$ , Extract(ID), Signcrypt $(m, ID_1, ID_2)$ , and De-signcrypt $(\phi, ID_2)$  oracles are forwarded to the corresponding challenger's oracles, and the responses from the challenger are forwarded back to the adversary.

#### $H_3(j)$ oracle:

If an entry of the form  $\langle j, h_3 \rangle$  already exists in  $L_3$ , then output  $h_3$ . Else, choose  $h_3 \in_R \mathbb{G}_1$ , enter  $\langle j, h_3 \rangle$  into  $L_3$  and output  $h_3$ .

#### Extract-rekey $(ID_1, ID_2)$

Case 1:  $ID_1 = ID_{B^*}$ 

- If there exists an entry of the form  $\langle ID_1, ID_2, R \rangle$  in the  $L_r$  list, then return R,
- Else choose  $R \in_R \mathbb{G}_1$ , make an entry  $\langle ID_1, ID_2, R \rangle$  to the  $L_r$  list, and return R

Case 2:  $ID_1 \neq ID_{B^*}$ 

• If there exists an entry of the form  $\langle ID_1, ID_2, R \rangle$  in the  $L_r$  list then return R

• Else compute  $S_1 = \text{Extract}(ID_1)$  and  $h = H_3(e(S_1, Q_{2||delegatee}))$ ; then enter  $\langle ID_1, ID_2, -S_1 + h \rangle$  in the  $L_r$  list and return  $(-S_1 + h)$ .

De-re-encrypt( $\phi' = \langle X, y, \lambda', ID_1, ID_2 \rangle, ID_3$ ) oracle:

Case 1:  $ID_2 \neq ID_{B^*}$ 

- 1.  $S_{3||delegatee}$  = Extract $(ID_3||"delegatee")$
- 2. Return De re encrypt $(\phi', S_{3||delegatee})$

Case 2:  $ID_2 = ID_{B^*}$ 

- 1. If there exists an entry  $\langle ID_{B^*}, ID_3, R \rangle$  from  $L_r$  list
  - (a) Compute  $\lambda = \lambda' \cdot e(X, -R)$
  - (b) If De-signcrypt( $\phi = \langle X, y, \lambda, ID_1 \rangle$ ,  $ID_{B^*}$ ) returns a valid  $\langle m, (X, Z), ID_1 \rangle$  then forward this to the adversary
- 2.  $S_{3||delegatee}$  = Extract $(ID_3||"delegatee")$
- 3. Return De re encrypt $(\phi', S_{3||delegatee})$

#### Reduction

In phase 1, the adversary can query the above oracles any polynomial number of times, but with the restrictions specified in the security model.

At the end of phase 1, adversary outputs two messages,  $m_0, m_1$  and a sender identity  $ID_A$ . The simulator forwards this to the challenger and gets the challenge ciphertext  $\phi^*$  addressed to  $ID_{B^*}$ . It then forwards this to adversary as its challenge ciphertext.

In phase 2, the adversary can query the oracles any polynomial number of times, again with the restrictions specified in the security model.

Finally, at the end of phase 2, the adversary outputs its guess b' as mentioned in the game, which is sent as the simulator's guess to the challenger.

#### 5.2.2 Analysis

We show that the simulated world is indistinguishable from the real world, for any adversary.

It can be seen that all the oracles and the challenge, except the  $H_3$ , Extract-rekey and De-re-encrypt oracles are simulated by forwarding to the IND-CCA2 challenger of section 5.1, and hence are indistinguishable from the real world, by the IND-CCA2 proof. It can also be seen that the random oracle  $H_3$  is simulated as in the real world.

#### De-re-encrypt oracle's indistinguishability

We now show that the de-re-encrypt oracle simulation is indistinguishable from the real world oracle.

When  $ID_2 \neq ID_{B^*}$ , it behaves as in the real world. When  $ID_2 = ID_{B^*}$ , if the second level signcryption was created using the random rekey, simulator will decrypt it properly. If the adversary has not queried the rekey and produced a valid second level signcryption, then step 2 will decrypt it properly. If the adversary has queried the rekey and produced a valid second level signcryption, then with very high probability the first step will fail, and the second step will decrypt it properly. It can also be seen that any other signcryption will be handled in the same way as the real world de-re-encrypt oracle. Hence, the de-re-encryption oracle is simulated as in the real world, except for a negligible probability  $\alpha_2(l)$ .

#### Extract-rekey indistinguishability

We show that the extract-rekey oracle is indistinguishable from the real world oracle, by simulating an environment that is indistinguishable to the environment simulated in our simulation, and then showing that the extract-rekey oracle is indistinguishable in this new simulation. In our simulation we assume that the adversary distinguishes that the rekey from,  $ID_{B^*}$  to say,  $ID_C$ , is invalid. Note that this proof will hold for any  $ID_C$ .

## Setup

The simulator chooses a bilinear pairing  $e: \mathbb{G}_1 \times \mathbb{G}_1 \to \mathbb{G}_2$  where  $\mathbb{G}_1$  and  $\mathbb{G}_2$  are groups of same prime order q. We make the Bilinear Computational Diffie Hellman (BCDH) assumption in  $\langle e, \mathbb{G}_1, \mathbb{G}_2 \rangle$ .

Initially the simulator is given (P, aP, bP, cP). As before, it sets up the public parameters,  $Param = \langle e, \mathbb{G}_1, \mathbb{G}_2, q, P, P_{pub} = aP, n, H_0, H_1, H_2, H_3 \rangle$  where  $H_0, H_1, H_2$  and  $H_3$  are defined as random oracles, and n is the length of the messages.

#### Oracle simulations

## $\underline{H_0(ID)}$ oracle:

If  $ID = ID_B$  then output bP

If  $ID = ID_C$ ||"delegatee" then output cP

Else, if there is entry of the form  $\langle x, ID, Q, S \rangle$  in the  $L_0$ , output Q. Else choose an  $x \in_R \mathbb{Z}_q^*$ , set Q = xP and  $S = xP_{pub}$ . Make an entry  $\langle x, ID, Q, S \rangle$  into  $L_0$  list, and output Q.

The  $H_1(X||m)$ ,  $H_2(k)$ , Extract(ID), Signcrypt $(m, ID_1, ID_2)$ , and De-signcrypt $(\phi, ID_2)$  oracles are simulated in the same way as in the CCA2 proof of our signcryption scheme.

The  $H_3(j)$ , and Extract-rekey $(ID_1,ID_2)$  oracles are simulated as in part 1 of proof.

De-re-encrypt( $\phi' = \langle X, y, \lambda', ID_1, ID_2 \rangle, ID_3$ ) oracle:

Case 1:  $ID_2 \neq ID_{B^*}$ 

- 1.  $S_{3||delegatee}$  = Extract $(ID_3||"delegatee")$
- 2. Return De re encrypt $(\phi', S_{3||delegatee})$

Case 2:  $ID_2 = ID_{B^*}$ 

- 1. If there exists an entry  $\langle ID_{B^*}, ID_3, R \rangle$  in the  $L_r$  list
  - (a) Compute  $\lambda = \lambda' . e(X, -R)$
  - (b) If De-signcrypt( $\phi = \langle X, y, \lambda, ID_1 \rangle$ ,  $ID_{B^*}$ ) returns a valid signature  $\langle m, (X, Z), ID_1 \rangle$  then forward this to the adversary. Else goto step 2.
- 2. If  $ID_3 \neq ID_C$

- (a)  $S_{3||delegatee}$  = Extract $(ID_3||"delegatee")$
- (b) Return De re encrypt $(\phi', S_{3||delegatee})$

The above oracle's simulation is as in the simulation of part 1 proof except for the case when the adversary creates a valid level 2 signcryption to  $ID_C$  with  $ID_{B^*}$  as delegator. For the adversary to be able to create such a signcryption, it should have queried  $H_3(e(S_{B^*}, Q_{C||delegatee}))$ . Then, the simulator can solve BCDH with probability  $1/q_3$  by choosing a random entry from the  $L_3$  list, and giving it as the solution.

In phase 1, the adversary can query the above oracles (a polynomial number of times) with the restrictions specified in the security model. At the end of phase 1, the adversary outputs two messages,  $m_0, m_1$  and a sender identity  $ID_A$ . The simulator chooses  $x \in_R \mathbb{Z}_q^*$ ,  $X^* = xP$ ,  $y^* \in_R \{0,1\}^{n+t}$  and  $\lambda^* \in_R \mathbb{G}_2$ , and outputs  $\langle X^*, y^*, \lambda^*, ID_A \rangle$ .

#### Analysis

Now, we show that this simulation is indistinguishable from the simulation in part 1 proof. It can be easily seen that all the oracles are simulated as in the original simulation. Now, it can also be easily seen that the challenge is as in the part 1 proof's simulation, since the challenge is formed in exactly the same way as in the part 1 proof's simulation.

Hence, this simulation is indistinguishable from the simulation of part 1 proof.

#### Reduction

It can be seen from that when the adversary queries the rekey from  $ID_{B^*}$  to  $ID_C$  the simulator responds with a random  $R \in_R \mathbb{G}_1$ . This R can be written as  $R = -S_{B^*} + T$ .

We say that the adversary has distinguished the Extract-rekey oracle if it has distinguished that at least one of the rekeys is invalid. First note that as long as  $H_3(e(S_{B^*}, Q_{C||delegatee})) = T$ , the rekey is valid. So, what  $H_3(e(S_{B^*}, Q_{C||delegatee}))$  is set to, determines whether a rekey is valid or not. Thus, if the adversary determines that one of the rekeys (of course, from  $B^*$ , say, to C) is invalid, it could not have done it without ensuring that  $H_3(e(S_{B^*}, Q_{C||delegatee}))$  is incorrectly set. Also, we can see that  $H_3(e(S_{B^*}, Q_{C||delegatee}))$  is random and independent for a rekey from  $ID_{B^*}$  to  $ID_C$ . Thus, distinguishing the Extract-rekey oracle could have been done

in only by querying  $H_3(e(S_{B^*}, Q_{C||delegatee}))$  and simulator returns a value  $\tau \neq T$ . The adversary now checks if  $e(\tau - R, P) = e(Q_{B^*}, P_{pub})$ , which will not be true, and hence determine that the rekey is invalid. But in that case, the simulator can solve BCDH with  $1/q_3$  probability by choosing a random entry in the  $L_3$  list. The probability with which this can happen, because of the BCDH assumption, is also negligible in l.

Thus, the rekey oracle is distinguishable with probability at most  $\alpha_3(l)$ , where  $\alpha_3(l)$  is negligible in l.

Thus, the probability that the adversary does *not* distinguish the simulation from the real world is

$$\geq 1 - \left(q_s\left(\frac{q_s + q_1}{q}\right) + \sum_{i=1}^{3} \alpha_i(l)\right)$$

which can be written as  $1 - \nu'(l)$ , where  $\nu'(l)$  is negligible in l.

#### 5.3 Proof: Part 2

Now we prove the complete IND-SCPRE-CCA2 security of our system.

#### 5.3.1 Simulation and Reduction

#### Setup

The simulator interacts with the challenger of the part 1 game. It is given the public parameters,  $Param = \langle e, \mathbb{G}_1, \mathbb{G}_2, q, P, P_{pub}, n, H'_0, H'_1, H'_2, H'_3 \rangle$  where  $H'_0, H'_1, H'_2, H'_3$  are defined as random oracles and n is the length of the message. Here,  $e: \mathbb{G}_1 \times \mathbb{G}_1 \to \mathbb{G}_2$  where  $\mathbb{G}_1$  and  $\mathbb{G}_2$  are groups of same prime order q, is a bilinear pairing, where the BCDH assumption holds. The simulator also gets an identity,  $ID_{B^*}$ , as its challenge identity.

The simulator now sets up the public parameters for the adversary as  $Param = < e, \mathbb{G}_1, \mathbb{G}_2, q, P, P_{pub}, n, H_0, H_1, H_2, H_3 >$ , and also gives the same  $ID_{B^*}$  as its challenge identity.

#### **Oracle Simulations**

Queries to all oracles except the Re-encrypt, are forwarded to the challenger's corresponding oracles, and the responses are forwarded back to the adversary.

Re-encryption $(\phi, ID_2, ID_3)$  oracle:

Return Re – encrypt( $\phi$ , Extract-rekey( $ID_2$ ,  $ID_3$ ), $ID_2$ ,  $ID_3$ )

### Reduction

In phase 1, the adversary can query the oracles with the restrictions specified in the security model, any polynomial number of times.

At the end of phase 1, the adversary outputs two messages,  $m_0$ , and  $m_1$ , and a sender identity  $ID_A$ . The simulator forwards this to the challenger and gets the challenge ciphertext  $\phi^*$  addressed to  $ID_{B^*}$ . It then forwards this to adversary as its challenge.

In phase 2, the adversary queries the oracles a polynomial number of times, again with the restrictions specified in the security model. At the end of the phase, the adversary outputs its guess b', which is the simulator sends as its guess, to the challenger.

#### 5.3.2 Analysis

All the random oracles  $H_0$ ,  $H_1$ ,  $H_2$ ,  $H_3$ , the Extract oracle, the Extractrekey oracle, the Signcrypt oracle, and the De-signcrypt oracle queries are simulated by forwarding them to the challenger and returning the responses. Since the challenger simulates these oracles indistinguishably from real world, our simulation of these oracles are also indistinguishable from real world. Also, it can be seen that the Re-encrypt and De-re-encrypt oracles are simulated as specified in the algorithm and hence the whole simulation is indistinguishable from real world.

Thus, the probability that the adversary does *not* distinguish the simulation from the real world is

$$\geq 1 - \left(q_s\left(\frac{q_s + q_1}{q}\right) + \sum_{i=1}^3 \alpha_i(l)\right)$$

which can be seen as  $1 - \nu'(l)$ , where  $\nu'(l)$  is negligible in l.

Thus, with probability at least  $1 - \left(q_s\left(\frac{q_s + q_1}{q}\right) + \sum_{i=1}^{3} \alpha_i(l)\right)$ , the adversary will not distinguish the oracle simulation. Now, any adversary that does not distinguish the oracle simulations should either distinguish that the challenge ciphertext is invalid, or should attempt to win the game by computing b, with  $\epsilon$  probability. It can also be noted that our simulator does to its challenger whatever the adversary does to it (both forwarding b, or aborting claiming distinguishability of simulations). Hence, by the structure of the proof, BCDH will be solved in either case, with probability  $1/q_2$ . Hence, we have the theorem 1 for the confidentiality of our system.

## 6 Proof of Non-repudiation (EUF-SCPRE-CMA)

## 6.1 Security as a signcryption scheme

## 6.1.1 Simulation & Reduction

#### Setup

The simulator interacts with the challenger in the non-repudiation game of Chen et al's scheme [7]. It initially gets the public parameters  $param' = < e, \mathbb{G}_1, \mathbb{G}_2, q, n, P, P_{pub}, H'_0, H'_1, H'_2 > \text{from the challenger, and sets the public parameters to the adversary as } param = < e, \mathbb{G}_1, \mathbb{G}_2, q, n, P, P_{pub}, H_0, H_1, H_2 > < e, \mathbb{G}_1, \mathbb{G}_2, q, n, P, P_{pub}, H_0, H_1, H_2 > < e, \mathbb{G}_1, \mathbb{G}_2, q, n, P, P_{pub}, H_0, H_1, H_2 > < e, \mathbb{G}_1, \mathbb{G}_2, q, n, P, P_{pub}, H_0, H_1, H_2 > < e, \mathbb{G}_1, \mathbb{G}_2, q, n, P, P_{pub}, H_0, H_1, H_2 > < e, \mathbb{G}_1, \mathbb{G}_2, q, n, P, P_{pub}, H_0, H_1, H_2 > < e, \mathbb{G}_1, \mathbb{G}_2, q, n, P, P_{pub}, H_0, H_1, H_2 > < e, \mathbb{G}_1, \mathbb{G}_2, q, n, P, P_{pub}, H_0, H_1, H_2 > < e, \mathbb{G}_1, \mathbb{G}_2, q, n, P, P_{pub}, H_0, H_1, H_2 > < e, \mathbb{G}_1, \mathbb{G}_2, q, n, P, P_{pub}, H_0, H_1, H_2 > < e, \mathbb{G}_1, \mathbb{G}_2, q, n, P, P_{pub}, H_0, H_1, H_2 > < e, \mathbb{G}_1, \mathbb{G}_2, q, n, P, P_{pub}, H_0, H_1, H_2 > < e, \mathbb{G}_1, \mathbb{G}_2, q, n, P, P_{pub}, H_0, H_1, H_2 > < e, \mathbb{G}_1, \mathbb{G}_2, q, n, P, P_{pub}, H_0, H_1, H_2 > < e, \mathbb{G}_1, \mathbb{G}_2, q, n, P, P_{pub}, H_0, H_1, H_2 > < e, \mathbb{G}_1, \mathbb{G}_2, q, n, P, P_{pub}, H_0, H_1, H_2 > < e, \mathbb{G}_1, \mathbb{G}_2, q, n, P, P_{pub}, H_0, H_1, H_2 > < e, \mathbb{G}_1, \mathbb{G}_2, q, n, P, P_{pub}, H_0, H_1, H_2 > < e, \mathbb{G}_1, \mathbb{G}_2, q, n, P, P_{pub}, H_0, H_1, H_2 > < e, \mathbb{G}_1, \mathbb{G}_2, q, n, P, P_{pub}, H_1, H_2 > < e, \mathbb{G}_1, \mathbb{G}_2, q, n, P, P_{pub}, H_1, H_2 > < e, \mathbb{G}_1, \mathbb{G}_2, q, n, P, P_{pub}, H_1, H_2 > < e, \mathbb{G}_1, \mathbb{G}_2, q, n, P, P_{pub}, H_1, H_2 > < e, \mathbb{G}_1, \mathbb{G}_2, q, n, P, P_{pub}, H_1, H_2 > < e, \mathbb{G}_2, q, n, P, P_{pub}, H_1, H_2 > < e, \mathbb{G}_2, q, n, P, P_{pub}, H_1, H_2 > < e, \mathbb{G}_2, q, n, P, P_{pub}, H_1, H_2 > < e, \mathbb{G}_2, q, n, P, P_{pub}, H_1, H_2 > < e, \mathbb{G}_2, q, n, P, P_{pub}, H_1, H_2 > < e, \mathbb{G}_2, q, n, P, P_{pub}, H_1, H_2 > < e, \mathbb{G}_2, q, n, P, P_{pub}, H_1, H_2 > < e, \mathbb{G}_2, q, n, P, P_{pub}, H_1, H_2 > < e, \mathbb{G}_2, q, n, P, P_{pub}, H_1, H_2 > < e, \mathbb{G}_2, q, n, P, P_{pub}, H_1, H_2 > < e, \mathbb{G}_2, q, n, P, P_{pub}, H_1, H_$ 

The simulator then chooses an identity,  $ID_{A^*}$ , as the challenge identity and gives it to the adversary.

#### Oracle simulations

Queries to the  $H_0(ID)$ ,  $H_1(X||m)$ , and  $\operatorname{Extract}(ID)$  oracles are forwarded to the challenger's corresponding oracles, and the responses are forwarded back to the adversary.

Of the random oracles,  $H_2$  alone is simulated by the simulator without forwarding to challenger, as shown below.

#### $H_2(k)$ oracle:

If there exists an entry of the form  $\langle k, h_2 \rangle$  in  $L_2$ , then output  $h_2$ . Else,

choose  $h_2 \in_R \{0,1\}^{n+t}$ , enter  $\langle k, h_2 \rangle$  into  $L_2$  and output  $h_2$ .

Signcrypt $(m, ID_1, ID_2)$  oracle:

Case 1:  $ID_1 \neq ID_{A^*}$ 

- 1.  $S_1 = \text{Extract}(ID_1)$
- 2. Output Signcrypt $(m, S_1, ID_2)$

Case 2:  $ID_1 = ID_{A^*}$ 

In steps 1 and 2, a signature on m from  $ID_1$  is created using the oracles provided by the challenger in an appropriate way. Then, in steps 3 to 7, the normal encryption routine is carried out as in the algorithm.

- 1.  $\phi = \text{Signcrypt}'(m, ID_1, ID_2)$
- 2.  $\langle m, (X, Z), ID_{A^*} \rangle = \text{De-signcrypt'}(\phi, ID_2)$
- 3.  $S_2 = \text{Extract}(ID_2)$
- 4. Compute  $w = e(X, S_2)$
- 5. Choose  $k \in_R \mathbb{Z}_q^*$  and compute  $\lambda = k \cdot w$
- 6.  $y = H_2(k) \oplus (m||Z)$
- 7. Output  $\langle X, y, \lambda, ID_{A^*} \rangle$

De-signcrypt( $\phi = (X, y, \lambda, ID_1), ID_2$ ) oracle:

Case 1:  $ID_2 \neq ID_{A^*}$ 

- 1.  $S_2 = \text{Extract}(ID_2)$
- 2. Output  $De signcrypt(\phi, S_2)$

Case 2:  $ID_2 = ID_{A^*}$ 

With very high probability, the adversary could have computed a valid sign-cryption only by querying the appropriate k to the  $H_2$  oracle. Thus, the simulator does the following.

For each entry  $\langle k, h_2 \rangle$  in  $L_2$  list do:

- 1. Compute  $(m||Z) = y \oplus h_2$
- 2. Compute  $Q_1 = H_0(ID_1)$ ,  $Q_2 = H_0(ID_2)$  and  $h_1 = H_1(X||m)$
- 3. Validate the sign by checking if  $e(Z, P) = e(P_{pub}, X + h_1Q_1)$ . If failed, go to next entry in  $L_2$ . Else go to next step.
- 4. Now, validate the correctness of k by checking if  $\lambda = k \cdot e(Z h_1 S_1, Q_2)$ . If failed, go to next entry in  $L_2$ . Else, output  $\langle m, (X, Z), ID_1 \rangle$ .
- 5. Finally, if no message was returned, output  $\perp$

#### 6.1.2 Reduction

The adversary can query the above oracles any polynomial number of times, with the restrictions as specified in the security model. Finally it produces a signature  $\langle m, (X, Z), ID_{A^*} \rangle$ . Now the simulator will produce a forgery for Chen et al scheme's non-repudiation game as follows:

- 1. Choose an identity  $ID_2 \neq ID_{A^*}$ ; compute  $S_2 = \text{Extract}(ID_2)$ .
- 2. Compute  $w = e(X, S_2)$ .
- 3.  $y = H'_2(w) \oplus (m||Z||ID_{A^*}).$
- 4. Output  $\langle X, y \rangle$ ,  $ID_2$  as forgery for Chen et al scheme's

It can be seen that, the simulator's forgery Chen et al's game is valid whenever the adversary's forgery to our non-repudiation game is valid.

#### 6.1.3 Analysis

We analyze the simulation of each oracles and show that the simulation is indistinguishable from the real world.

#### Random oracles' indistinguishability

It can be easily seen that the random oracles  $H_0, H_1$  and  $H_2$  are simulated indistinguishably from the real world, due to Chen et al's proof.

### Extract and Signcrypt oracle's indistinguishability

Extract oracle and signcrypt oracle are simulated indistinguishably from the real world, due to Chen et al's proof. It can be noted that the probability of distinguishing the Signcrypt oracle, in Chen et al's proof and hence our proof, is at most  $q_s\left(\frac{q_s+q_1}{q}\right)$ .

#### De-signcrypt oracle's indistinguishability

It can be observed that the de-signcrypt oracle is simulated correctly in all cases, except for the case when the adversary has produced a valid signcryption without querying  $H_2(k)$ , but the De-signcryption oracle rejects it and returns  $\perp$ .

But as  $H_2$  is a random oracle and because only a negligible fraction of the signature space has valid signatures, the probability that the adversary can create a valid signcryption without querying  $H_2(k)$  is negligible in l. Thus, the probability that this bad event happens in at least one of the  $q_d$  designcrypt queries, is also negligible, say,  $\beta_1(l)$ .

Thus, the adversary does not distinguish the simulation from the real world with probability

$$\geq \left(1 - \left(q_s\left(\frac{q_s + q_1}{q}\right) + \beta_1(l)\right)\right)$$

#### 6.2 Proof: Part 1

We prove that the security of our system as a signcryption scheme in the EUF-CMA model under given-ID attacks by probabilistic polynomial time adversaries, is maintained even in the presence of Extract-rekey and De-reencrypt oracles.

The security model is same as that of the CMA proof (of course, the Extract-rekey and De-re-encrypt oracles are provided additionally), except there is one additional restriction:

• It cannot query both Extract-rekey  $(ID_{A^*}, ID_3)$  and Extract $(ID_3||$ "delegatee"), for any  $ID_3$ .

Finally, the adversary produces a signature  $\langle m, (X, Z), ID_{A^*} \rangle$  such that Signcrypt $(m, ID_{A^*}, ID_3)$  was never queried, for any  $ID_3$ . The adversary succeeds if the signature is valid.

#### Simulation and Reduction

Everything is simulated as in the part 1 simulation of confidentiality game with  $ID_{A^*}$  instead of  $ID_{B^*}$ . Finally, the adversary will produce a forgery, which will be forwarded as the simulator's forgery, to the challenger.

#### **Analysis**

It can be seen that the indistinguishability of the Rekey-extract and De-reencrypt oracles will follow from the proofs given in the part 1 of the confidentiality game. Hence, the probability that the adversary cannot distinguish the simulation from the real world is

$$\geq \left(1 - \left(q_s\left(\frac{q_s + q_1}{q}\right) + \beta_1(l) + \sum_{i=2}^3 \alpha_i(l)\right)\right)$$

#### 6.3 Proof: Part 2

Now we prove the complete EUF-SCPRE-CMA security of our system.

#### Simulation and Reduction

Everything is simulated as in the part 2 simulation of confidentiality game with  $ID_{A^*}$  instead of  $ID_{B^*}$ . Finally, the adversary will produce a forgery, which will be forwarded as the simulator's forgery, to the challenger.

## Analysis

Obviously, the probability that the adversary cannot distinguish the simulation from the real world is

$$\geq \left(1 - \left(q_s\left(\frac{q_s + q_1}{q}\right) + \beta_1(l) + \sum_{i=2}^3 \alpha_i(l)\right)\right)$$

Hence theorem 2 follows.

## 7 Conclusion

In this work, we have introduced the notion of signcryption with proxy reencryption (SCPRE). We have also provided the first concrete construction of an SCPRE scheme, which is non-interactive and unidirectional. We have further defined the security models for confidentiality and non-repudiation, and formally prove the security of the system in the models defined.

#### 7.1 Costs

## Computation costs

We represent the costs as (no. of: pairings,  $\mathbb{G}_1$  multiplications,  $\mathbb{G}_2$  exponentiations). The sender of a signcryption requires (1,2,1), the receiver requires (3,1,1), the proxy (1,0,0), and the delegatee (4,2,1).

#### Communication costs

The communication cost are represented in the the number of bits required. A level 1 signcryption takes  $2|\mathbb{G}_1| + |\mathbb{G}_2| + |m| + |ID|$  bits, whereas a level 2 signcryption  $2|\mathbb{G}_1| + |\mathbb{G}_2| + |m| + 2|ID|$  bits. Here, |x| is used to represent the number of bits to represent x.

#### 7.2 Future Work

The problems open in the area of SCPRE include:

- An SCPRE scheme that is confidential and non-repudiable, in models that are a natural adaptation of the CCA2 model and the CMA model.
- An SCPRE scheme secure in the standard model.
- A collusion-resistant SCPRE scheme.

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