MV3: A new word based stream cipher using rapid mixing and revolving buffers

Nathan Keller[1*], Stephen D. Miller[1,2**], Ilya Mironov[3], and Ramarathnam Venkatesan[4]

[1] Einstein Institute of Mathematics, The Hebrew University, Givat Ram, Jerusalem 91904, Israel
`nkeller@math.huji.ac.il`

[2] Department of Mathematics, Rutgers University, Piscataway, NJ 08854
`miller@math.rutgers.edu`

[3] Microsoft Research, Silicon Valley Campus, 1065 La Avenida, Mountain View, CA 94043
`mironov@microsoft.com`

[4] Cryptography and Anti-piracy Group, Microsoft Research, 1 Microsoft Way, Redmond, WA 98052
and
Cryptography, Security and Algorithms Research Group, Microsoft Research India Scientia - 196/36 2nd Main, Sadashivnagar, Bangalore 560 080, India
`venkie@microsoft.com`

Abstract. MV3 is a new *word based* stream cipher for encrypting long streams of data. A direct adaptation of a byte based cipher such as RC4 into a 32- or 64-bit word version will obviously need vast amounts of memory. This scaling issue necessitates a look for new components and principles, as well as mathematical analysis to justify their use. Our approach, like RC4's, is based on rapidly mixing random walks on directed graphs (that is, walks which reach a random state quickly, from any starting point). We begin with some well understood walks, and then introduce nonlinearity in their steps in order to improve security and show long term statistical correlations are negligible. To minimize the short term correlations, as well as to deter attacks using equations involving successive outputs, we provide a method for sequencing the outputs derived from the walk using three revolving buffers. The cipher is fast — it runs at a speed of less than 5 cycles per byte on a Pentium IV processor. A word based cipher needs to output more bits per step, which exposes more correlations for attacks. Moreover we seek simplicity of construction and transparent analysis. To meet these requirements, we use a larger state and claim security corresponding to only a fraction of it. Our design is for an adequately secure word-based cipher; our very preliminary estimate puts the security close to exhaustive search for keys of size ≤ 256 bits.

Keywords: stream cipher, random walks, expander graph, cryptanalysis.

1 Introduction

Stream ciphers are widely used and essential in practical cryptography. Most are custom designed, e.g. alleged RC4 [Sch95, Ch. 16], SEAL [RC98], SCREAM [HCJ02], and LFSR-based NESSIE submissions such as LILI-128, SNOW, and SOBER [P+03, Ch. 3]. The VRA cipher [ARV95] has many provable properties, but requires more memory than the rest. We propose some new components and principles for stream cipher design, as well as their mathematical analysis, and present a concrete stream cipher called MV3.

To motivate our construction, we begin by considering RC4 in detail. It is an exceptionally short, byte-based algorithm that uses only 256 bytes of memory. It is based on random walks (card shuffles), and has no serious attacks. Modern personal computers are evolving from 32 to 64 bit words, while a growing number of smaller devices have different constraints on their word and memory sizes. Thus one may desire ciphers better suited to their architectures, and seek designs that scale nicely across these sizes. Here we focus on scaling up such random walk based ciphers. Clearly, a direct adaptation of RC4 would require vast amounts of memory.

The security properties of most stream ciphers are not based on some hard problem (e.g., as RSA is based on factoring). One would expect this to be the case in the foreseeable future. Nevertheless, they use components that – to varying degrees – are analyzable in some idealized sense. This analysis typically involves simple statistical parameters such as cycle length and mixing time. For example, one idealizes each iteration of the main loop of RC4 as a step in a random walk over its state space. This can be modeled by a graph G with nodes consisting of S_{256}, the permutations on 256 objects, and edges connecting nodes that differ by a transposition. Thus far no serious deviations from the random walk assumptions are known. Since storing an element of $S_{2^{32}}$ or $S_{2^{64}}$ is out of the question, one may try simulations using smaller permutations; however, this is nontrivial if we desire both competitive speeds and a clear analysis. It therefore is attractive to consider other options for the underlying graph G.

One of the most important parameters of RC4 is its mixing time. This denotes the number of steps one needs to start from an arbitrary state and achieve uniform distribution over the state space through a sequence of independent random moves. This parameter is typically not easy to determine. Moreover, RC4 keeps a loop counter that is incremented modulo 256, which introduces a memory over 256 steps. Thus its steps are not even Markovian (where a move from the current state is independent of earlier ones). Nevertheless, the independence of moves has been a helpful idealization (perhaps similar to Shannon's random permutation model for block ciphers), which we will also adhere to.

We identify and focus on the following problems:

- **Problem 1 – Graph Design.** How to design graphs whose random walks are suitable for stream ciphers that work on arbitrary word sizes.
- **Problem 2 – Extraction.** How to extract bits to output from (the labels of) the nodes visited by walk.

- **Problem 3 – Sequencing.** How to sequence the nodes visited by the walk so as to diminish any attacks that use relationships (e.g. equations) between successive outputs.

We now expand on these issues. At the outset, it is important to point out the desirability of simple register operations, such as additions, multiplications, shifts, and xor's. These are crucial for fast implementation, and preclude us from using many existing constructions of expander graphs (such as those in [LPS86,HLW06]). Thus part of the cipher design involves new mathematical proofs and constructions. The presentation of the cipher does not require these details, which may be found in Appendix A.

High level Design Principles: Clearly, a word based cipher has to output more bits per step of the algorithm. But this exposes more relationships on the output sequence, and to mitigate its effect we increase the state size and aim at security that is only a fraction of the log of the state size. We also tried to keep our analysis as transparent and construction as simple as possible. Our key initialization is a bit bulky and in some applications may require further simplifications, a topic for future research.

1.1   Graph Design: Statistical properties and Non-linearities

In the graph design, one wants to keep the mixing time τ small as a way to keep the long term correlations negligible. This is because many important properties are guaranteed for walks that are longer than τ. For example, such a walk visits any given set S nearly the expected number of times, with exponentially small deviations (see Theorem A.2). A corollary of this fact is that each output bit is unbiased.

Thus one desires the optimal mixing time, which is on the order of $\log N$, N being the size of the underlying state space. Graphs with this property have been well studied, but the requirements for stream ciphers are more complicated, and we are not aware of any work that focuses on this issue. For example, the graphs whose nodes are $\mathbb{Z}/2^n\mathbb{Z}$ (respectively $(\mathbb{Z}/2^n\mathbb{Z})^*$) and edges are $(x, x + g_i)$ (respectively $(x, x \cdot g_i)$), where g_i are randomly chosen and $i = O(n)$, have this property [AR94]. While these graphs are clearly very efficient to implement, their commutative operations are quite linear and hence the attacks mentioned in Problem 3 above can be effective.

To this end, we introduce some nonlinearities into our graphs. For example, in the graph on $\mathbb{Z}/2^n\mathbb{Z}$ from the previous paragraph, we can also add edges of the form (x, hx) or (x, x^r). This intuitively allows for more randomness, as well as disrupting relations between successive outputs. However, one still needs to prove that the mixing time of such a modified graph is still small. Typically this type of analysis is hard to come by, and in fact was previously believed to be false. However, we are able to give rigorous proofs in some cases, and empirically found the numerical evidence to be stronger yet in the other cases. More details can be found in the Appendices.

Mixing up the random walks on multiplicative and additive abelian groups offers a principled way to combine with nonlinearities for an effective defense. As a practical matter, it is necessary to ensure that our (asymptotic) analysis applies when parameters are small, which we have verified experimentally.

We remark here that introduction of nonlinearities was the main motivation behind the construction of the T-functions of Klimov and Shamir ([KS02]). They showed that the walk generated by a T-function deterministically visits every n-bit number once before repeating. A random walk does not go through all the nodes in the graph, but the probability that it returns to a previous node in m steps tends to the uniform probability at a rate that drops exponentially in m. It also allows us to analyze the statistical properties as indicated above. (See Appendix A for more background.)

1.2 Extraction

Obviously, if the nodes are visited truly randomly, one can simply output the LSB's of the node, and extraction is trivial. But when there are correlations among them, one can base an attack on studying equations involving successive outputs. One solution to this problem is to simultaneously hash a number of successive nodes using a suitable hashing function, but this will be expensive since the hash function has to work on very long inputs.

Our solution to the sequencing problem below allows us to instead hash a linear combination of the nodes in a faster way. A new aspect of our construction is that our hash function itself evolves on a random walk principle. We apply suitable rotations on the node labels (to alter the internal states) at the extraction step to ensure the top and bottom half of the words mix well.

1.3 Sequencing

As we just mentioned, the sequencing problem becomes significant if we wish to hash more bits to the output (in comparison to RC4). First we ensure that our graph is directed and has no short cycles. But this by itself is insufficient, since nodes visited at steps in an interval $[t, t + \Delta]$, where $\Delta \ll \tau$, can have strong correlations. Also, we wish to maximize the number of terms required in equations involved in the attacks mentioned in Problem 3. To this end, we store a short sequence of nodes visited by the walk in buffers, and sequence them properly. The buffers ensure that any relation among output bits is translated to a relation involving many nonconsecutive bits of the internal state. Hence, such relations cannot be used to mount efficient attacks on the internal state of the cipher.

The study of such designs appear to be of independent interest. We are able to justify their reduction of correlations via a theorem of [CHJ02] (see Section 4.5).

1.4 Analysis and Performance

We do not have a full analysis of the exact cipher that is implemented. However, we have ensured that our idealizations are in line with the ones that allow

RC4 be viewed via random walks. Of course some degree of idealization is necessary because random bits are required to implement any random walk; here our design resembles that of alleged RC4 [Sch95, Ch. 16]. Likewise, our cipher involves combining steps from different, independent random walks on the same underlying graph. We are able to separately analyze these processes, but although combining such steps should intuitively only enhance randomness, our exact mathematical models hold only for these separate components and hence we performed numerical tests as well.

Our cipher MV3 is fast on 32 bit processors — it runs at a speed of 4.8 cycles a byte on Pentium IV, while the speed of RC4 is about 10 cycles a byte. Only two of the eSTREAM candidates [DC06] are faster on similar architecture.

We evaluated it against some known attacks and we present the details in Section 4. We note that some of the guess-and-determine attacks against RC4 (e.g. [K+98]) are also applicable against MV3. However, the large size of the internal state of MV3 makes these attacks much slower than exhaustive key search, even for very long keys.

The security claim of MV3 is that no attack faster than exhaustive key search can be mounted for keys of length up to 256 bits.[5]

The paper is organized as follows: In Section 2 we give a description of MV3. Section 3 contains the design rationale of the cipher. In Section 4 we examine the security of MV3 with respect to various methods of cryptanalysis. Finally, Section 5 summarizes the paper. We have also included appendices giving some mathematical and historical background.

2   The Cipher MV3

In this section we describe the cipher algorithm and its basic ingredients. The letters in its name stand for "multi-vector", and the number refers to the three revolving buffers that the cipher is based upon.

Internal state. The main components of the internal state of MV3 are three revolving buffers A, B, and C of length 32 double words (unsigned 32-bit integers) each and a table T that consists of 256 double words. Additionally, there are publicly known indices i and u ($i \in [0\ldots31]$, $u \in [0\ldots255]$), and secret indices j, c, and x (c, x are double words, j is an unsigned byte).

Every 32 steps the buffers shift to the left: $A \leftarrow B$, $B \leftarrow C$, and C is emptied. In code, only the pointers get reassigned (hence the name "revolving", since the buffers are circularly rotated).

Updates. The internal state of the cipher gets constantly updated by means of pseudo-random walks. Table T gets refreshed one entry every 32 steps, via application of the following two operations:

$$u \leftarrow u + 1$$
$$T[u] \leftarrow T[u] + (T[j] \ggg 13).$$

---

[5] Note that MV3 supports various key sizes of up to 8192 bits. However, the security claims are only for keys of size up to 256 bits.

(Symbol $x \ggg a$ means a circular rotation to the right of the double word x by a bits).

In other words, the u-th element of the table, where u sweeps through the table in a round-robin fashion, gets updated using $T[j]$.

In its turn, index j walks (in every step, which can be idealized as a random walk) as follows:

$$j \leftarrow j \ + (B[i] \bmod 256),$$

where i is the index of the loop. Index j is also used to update x:

$$x \leftarrow x + T[j],$$

which is used to fill buffer C by $C[i] \leftarrow (x \ggg 8)$.

Also, every 32 steps the multiplier c is additively and multiplicatively refreshed as follows:

$$c \leftarrow c + (A[0] \ggg 16)$$
$$c \leftarrow c \vee 1$$
$$c \leftarrow c^2 \ \text{(can be replaced by } \ c \leftarrow c^3)$$

Main loop. The last ingredient of the cipher (except for the key setup) is the instruction for producing the output. This instruction takes the following form:

$$\text{output:} \qquad (x \cdot c) \oplus A[9i + 5] \oplus (B[7i + 18] \ggg 16).$$

The product $x \cdot c$ of two 32-bit numbers is taken modulo 2^{32}.

Putting it all together, the main loop of the cipher is the following:

Input: length *len*
Output: stream of length *len*
  repeat *len*/32 **times**
    for $i = 0$ **to** 31
      $j \leftarrow j + (B[i] \bmod 256)$
      $x \leftarrow x + T[j]$
      $C[i] \leftarrow (x \ggg 8)$
      output $(x \cdot c) \oplus A[9i + 5] \oplus (B[7i + 18] \ggg 16)$
    end for
    $u \leftarrow u + 1$
    $T[u] \leftarrow T[u] + (T[j] \ggg 13)$
    $c \leftarrow c + (A[0] \ggg 16)$
    $c \leftarrow c \vee 1$
    $c \leftarrow c^2$ (can be replaced by $c \leftarrow c^3$)
    $A \leftarrow B, B \leftarrow C$
  end repeat

Key initialization.

The key initialization algorithm accepts as inputs a key K of length *keylength*, which can be any multiple of 32 less than or equal to 8192 (we recommend at least 96 bits), and an initial vector IV of the same length as the key. The key remains the same throughout the entire encryption session, though the initial vector changes occasionally. The initial vector is publicly known, but should not be easily predictable. For example, it is possible to start with a "random" IV using a (possibly insecure) pseudo-random number generator known to the attacker, and then increment the IV by 1 every time (see Section 4.2).

The key initialization algorithm is the following:

> **Input:** key *key* and initial vector IV, both of length *keylength* double words
> **Output:** internal state that depends on the key and the IV
>
> $j, x, u \leftarrow 0$
> $c \leftarrow 1$
> fill A, B, C, T with $0x\texttt{EF}$
> **for** $i = 0$ **to** 3
>   **for** $l = 0$ **to** 255
> $T[i + l] \leftarrow T[i + l] + (key[l \bmod keylength] \ggg 8i) + l.$
>   **end for**
>   produce 1024 bytes of MV3 output
>   encrypt T with the resulting key stream
> **end for**
> **for** $i = 4$ **to** 7
>   **for** $l = 0$ **to** 255
> $T[i + l] \leftarrow T[i + l] + (IV[l \bmod keylength] \ggg 8i) + l.$
>   **end for**
>   produce 1024 bytes of MV3 output
>   encrypt T with the resulting key stream
> **end for**

Note that when only the IV is changed, only the second half of the key initialization is performed.

3 Design Rationale

In this section we describe more of the motivating principles behind the new cipher.

Internal state. The internal state of the cipher has a huge size of more than 11,000 bits. This makes guess-and-determine attacks on it (like the attack against RC4 in [K+98]) much slower than exhaustive key search, even for very long keys. In addition, it also secures the cipher from time/memory tradeoff attacks trying to invert the function $f : State \longrightarrow Output$, even for large key sizes. More detail on the security of the cipher with respect to these attacks appears in Section 4.

The buffers A, B, C and table T, as well as the indices j, c, and x should never be exposed. Since the key stream is available to the attacker and depends on this secret information, the cipher strictly adheres to the following design principles:

Principle 1. Output words must depend on as many secret words as possible.
Principle 2. Retire information faster than the adversary can exploit it.

As the main vehicle towards these goals, we use random walks (or, more precisely, pseudo-random walks, as the cipher is fully deterministic).

Updates. The updates of the internal state are based on several simultaneously applied random walks. On the one hand, these updates are very simple and can be efficiently implemented. On the other hand, as shown in Appendix A, the update mechanism allows one to mathematically prove some randomness properties of the sequence of internal states. Note that the random walks are interleaved, and the randomness of each one of them relies on the randomness of the others. Note also that the updates use addition in $\mathbb{Z}/2^n\mathbb{Z}$ and not a bitwise XOR operation. This partially resolves the problem of high-probability short correlations in random walks: In an undirected random walk, there is a high probability that after a short number of steps the state returns to a previous state, while in a directed random walk this phenomenon does not exist. For example, if we would use an update rule $x \leftarrow x \oplus T[j]$, then with probability 2^{-8} (rather than the trivial 2^{-32}) x would return to the same value after two steps. The usage of addition, which unlike XOR is not an involution, prevents this property. However, in the security proof for the idealized model we use the undirected case, since the known proofs of rapid mixing (like the theorem of Alon and Roichman [AR94]) refer to that case.

Introducing nonlinearity. In order to introduce some nonlinearity we use a multiplier c that affects the cipher output in a multiplicative way. The value of c is updated using an expander graph which involves both addition and multiplication, as explained in Appendix A. It is far from clear the squaring or cubing operation still leaves the mixing time small and our theorem addresses this.

Our update of c involves a step $c \leftarrow c \vee 1$. This operation may at a first seem odd, since it leaks LSB(c) to attacker, who may use it for a distinguishing attack based only on the LSB of outputs, ignoring c entirely. However, this operation is essential, since otherwise the attacker can exploit cases where $c = 0$, which occur with a relatively high probability of 2^{-16} due to the $c \leftarrow c^2$ operation (and last for 32 steps at a time). In this situation, they can disregard the term $x \cdot c$ and devise a guess-and-determine attack with a much lower time complexity than the currently possible one.

Sequencing rule. The goals of this step were explained in section 1.3. Our output rule is based on the following general structure: The underlying walk $x_0, x_1, \ldots, x_n, \ldots$ is transformed into the output $y_0, y_1, \ldots, y_n, \ldots$ via a linear transformation:

$$y_i \quad = \quad x_{n_{i1}} \oplus x_{n_{i2}} \oplus \cdots \oplus x_{n_{ik}}.$$

Without loss of generality, we assume that the indices are sorted $n_{i1} < n_{i2} < \cdots < n_{ik}$. Let $\mathcal{N} = \{n_{ij}\}$. The set $\mathcal{N}$ is chosen to optimize the following parameters:

1. Minimize the latency and the buffer size required to compute y_i. To this end, we require that there will be two constants m and C, between 64 and 256, such that $i - C \le n_{ij} \le i$ for each $i \ge m$ and $1 \le j \le k$. We additionally constrain $n_{ik} = i$ for all $i > m$;
2. Maximize the minimal size of a set of pairs x_i, x_{i+1} that can be expressed as a linear combination of y's. More precisely, we seek to maximize a such that the following holds for some $j_1, \ldots, j_b > m$ and $i_1, \ldots, i_a$:

$$(x_{i_1} \oplus x_{i_1+1}) \oplus (x_{i_2} \oplus x_{i_2+1}) \oplus \cdots \oplus (x_{i_a} \oplus x_{i_a+1}) \;=\; y_{j_1} \oplus y_{j_2} \oplus \cdots \oplus y_{j_b}. \quad (3.1)$$

Notice that the value of b has not been constrained, since usually this value is not too high and the attacker can obtain the required data.

Intuitively speaking, the second constraint ensures that if the smallest feasible a is large enough, no linear properties of the x walk propagate to the y walk. Indeed, any linear function on the y walk can be expressed as a function on the x walk. Since the x walk is memoryless, any linear function on a subset of x's can be written as a XOR of linear functions on the intervals of the walk. Each such interval can in turn be broken down as a sum of pairs. If a is large enough, no linear function can be a good distinguisher. Note that we concentrate on the relation between consecutive values of the state x, since in a directed random walk such pairs of states seem to be the most correlated ones.

Constructing the set $\mathcal{N}$ can be greatly simplified if $\mathcal{N}$ has periodic structure. Experiments demonstrate that for sequences with period 32 and $k = 3$, a can be as large as 12. Moreover, the best sequences have a highly regular structure, such as $n_{i1} = i - (5k \bmod 16)$ and $n_{i2} = i - 16 - (3k \bmod 16)$, where $k = i \bmod 16$. For larger periods a cannot be computed directly; an analytical approach is desirable.

As soon as the set of indices is fixed, y_i for $i > m$ can be output once x_i becomes available. The size of the buffer should be at least $i - n_{ij}$ for any $i > m$ and j. If $\mathcal{N}$ is periodic, retiring older elements can be trivially implemented by keeping several buffers and rotating between them. We note that somewhat similar buffers where used recently in the design of the stream cipher PY [BS05].

More precisely, if we choose the period $P = 32$ and $k = 3$, i.e. every output element is an XOR of three elements of the walk, the output rule can be implemented by keeping three P-word buffers, A, B, and C. Their content is shifted to the left every P cycles: A is discarded, B moves to A, and C moves to B. The last operation can be efficiently implemented by rotating pointers to the three buffers.

The exact constants chosen for n_{ij} in the output rule are chosen to maximize the girth and other useful properties of the graph of dependencies between internal variables and the output, which is available to the attacker.

Rotations.

9

Another operation used both in the output rule and in the update of the internal state is bit rotation. The motivation behind this is as follows: all the operations used in MV3 except for the rotation (that is, bitwise XOR, modular addition and multiplication) have the property that in order to know the k least significant bits of the output of the operation, it is sufficient to know the k least significant bits of the input. An attacker can use this property to devise an attack based on examining only the k least significant bits of the output words, and disregard all the other bits. This would dramatically reduce the time complexity of guess-and-determine attacks. For example, if no rotations were used in the cipher, then a variant of the standard guess-and-determine attack presented in Section 4 would apply. This variant examines only the least significant byte of every word, and reduces the time complexity of the attack to the fourth root of the original time complexity.

One possible way to overcome this problem is to use additional operations that do not have this problematic property, like multiplication in some other modular group. However, such operations slow the cipher significantly. The rotations used in MV3 can be efficiently implemented and prevent the attacker from tracing only the several least significant bits of the words. We note that similar techniques were used in the stream cipher SOSEMANUK [B+05] and in other ciphers as well.

Key setup. Since the bulk of the internal state is the table T, we concentrate on intermingling T and the pair (key, IV). Once T is fully dependent on the key and the IV, the revolving buffers and other internal variables will necessarily follow suit.

We have specified that the IV be as long as the key in order to prevent time/memory tradeoff attacks that try to invert the function $g : (key, IV) \longrightarrow Output$. The IV is known to the attacker but should not be easily predictable. One should avoid initializing the IV to zero at the beginning of every encryption session (as is frequently done in other applications), since this reduces the effective size of the IV and allows for better time/memory tradeoff attacks. A more comprehensive study of the security of MV3 with respect to time/memory tradeoff attacks is presented in Section 4.

We note that the key initialization phase is relatively slow. However, since the cipher is intended for encrypting long streams of data, the fast speed of the output stream generation compensates for it. We note that since the IV initialization phase is also quite slow, the IV should not be re-initialized too frequently.

4   Security

MV3 is designed to be a fast and very secure cipher. We are not aware of any attacks on MV3 faster than exhaustive key search even for huge key sizes of more than 1000 bits (except for the related key attacks in Section 4.6), but have only made security claims up to a 256-bit key size. In this section we analyze the security of MV3 against various kinds of cryptanalytic attacks.

4.1 Tests

We ran the cipher through several tests. First, we used two well-known batteries of general tests. One is Marsaglia's time-tested DIEHARD collection [Mar97], and the other is the NIST set of tests used to assess AES candidates [R+01] (with corrections as per [KUH04]). Both test suites were easily cleared by MV3.

In light of attacks on the first few output bytes of RC4 [MS01,Mir02], the most popular stream cipher, we tested the distribution of the initial double words of MV3 (by choosing a random 160-bit key and generating the first double word of the output). No anomalies were found.

RC4's key stream is also known to have correlations between the least significant bits of bytes one step away from each other [Gol97]. Neither of the two collections of tests specifically targets bits in similar positions of the output's double words. To compensate for that, we ran both DIEHARD and NIST's tests on the most and the least significant bits of 32-bit words of the key stream. Again, none of the tests raised a flag.

4.2 Time/Memory/Data Tradeoff Attacks

There are two main types of TMDTO (time/memory/data tradeoff) attacks on stream ciphers.

The first type consists of attacks that try to invert the function $f : State \longrightarrow Output$ (see, for example, [BS00]). In order to prevent attacks of this type, the size of the internal state should be at least twice larger than the key length. In MV3, the size of the internal state is more than 11,000 bits, and hence there are no TMDTO attacks of this type faster than exhaustive key search for keys of less than 5,500 bits length. Our table sizes are larger than what one may expect to be necessary to make adequate security claims, but we have chosen our designs so that we can keep our analysis of the components transparent, and computational overhead per word of output minimal. We intend to return to this in a future paper and propose an algorithm where the memory is premium, based on different principles for light weight applications.

The second type consists of attacks that try to invert the function $g : (Key, IV) \longrightarrow Output$ (see, for example, [HS05]). The IV should be at least as long as the key – as we have mandated in our key initialization – in order to prevent such attacks faster than exhaustive key search. We note again that if the IV's are used in some predictable way (for example, initialized to zero at the beginning of the encryption session and then incremented sequentially), then the effective size of the IV is much smaller, and this may enable a faster TMDTO attack. However, in order to overcome this problem the IV does not have to be "very random". The only thing needed is that the attacker will not be able to know which IV will be used in every encryption session. This can be achieved by initializing the IV in the beginning of the session using some (possibly insecure) publicly known pseudo-random number generator and then incrementing it sequentially.

4.3 Guess-and-Determine Attacks

A guess-and-determine attack against RC4 appears in [K+98]. The attack, adapted to MV3, has the following form:

1. The attacker guesses the values of all the 32 words in buffers A and B in some loop of MV3, and the values of j, c, x in the beginning of the loop.
2. Using the guessed values of the words in B, the attacker traces the value of j during the whole loop.
3. Using the output stream and the guessed values, the attacker traces the value of x during the whole loop.
4. Using the update rule of x and the knowledge of j, the attacker gets the values of 32 words in the T array. If the attacker encounters a word whose value is already known to her, she checks whether the values match, and if not, discards the initial guess.
5. The attacker moves on to the next loop. Note that due to the knowledge of buffer A and some of the words $T[j]$, the attacker can trace the update of c and of the T register.
6. Each "collision" in the T array supplies the attacker with a 32-bit filtering condition. Since the attacker started by guessing 66 32-bit words, finding 70 collisions should be sufficient to discard all the wrong guesses and find the right one. In 10 loops we expect to find more than 70 such collisions, and hence 2^{14} bits of key stream will be sufficient for the attacker to find the internal state of the cipher.
7. Once the attacker knows the internal state, she can compute the entire output stream without knowing the key.

However, the time complexity of this attack is quite large – more than 2^{2000}, since the attacker starts with guessing more than 2000 bits of the state. Hence, this attack is slower than exhaustive key search for keys of less then 2000 bits length.

4.4 Guess-and-Determine Attacks Using the Several Least Significant Bits of the Words

Most of the operations in MV3 allow the attacker to focus the attack on the k least significant bits, thus dramatically reducing the number of bits guessed in the beginning of the attack. We consider two reasonable attacks along these lines.

The first attack concentrates on the least significant bit of the output words. In this case, since the least significant bit of c is fixed to 1, the attacker can disregard c at all. However, in this case the attacker cannot trace the values of j, and guessing them all the time will require a too high time complexity. Hence, it seems that this attack is not applicable to MV3.

The second attack concentrates on the eight least significant bits of every output word. If there were no rotations in the update and output rules, the

attacker would indeed be able to use her guess to trace the values of j and the eight least significant bits in all the words of the internal state. This would result in an attack with time complexity of about 2^{600}. However, the rotations cause several difficulties for such an attack:

1. Due to the rotations, the values the attacker knows after her initial guess are bits $24 - 31$ of the words in buffer C, bits $16 - 23$ of the words in buffer B, and bits $0 - 7$ of the words in buffer A (these are the bits that affect the eight LSB's of the output words). Yet the attacker still does not know the eight LSB's of the words in buffer B and hence cannot find the value of j.
2. If the attacker rolls the arrays to the previous loop, she can find the eight LSB's of the words in buffer B. However, the attacker cannot use her guess to get information from the previous loop. In that loop she knows bits $0 - 7$ of the words in buffer B and bits $16 - 23$ of the words in buffer C, but in order to compare the information with the eight LSB's of the output stream, she needs bits $16 - 23$ of the words in buffer B and bits $24 - 31$ of the words in buffer C. Therefore, the guesses in consecutive loops cannot be combined together.

Hence, it seems that both of the attacks cannot be applied, unless the attacker guesses the full values of all the words in two buffers, which leads to the attack described in subsection 4.3 (with a time complexity of more than 2^{2000}).

4.5   Linear Distinguishing Attacks

Linear distinguishing attacks aim at distinguishing the cipher output from random streams, using linear approximations of the non-linear function used in the cipher – in our case, the random walk.

In [CHJ02], Coppersmith et al. developed a general framework to evaluate the security of several types of stream ciphers with respect to these attacks. It appears that the structure of MV3 falls into this framework, to which [CHJ02, Theorem 6] directly applies:

Theorem 1. *Let ϵ be the bias of the best linear approximation one can find for pairs x_i, x_{i+1}, and let $A_N(a)$ be the number of equations of type (3.1) that hold for the sequence $y_m, y_{m+1}, \dots$. Then the statistical distance between the cipher and the random string is bounded from above by*

$$\sqrt{\sum_{a=1}^{N} A_N(a)\epsilon^{2a}}. \tag{4.1}$$

Note that for $\epsilon \ll 1/2$, the bound (4.1) is dominated by the term with the smallest a, which equals to 12 in our case. Since the relation between x_i and x_{i+1} is based on a random walk, ϵ is expected to be very small. Since the statistical distance is of order ϵ^{24}, we expect that the cipher cannot be distinguished from a random string using a linear attack, even if the attacker uses a very long output stream for the analysis.

4.6 Related-Key Attacks and Key Schedule Considerations

Related key attacks study the relation between the key streams derived from two unknown, but related, secret keys. These attacks can be classified into distinguishing attacks, that merely try to distinguish between the key stream and a random stream, and key recovery attacks, that try to find the actual values of the secret keys.

One of the main difficulties in designing the key schedule of a stream cipher with a very large state is the vulnerability to related-key distinguishing attacks. Indeed, if the key schedule is not very complicated and time consuming, an attacker may be able to find a relation between two keys that propagates to a very small difference in the generated states. Such small differences can be easily detected by observing the first few words of the output stream.

It appears that this difficulty applies to the current key schedule of MV3. For long keys, an attacker can mount a simple related-key distinguishing attack on the cipher. Assume that $keylength = 8192/t$. Then in any step of the key initialization phase, every word of the key affects exactly t words in the T array, after which the main loop of the cipher is run eight times and the output stream is XORed (bit-wise) to the content of the T array. The same is repeated with the IV replacing the key in the IV initialization phase.

The attacker considers encryption under the same key with two IVs that differ only in one word. Since the key is the same in the two encryptions, the entire key initialization phase is also the same. After the first step of the IV initialization, the intermediate values differ in exactly t words in the T array. Then, the main loop is run eight times. Using the random walk assumption, we estimate that, with probability $(1 - t/256)^{256}$, each of the corresponding words in the respective T arrays used in these eight loops are equal, making the output stream equal in both encryptions. Hence, with probability $(1 - t/256)^{256}$, after the first step of the IV initialization the arrays A, B, and C are equal in both encryptions and the respective T arrays differ only in t words.

The same situation occurs in the following three steps of the IV initialization. Therefore, with probability

$$(1 - t/256)^{256} \cdot (1 - 2t/256)^{256} \cdot (1 - 3t/256)^{256} \cdot (1 - 4t/256)^{256} \qquad (4.2)$$

all of the corresponding words used during the entire initialization phase are equal in the two encryptions. Then with probability $(1 - 4t/256)^{32}$ all of the corresponding words used in the first loop of the key stream generation are also equal in the two encryptions, resulting in two equal key streams. Surely this can be easily recognized by the attacker after observing the key stream generated in the first loop.

In order to distinguish between MV3 and a random cipher, the attacker has to observe about

$$M = (1-t/256)^{-256} \cdot (1-2t/256)^{-256} \cdot (1-3t/256)^{-256} \cdot (1-4t/256)^{-256} \cdot (1-4t/256)^{-32}$$
$$(4.3)$$

pairs of related IVs, and for each pair she has to check whether there is equality in the first 32 key stream words. Hence, the data and time complexities of the attack are about $2^{10}M$. For keys of length at least 384 bits, this attack is faster than exhaustive key search. Note that (somewhat counter intuitively) the attack becomes more efficient as the length of the key is increased. The attack is most efficient for 8192-bit keys, where the data complexity is about 2^{10} bits of key stream encrypted under the same key and 2^{15} pairs of related IVs, and the time complexity is less than 2^{32} cycles. For keys of length at most 256 bits, the data and time complexities of the attack are at least 2^{618} and hence the related-key attack is much slower than exhaustive key search.

If we try to speed up the key schedule by reducing the number of loops performed at each step of the key schedule, the complexity of the related-key attack is reduced considerably. For example, if the number of loops is reduced to four (instead of eight), the complexity of the related-key attack becomes

$$M' = (1-t/256)^{-128} \cdot (1-2t/256)^{-128} \cdot (1-3t/256)^{-128} \cdot (1-4t/256)^{-128} \cdot (1-4t/256)^{-32}$$
(4.4)

In this case, the attack is faster than exhaustive key search for keys of length at least 320 bits. If the number of loops is further reduced to two, the complexity of the attack becomes

$$M'' = (1-t/256)^{-64} \cdot (1-2t/256)^{-64} \cdot (1-3t/256)^{-64} \cdot (1-4t/256)^{-64} \cdot (1-4t/256)^{-32}$$
(4.5)

and then the attack is faster than exhaustive search for keys of length at least 224 bits.

If the key schedule is speed up by inserting the output of the eight loops into the T array, instead of XORing it bit-wise to the content of the T array (as was proposed in a previous variant of the cipher), the complexity of the related-key attack drops to

$$M''' = ((1 - t/256)^{-256})^4$$
(4.6)

In this case, the attack is faster than exhaustive key search even for 256-bit keys.

Hence, the related-key attack described above is a serious obstacle to speeding up the key schedule. However, we note that the related-key model in general, and in particular its requirement of obtaining a huge number of encryptions under different related-IV pairs, is quite unrealistic.

4.7   Other Kinds of Attacks

We subjected the cipher to other kinds of attacks, including algebraic attacks and attacks exploiting classes of weak keys. We did not find any discrepancies in these cases.

5   Summary

We have proposed a new fast and secure stream cipher, MV3. The main attributes of the cipher are efficiency in software, high security, and its basis upon clearly analyzable components.

The cipher makes use of new rapidly mixing random walks, to ensure the randomness in the long run. The randomness in the short run is achieved by revolving buffers that are easily implemented in software, and break short correlations between the words of the internal state.

The cipher is word-based, and hence is most efficient on 32-bit processors. On a Pentium IV, the cipher runs with a speed of 4.8 clocks a byte.

Acknowledgements

References

[ARV95]   W. Aiello, S. Rajagopalan, and R. Venkatesan, *Design of Practical and Provably Good Random Number Generators*, Proc. of SODA'95, pp. 1–9, 1995.

[AR94]   N. Alon and Y. Roichman, *Random Cayley Graphs and Expanders*, Rand. Str. Alg. vol. 5(2), pp. 271–284, 1994.

[And94]   R. Anderson, *On Fibonacci Keystream Generators*, Proc. of FSE'94, pp. 346–352, 1994.

[BD76]   C. Bays and S. Durham, *Improving a Poor Random Number Generator*, ACM Trans. on Math. Software 2(1), pp. 59–64, 1976.

[B+05]   C. Berbain, O. Billet, A. Canteaut, N. Courtois, H. Gilbert, L. Goubin, A. Gouget, L. Granboulan, C. Lauradoux, M. Minier, T. Pornin, and H. Sibert, *Sosemanuk, a Fast Software-Oriented Stream Cipher*, submitted to Ecrypt, 2005.

[BS05]   E. Biham and J. Seberry, *Py (Roo): A fast and secure stream cipher using rolling arrays*, submitted to Ecrypt, 2005.

[BS00]   A. Biryukov and A. Shamir, *Cryptanalytic Time/Memory/Data Tradeoffs for Stream Ciphers*, Proc. of Asiacrypt'00, pp. 1–13, 2000.

[BS87]   A. Broder and E. Shamir, *On the Second Eigenvalue of Random Regular Graphs* (Preliminary Version), Proc. of FOCS'87, pp. 286–294, 1987.

[Bru84]   J. Bruer, *On Pseudo-Random Sequences as Crypto Generators*, Proc. of the Int. Zurich Seminar on Digital Comm., pp. 157–161, 1984.

[CHJ02]   D. Coppersmith, S. Halevi, and C. S. Jutla, *Cryptanalysis of stream ciphers with linear masking*, Proc. of CRYPTO'02, pp. 515–532, 2002.

[CKM93]   D. Coppersmith, H. Krawczyk, and Y. Mansour, *The Shrinking Generator*, Proc. of CRYPTO'93, pp. 22–39, 1993.

[Cou02]   N. Courtois, *Higher Order Correlation Attacks, XL Algorithm and Cryptanalysis of Toyocrypt*, Proc. of ICISC'02, pp. 182–199, 2002.

[Cou04]   N. Courtois, *Algebraic Attacks on Combiners with Memory and Several Outputs*, Proc. of ICISC'04, pp. 3–20, 2004.

[DC06]   C. De Canniere, *eSTREAM testing framework*, 2006, avalable on-line at http://www.ecrypt.eu.org/stream.

[GG79]   O. Gabber and Z. Galil, *Explicit constructions of linear size superconcentrators*, Proc. of FOCS'79, pp. 364–370, 1979.

[Gef73]   P. Geffe, *How to Protect Data with Ciphers that are Really Hard to Break*,
           Electronics, 46, pp. 99–101, 1973.

[Gi98]   D. Gillman, *A Chernoff bound for random walks on expander graphs*, SIAM
           J. Comput. 27 (4), pp.1203–1220 (1998).

[GL89]   O. Goldreich and L. Levin, *A Hard-Core Predicate for all One-Way Func-
           tions*, Proc. of STOC'89, pp. 25–32, 1989.

[Gol97]   J. Golić, *Linear statistical weakness of alleged RC4 keystream generator*,
           Proc. of Eurocrypt'97, pp. 226–238, 1997.

[Goli97]   J. Golić, *Cryptanalysis of Alleged A5 Stream Cipher*, Proc. of Eurocrypt'97,
           pp. 239–255, 1997.

[Golo67]   S. Golomb, *Shift Register Sequences*, Holden-Day, San Francisco, 1967.

[HCJ02]   S. Halevi, D. Coppersmith, and C. Jutla, *Scream: a Software-Efficient
           Stream Cipher*, Proc. of FSE'02, pp. 195–209, 2002.

[H+99]   J. Håstad, R. Impagliazzo, L. Levin, and M. Luby, *A Pseudorandom Gener-
           ator from Any One-way Function*, SIAM J. Comput. 28(4), pp. 1364–1396,
           1999.

[HS05]   J. Hong, P. Sarkar, *Rediscovery of Time Memory Tradeoffs*, 2005. Available
           online at http://eprint.iacr.org/2005/090.

[HLW06]   S. Hoory, N. Linial, and A. Wigderson, *Expander graphs and their applica-
           tions*, Bull. Amer. Math. Soc. **43** (2006), pp. 439–561.

[JMV05]   D. Jao, S. Miller, and R. Venkatesan, *Do All Elliptic Curves of the Same
           Order Have the Same Difficulty of Discrete Log?*, Proc. of Asiacrypt'05,
           pp. 21–40, 2005.

[JM85]   S. Jimbo, A. Maruoka, *Expanders obtained from affine transformations*,
           Proc. of STOC'85, pp. 88–97, 1985.

[KUH04]   S. Kim, K. Umeno, and A. Hasegawa, *Corrections of the NIST statistical test
           suite for randomness*, Cryptology ePrint Archive, Report 2004/018, 2004.

[Kla81]   M. Klawe, *Non-existence of one-dimensional expanding graphs*, Proc. of
           FOCS'81, pp. 109–114, 1981.

[KS02]   A. Klimov and A. Shamir, *A New Class of Invertible Mappings*, Proc. of
           CHES'02, pp. 470–483, 2002.

[KS04]   A. Klimov and A. Shamir, *New Cryptographic Primitives Based on Multi-
           word T-Functions*, Proc. of FSE'04, pp. 1–15, 2004.

[K+98]   L. Knudsen, W. Meier, B. Preneel, V. Rijmen, and S. Verdoolaege, *Analysis
           Methods for (Alleged) RC4*, Proc. of ASIACRYPT'98, pp.327-341, 1998.

[Knu97]   D. Knuth, *The Art of Computer Programming*, vol. 2, 3rd Ed., Addison
           Wesley, 1997.

[LPS86]   A. Lubotzky, R. Phillips, and P. Sarnak, *Explicit expanders and the Ra-
           manujan conjectures*, Proc. of STOC'86, pp. 240–246, 1986.

[MM65]   M. MacLaren and G. Marsaglia, *Uniform Random Number Generators*,
           J. of the ACM 12(1), pp. 83–89, 1965.

[MS01]   I. Mantin and A. Shamir, *A practical attack on broadcast RC4*, Proc. of
           FSE'01, pp. 152–164, 2001.

[Mar73]   G.A. Margulis *Explicit constructions of expanders*, Problemy Peredači In-
           formacii, 9, pp. 71–80, 1973 (Russian).

[Mar97]   G. Marsaglia, *DIEHARD battery of tests*, available from
           http://stat.fsu.edu/~geo/.

[Mas69]   J. Massey, *Shift-register Synthesis and BCH decoding*, IEEE Transactions
           on Information Theory, 15, pp. 122–127, 1969.

[Mir02]   I. Mironov, *(Not so) random shuffles of RC4*, Proc. of CRYPTO'02, pp. 304–
           319, 2002.

[Neu51]   J. von Neumann. *Various Techniques Used in Connection with Random Digits*, In A. Traub, editor, John von Neumann, Collected Works, vol. 5. Macmillan, 1963.

[Ple77]   V. Pless, *Encryption Schemes for Computer Confidentiality*, IEEE Transactions on Computers C-26(11), pp. 1133–1136, 1977.

[P+03]   B. Preneel et al., *NESSIE Security Report, version 2.0*, 2003.

[RC98]   P. Rogaway and D. Coppersmith, *A Software-Optimized Encryption Algorithm*, J. of Cryptology 11(4), pp. 273–287, 1998.

[Rub79]   F. Rubin, *Decrypting a Stream Cipher Based on J-K Flip-Flops*, IEEE Transactions on Computers C-28(7), pp. 483–487, 1979.

[R+01]   A. Rukhin, J. Soto, J. Nechvatal, M. Smid, E. Barker, S. Leigh, M. Levenson, M. Vangel, D. Banks, A. Heckert, J. Dray, and S. Vo, *A statistical test suite for random and pseudorandom number generators for cryptographic applications*, NIST Special Publication 800-22, `http://www.nist.gov`, 2001.

[Sch95]   B. Schneier, *Applied Cryptography: Protocols, Algorithms, and Source Code in C*, 2nd Ed., John Wiley & Sons, 1995.

[Sie84]   T. Siegenthaler, *Correlation-Immunity of Nonlinear Combining Functions for Cryptographic Applications*, IEEE Transactions on Information Theory 30(5), pp. 776–780, 1984.

A   Appendix: Mathematical Background

The good long term randomness properties of the internal state of MV3 are achieved by updates using rapidly mixing random walks. Actually, the walks are only pseudo-random since the cipher is fully deterministic, but we desire the update rule to be as close as possible to a random walk. In this appendix we recall some mathematics used to study random walks, such as expander graphs and the rapid mixing property. Afterwards, we describe two particular types of random walks used in the MV3 cipher: a well-known random walk in the additive group $\mathbb{Z}/2^n\mathbb{Z}$, and a novel random walk that mixes addition with multiplication operations.

A.1   Rapidly Mixing Random Walks and Expander Graphs

Recall that a random walk on a graph starts at a node z_0, and at each step moves to a node connected by one of its adjacent edges at random. A lazy random walk is the same, except that it stays at the same node with probability $1/2$, and otherwise moves to an adjacent node at random. Intuitively, a random walk is called "rapidly mixing" if, after a relatively short time, the distribution of the state of the walk is close to the uniform distribution — regardless of the initial distribution of the walk.

Next, we come to the notion of expander graph. Let Γ be an undirected k-regular graph on $N < \infty$ vertices. Its adjacency operator acts on $L^2(\Gamma)$ by summing the values of a function at the neighbors of a given vertex:

$$(Af)(x) \;=\; \sum_{x \sim y} f(y)\,. \qquad\qquad (A.1)$$

The spectrum of A is contained in the interval $[-k, k]$. The *trivial eigenvalue* $\lambda = k$ is achieved by the constant eigenvector; if the graph is connected then this eigenvalue has multiplicity 1, and all other eigenvalues are strictly less than k. A sequence of k-regular graphs (where the number of vertices tends to infinity) is customarily called a sequence of *expanders* if all nontrivial eigenvalues λ of all the graphs in the sequence satisfy the bound $|\lambda| \leq k - c$ for an absolute constant c. We shall take a slightly more liberal tack here and consider graphs which satisfy the weaker eigenvalue bound $|\lambda| \leq k - c(\log N)^{-A}$ for some constant $A \geq 0$.

The importance of allowing the lenient eigenvalue bound $|\lambda| \leq k - c(\log N)^{-A}$ is that a random walk on such a graph mixes in $\mathrm{polylog}(N)$ time, even if $A > 0$. More precisely, we have the following estimate (see, for example, [JMV05, Proposition 3.1]).

Proposition A.1 *Let Γ be a regular graph of degree k on N vertices. Suppose that the eigenvalue λ of any nonconstant eigenvector satisfies the bound $|\lambda| \leq \sigma$ for some $\sigma < k$. Let S be any subset of the vertices of Γ, and x be any vertex in Γ. Then a random walk of any length at least $\frac{\log 2N/|S|^{1/2}}{\log k/\sigma}$ starting from x will land in S with probability at least $\frac{|S|}{2N} = \frac{|S|}{2|\Gamma|}$.*

Indeed, with $\sigma = k - c(\log N)^{-A}$, the random walk becomes evenly distributed in the above sense after $O((\log N)^{A+1})$ steps.

Next, we come to the issue of estimating the probability that the random walk returns to a previously visited node. This is very important for cryptographic purposes, since short cycles lead to relations which an attacker can exploit. The following result gives a very precise estimate of how unlikely it is that a random walk returns to the vertex it starts from. More generally, it shows that if one has *any* set S consisting of, say, one quarter of all nodes, then the number of visits of the random walk to this set will be exceptionally close to that of a purely random walk in the sense that it will obey a Chernoff type bound. This in turn allows one to show that the idealized cipher passes all the moment tests.

Theorem A.2 *([Gi98, Theorem 2.1]) Consider a random walk on a k-regular graph Γ on N vertices for which the second-largest eigenvalue of the adjacency operator A equals $k - \varepsilon k$, $\varepsilon > 0$. Let S be a subset of the vertices of Γ, and t_n the random variable of how many times a particular walk of n steps along the graph lands in S. Then, as sampled over all random walks, one has the following estimate for any $x > 0$:*

$$\mathrm{Prob}\left[\left| t_n - n\frac{|S|}{|\Gamma|} \right| \geq x \right] \leq \left(1 + \frac{x\varepsilon}{10n}\right) e^{-x^2 \varepsilon/(20n)}. \qquad (A.2)$$

Thus even with a moderately small value of ε, the random walk avoids dwelling in any one place overly long. The strength of the Chernoff type bound (A.2) is also useful for ruling out other substitutes for random walks because of their non-random behavior. For example, it has been shown by Klimov and

Shamir [KS04] that iterates of their T-functions on n-bit numbers cycle through all n-bit numbers exactly once, whereas our random walks will have very large expected return times.

In practice, algorithms often actually consider random walks on *directed* graphs. The connection between rapid mixing of directed graphs (with corresponding adjacency/transition matrix M) and undirected graphs is as follows. A result of J. Fill shows that if the additive reversalization (whose adjacency matrix is $M + M^t$) or multiplicative reversalization (whose adjacency matrix is MM^t) rapidly mixes, then the lazy random walk on the directed version also rapidly mixes. From this it is easy to derive the effect of having no self-loops as well. Moreover, if the undirected graph has expansion, then so does the directed graph — provided it has an Eulerian orientation. It is important to note that this implication can also be used to greatly improve poorly mixing graphs. For example, we will present a graph in Theorem A.4 which involves additive reversalization in an extreme case: where the original graph is definitely not an expander (the random walk mixes only in time proportional to the number of vertices N), yet the random walk on the additive reversalization mixes in polylog(N) time.

Expander graphs are natural sources of (pseudo)randomness, and have numerous applications as extractors, de-randomizers, etc. (see [HLW06]). However, there are a few practical problems that have to be resolved before expanders can be used in cryptographic applications. One of these, as mentioned above, is a serious security weakness: the walks in such a graph have a constant probability of returning to an earlier node in constant number of steps. It is possible to solve this problem by adding the current state (as a binary string) to that of another process which has good short term properties, but this increases the cache size. In addition, if the graph has large directed girth (i.e. no short cycles), then the short term return probabilities can be minimized or even eliminated.

A.2 Additive Random Walks on $\mathbb{Z}/2^n\mathbb{Z}$

Most of the random walks used in the cipher, namely the random walks used in the updates of j, x, and T, are performed in the additive group $\mathbb{Z}/2^n\mathbb{Z}$. The mixing properties of these walks can be studied using results on *Cayley graphs* of this group. In general, given a group G with a set of generators S, the Cayley graph $X(G, S)$ of G with respect to S is the graph whose vertices consist of elements of G, and whose edges connect pairs (g, gs_i), for all $g \in G$ and $s_i \in S$.

Alon and Roichman [AR94] gave a detailed study of the expansion properties of abelian Cayley graphs, viewed as undirected graphs. They showed that $X(G, S)$ is an expander when S is a randomly chosen subset of G whose size is proportional to $\log |G|$. More precisely, they have shown the following:

Theorem A.3 *([AR94]). For every $0 < \delta < 1$ there is a positive constant $c = c(\delta)$ such that the following assertion holds. Let G be a finite abelian group, and let S be a random set of $c \log |G|$ elements of G. Then the expected value of*

the second largest eigenvalue of the normalized adjacency matrix of $X(G, S)$ is at most $1 - \delta$.

The normalized adjacency matrix is simply the adjacency matrix, divided by the degree of the graph. Thus, in light of the results of the last section, the proposition implies that these random abelian Cayley graphs are expanders, and hence random walks on them mix rapidly.

Using second-moment methods it can be shown that the graph is ergodic (and also that the length of the shortest cycle is within a constant factor of $\log |\Gamma|$) with overwhelming probability over the choice of generators. The significance of this is that we need not perform a lazy random walk, which would introduce undesirable short term correlations as well as waste cycles and compromise the cryptographic strength.

In MV3, the rapid mixing of the random walks updating x, j and T follows from the theorem of Alon and Roichman. For example, consider the update rule of x:

$$x \leftarrow x + T[j],$$

The update rule corresponds to a random walk on the Cayley graph $X(G, S)$ where G is the additive group $\mathbb{Z}/2^n\mathbb{Z}$ and S consists of the 256 elements of the T register. Note that we have $|S| = 4 \log_2(|G|)$. In order to apply the theorem of Alon and Roichman we need that the elements of the T array will be random and that the walk will be random, that is, that j will be chosen each time randomly in $\{0, \ldots, 255\}$. Hence, assuming that j and T are uniformly distributed, we have a rapid mixing property for x. Similarly, one can get rapid mixing property for j using the randomness of x.

A.3   Non-linear Random Walks

In order to introduce some nonlinearity to the cipher, we use a multiplier c that affects the cipher output in a multiplicative way. The multiplier itself is updated using a nonlinear random walk that mixes addition and multiplication operations. The idealized model of this random walk is described in the following theorem:

Theorem A.4 *Let N and r be relatively prime positive integers greater than 1, and $\bar{r}$ an integer such that $r\bar{r} \equiv 1 \pmod{N}$. Let Γ be the 4-valent graph on $\mathbb{Z}/N\mathbb{Z}$ in which each vertex x is connected to the vertices $r(x + 1)$, $r(x - 1)$, $\bar{r}x + 1$, and $\bar{r}x - 1$. Then there exists a positive constant $c > 0$, depending only on r, such that all nontrivial eigenvalues λ of the adjacency matrix of Γ satisfy the bound*

$$|\lambda| \;\leq\; 4 \;-\; \frac{c}{(\log N)^2}\,, \tag{A.3}$$

or are of the form

$$\lambda \;=\; 4\cos(2\pi k/N) \quad \text{for k satisfying } \; rk \equiv k \pmod{N}.$$

In particular, if N is a power of 2 and $(r - 1, N) = 2$, then Γ is a bipartite graph for which all eigenvalues not equal to ± 4 satisfy (A.3).

The proof of the theorem can be found in Appendix B. The result means that for a fixed r, Γ is an expander graph in the looser sense that its eigenvalue separation is at least $c/(\log N)^2$ for N large. This is still enough to guarantee that the random walk on the graph mixes rapidly (i.e. in polylog(N) time).

We note that although we use an additive notation, the theorem holds for any cyclic group, for example a multiplicative group in which the multiplication by r corresponds to exponentiation (this is the non-linearity we are referring to). Also the expressions $r(x \pm 1)$, $\bar{r}x \pm 1$ may be replaced by $r(x \pm g)$, $\bar{r}x \pm g$ for any integer g relatively prime to N. Additionally, the expansion remains valid if a finite number of extra relations of this form are added.

We also note that it was observed by Klawe [Kla81] that graphs of the form described in the theorem cannot be expanders with a *constant* eigenvalue separation, i.e. the assertion of the theorem is false without the logarithmic terms in the denominator. Even so, this would not change the polynomial dependence of $\log N$ in the mixing time, but only improve its exponent.

The operation used in the MV3 cipher algorithm itself is slightly different: it involves not only addition steps, but also a squaring or cubing step. Though this is not covered directly the Theorem, it is similar in spirit. We have run extensive numerical tests and found that this operation can in fact greatly enhance the eigenvalue separation, apparently giving eigenvalue bounds of the form $|\lambda| \leq \sigma$ for some constant $\sigma < 4$ (Klawe's theorem does not apply to this graph). Thus the squaring or cubing operations are not covered by the theoretical bound (A.3), but empirically give stronger results anyhow.

B   Appendix: Proof of Theorem A.4

We begin with some considerations in harmonic analysis. We may write the adjacency operator on $L^2(\Gamma) = L^2(\mathbb{Z}/N\mathbb{Z})$ as

$$A \;=\; MP + P^t M \;=\; (MP) \;+\; (MP)^t, \tag{B.1}$$

where

$$(Mf)(a) \;=\; f(a+1) \;+\; f(a-1) \tag{B.2}$$

and

$$(Pf)(a) \;=\; f(ra)\,, \quad (P^t f)(a) \;=\; f(\bar{r}a)\,. \tag{B.3}$$

The additive characters of $\mathbb{Z}/N\mathbb{Z}$ play an important role. They are indexed by integers $k \in \mathbb{Z}/N\mathbb{Z}$ as follows:

$$\chi = \chi_k \;:\; a \;\mapsto\; e^{2\pi i k a/N}\,. \tag{B.4}$$

These characters are eigenfunctions of M with eigenvalue $\lambda_\chi = \chi(1) + \chi(-1)$, so that $\lambda_{\chi_k} = 2\cos(2\pi k/N)$. Furthermore $P\chi = \chi^r$, which means $P\chi_k = \chi_{rk}$ and $P^t \chi_k = \chi_{\bar{r}k}$.

The operator A is self-adjoint, so its spectrum may be analyzed by means of the Rayleigh quotient. To prove the theorem, it suffices to show the existence of a constant $c > 0$ such that

$$\max_{v \perp \mathbf{1}} \left| \frac{\langle Av, v \rangle}{\langle v, v \rangle} \right| \;\; \leq \;\; 4 \; - \; \frac{c}{(\log N)^2} \, . \tag{B.5}$$

Here $\mathbf{1}$ denotes the constant function on the graph, which is the trivial character χ_0, and $\langle v, w \rangle = \sum_{j=1}^{N} v_j \overline{w_j}$ denotes the L^2-inner product of functions on Γ. Every vector $v \in L^2(\Gamma)$ has an expansion of the form $v = \sum c_\chi \cdot \chi$ in terms of the basis of characters χ_k ; the condition that $v \perp \mathbf{1}$ is simply equivalent to requiring that $c_{\chi_0} = 0$.

Let us now calculate the inner products in (B.5) for $v = \sum_{\chi \neq \mathbf{1}} c_\chi \cdot \chi$, using the fact that $\langle \chi, \chi' \rangle = N$ if $\chi = \chi'$, and 0 otherwise. First, $\langle v, v \rangle = \sum N |c_\chi|^2$. As

$$A\chi_k \;\; = \;\; MP\chi_k + P^t M \chi_k \;\; = \;\; M\chi_{rk} + P^t \lambda_k \chi_k \;\; = \;\; \lambda_{rk} \chi_{rk} + \lambda_k \chi_{\bar{r}k} \, , \tag{B.6}$$

χ_k is an eigenfunction of A with eigenvalue $2\lambda_k$ if $k \equiv rk \pmod{N}$. This accounts for the explicit eigenvalues which are mentioned in the statement of the theorem. We have that

$$Av \;\; = \;\; \sum_{k=1}^{N-1} c_k \, \lambda_{rk} \, \chi_{rk} \; + \; \sum_{k=1}^{N-1} c_k \, \lambda_k \, \chi_{\bar{r}k} \, , \tag{B.7}$$

where we have set $c_k = c_{\chi_k}$ for notational convenience. The inner product $\langle Av, v \rangle$ satisfies

$$
\begin{aligned}
\langle Av, v \rangle \;\; &= \;\; \sum_{k,\ell=1}^{N-1} c_k \, \overline{c_\ell} \left[\lambda_{rk} \langle \chi_{rk}, \chi_\ell \rangle \; + \; \lambda_k \langle \chi_{\bar{r}k}, \chi_\ell \rangle \right] \\
&= \;\; N \sum_{k=1}^{N-1} c_k \, \overline{c_{rk}} \, \lambda_{rk} \; + \; N \sum_{\ell=1}^{N-1} \overline{c_\ell} \, c_{r\ell} \, \lambda_{r\ell} \\
&\leq \;\; N \sum_{k=1}^{N-1} |c_k| \, |c_{rk}| \, |\lambda_{rk}| \; + \; N \sum_{\ell=1}^{N-1} |c_{r\ell}| \, |c_\ell| \, |\lambda_{r\ell}| \, .
\end{aligned}
\tag{B.8}
$$

We are now reduced to a problem about quadratic forms. For $1 \leq k, \ell \leq N - 1$, let

$$
a_{k,\ell} \;\; = \;\; \begin{cases} |\lambda_k| + |\lambda_\ell|, & k \equiv r\ell \pmod{N} \quad \text{and} \quad \ell \equiv rk \pmod{N} \\ |\lambda_k|, & k \equiv r\ell \pmod{N} \quad \text{and} \quad \ell \not\equiv rk \pmod{N} \\ |\lambda_\ell|, & k \not\equiv r\ell \pmod{N} \quad \text{and} \quad \ell \equiv rk \pmod{N} \\ 0, & \text{otherwise.} \end{cases}
\tag{B.9}
$$

We need to show the existence of a constant $c > 0$ for which

$$\sum_{k,\ell=1}^{N-1} a_{k,\ell} \, y_k \, y_\ell \;\; \leq \;\; \left(4 \; - \; \frac{c}{(\log N)^2} \right) \sum_{k=1}^{N-1} y_k^2 \tag{B.10}$$

23

for any $N-1$ real numbers $y_1, \ldots, y_{N-1}$. Since the spectrum coming from the characters χ_k for which $rk \equiv k \pmod{N}$ has already been accounted form, we may assume $y_k = 0$ for such k, and modify (B.9) so that

$$a_{k,\ell} \;=\; a_{\ell,k} \;=\; 0 \quad \text{if } rk \equiv k \pmod{N}. \tag{B.11}$$

For this we use the following inequality.

Lemma B.1 *(Proposition 8 in [JM85]) Let (a_{ij}) be a symmetric $n \times n$ real matrix whose entries are nonnegative. Let (γ_{ij}) be an $n \times n$ real matrix with positive entries for which $\gamma_{ij}\gamma_{ji} = 1$. Then*

$$\left| \sum_{i,j \le n} a_{ij}\, y_i\, y_j \right| \;\le\; \max_{i \le n} \left(\sum_{j \le n} \gamma_{ij}\, a_{ij} \right) \sum_{i \le n} |y_i|^2 . \tag{B.12}$$

Since the proof is short, we have included it here.

Proof: Since $0 \le (\gamma^{1/2}y_i \pm \gamma^{-1/2}y_j)^2 = \gamma y_i^2 + \gamma^{-1} y_j^2 \pm 2y_iy_j$, we may bound

$$
\begin{aligned}
\left| \sum_{i,j \le n} a_{ij}\, y_i\, y_j \right|
&\le\; \frac{1}{2} \sum_{i,j \le n} 2\, a_{ij}\, |y_i|\, |y_j| \\
&\le\; \frac{1}{2} \sum_{i,j \le n} a_{ij}\, (\gamma_{ij}\, y_i^2 \,+\, \gamma_{ji}\, y_j^2) \\
&=\; \sum_{i,j \le n} a_{ij}\, \gamma_{ij}\, y_i^2 \\
&\le\; \max_{i \le n} \left(\sum_{j \le n} \gamma_{ij}\, a_{ij} \right) \sum_{i \le n} |y_i|^2 . \qquad \square
\end{aligned}
\tag{B.13}
$$

Now we specify which γ_{ij} to use in bounding our sequence. (In what follows we closely follow the technique of Jimbo-Maruoka from a different example in [JM85].) Given an element $i \in \mathbb{Z}/N\mathbb{Z}$, we let $||i||$ denote the distance from i to $N\mathbb{Z}$. In other words, if i is represented by a residue between 0 and N, $||i|| = \min\{i, N-i\}$. For $s \ge 1$ set

$$a_s \;=\; 1 \;-\; s\, \frac{d}{(\log N)^2} , \tag{B.14}$$

where d is a small constant (depending on r) which shall be chosen later. Given an integer m relatively prime to N, we define s_m to be the largest integer s such that r^s divides $||2m||$. Since $||2m|| \le N/2$, $s = O(\log N)$ and $a_s > 0$ provided d is sufficiently small. We set $\gamma_{k\ell} = 1$ **except** in the following cases:

	$\|2k\| < N/(2r)$	$\|2k\| \ge N/(2r)$
$\|2\ell\| < N/(2r)$	$\gamma_{k,rk} = a_{s_k}$ $\gamma_{r\ell,\ell} = a_{s_\ell}^{-1}$	$\gamma_{r\ell,\ell} = a_{s_\ell}^{-1}$
$\|2\ell\| \ge N/(2r)$	$\gamma_{k,rk} = a_{s_k}$	(no exceptions)

This satisfies the requirement that $\gamma_{k,\ell}\,\gamma_{\ell,k} = 1$. We will choose the constant d to be smaller yet so that each $\gamma_{k\ell} \leq 1 + \frac{1}{2}(1 - \cos \pi/(2r))$, as we may do. To finish the proof we must now show the existence of a constant $c > 0$ so that

$$\sum_{\ell=1}^{N-1} \gamma_{k,\ell}\, a_{k,\ell} \;=\; \gamma_{k,rk}\,|\lambda_{rk}| \;+\; \gamma_{k,\bar{r}k}\,|\lambda_k| \;\leq\; 4 \;-\; \frac{c}{(\log N)^2} \tag{B.15}$$

for each $1 \leq k \leq N - 1$ which does not satisfy $rk \equiv k \pmod{N}$.

Case I: Assume that $||2k|| \geq N/(2r)$. Then

$$k \;\in\; \left[\frac{N}{4r}, \frac{N}{2} - \frac{N}{4r}\right] \cup \left[\frac{N}{2} + \frac{N}{4r}, N - \frac{N}{4r}\right]$$
$$2\pi k/N \;\in\; \left[\frac{\pi}{2r}, \pi - \frac{\pi}{2r}\right] \cup \left[\pi + \frac{\pi}{2r}, 2\pi - \frac{\pi}{2r}\right] \tag{B.16}$$

and $|\lambda_k| = 2|\cos(\frac{2\pi k}{N})| \leq 2\cos(\frac{\pi}{2r})$. Now the lefthand side of (B.15) is bounded by $(1 + \frac{1}{2}(1 - \cos\frac{\pi}{2r})(2 + 2\cos(\frac{\pi}{2r})) = 4 - 4\sin(\frac{\pi}{4r})^4$, which is bounded away from 4 by an positive constant depending only on r.

Case II: Now assume that $||2k|| < N/(2r)$, and that rk is not congruent to k modulo N. Using the trivial bound that $|\lambda_k|, |\lambda_{rk}| \leq 2$, the lefthand side of (B.15) is bounded by $2(\gamma_{k,rk} + \gamma_{k,\bar{r}k})$. Both cases in the first column of the table have $\gamma_{k,rk} = a_{s_k}$. If $||2\bar{r}k|| \geq N/(2r)$, then $\gamma_{k,\bar{r}k} = 1$ and $1 + a_{s_k} \leq 2 - d/(\log N)^2$, so that the bound in (B.15) is satisfied so long as $c < 2d$, which it may be chosen to be.

The only remaining situation is when both $||2k|| < N/(2r)$ and $||2\bar{r}k|| < N/(2r)$, where the left hand side of (B.15) is bounded by $2(a_{s_k} + a_{s_{\bar{r}k}}^{-1})$. Let $-N/(2r) < m < N/(2r)$ be the integer congruent to $2\bar{r}k$ modulo N, i.e. so that $||2\bar{r}k|| = |m|$. Then $rm \equiv 2k \pmod{N}$. Yet since $-N/2 < rm < N/2$, $||2k|| = |rm|$. We may assume that $m \neq 0$, for otherwise $2k \equiv 2rk \equiv 0 \pmod{N}$; this implies $k \equiv rk \pmod{N}$ if N is odd, and $k \equiv rk \equiv N/2$ if N is even (since then r is odd). Therefore r divides $||2k|| = |rm|$ to exactly one more power than it divides $||2\bar{r}k|| = |m|$. Thus $s_{\bar{r}k} = s_k - 1$. Now

$$a_s^{-1} \;=\; \left(1 - \frac{s\,d}{(\log N)^2}\right)^{-1} \;=\; 1 + \frac{s\,d}{(\log N)^2} + O\left(\left(\frac{s\,d}{(\log N)^2}\right)^2\right)$$
$$=\; 1 + \frac{s\,d}{(\log N)^2} + O\left(\frac{d^2}{(\log N)^2}\right)$$

since $s = O(\log N)$. Therefore $(a_{s_k} + a_{s_{\bar{r}k}}^{-1})$ equals

$$1 \;-\; s_k\frac{d}{(\log N)^2} \;+\; \left(1 - s_{\bar{r}k}\frac{d}{(\log N)^2}\right)^{-1}$$
$$\leq\; 2 \;-\; (s_k - s_{\bar{r}k})\frac{d}{(\log N)^2} \;+\; O\left(\frac{d^2}{(\log N)^2}\right),$$

which is smaller than $2 - c/(\log N)^2$ for some sufficiently small $c > 0$. This concludes the proof of Theorem A.4.

C   Appendix: Related Work

Theoretically, the requirements for stream ciphers are well understood: cryptographically secure pseudo-random number generators (PRNG) exist if and only if one-way functions exist [H+99], and such a generator would be ideal as a stream cipher. However such known constructions would yield prohibitively slow implementations in practice. The heart of such constructions involves a one-way function f and a hard-core bit extractor $B(x)$. If f is based on an algebraic problem such as DLOG or FACTORING, the resulting cipher is quite slow even when $B(x)$ is simple and constitutes outputting some bits of x. If f is based on block ciphers, then $B(x, r) = parity(x \wedge r)$ is often based on the Goldreich-Levin theorem [GL89]. Computing this parity bit takes on the average $\frac{n}{2}$ cycles, where n is the machine word size. One can speed this up with some precomputations and make it into a practical algorithm with provable properties (e.g. the VRA cipher [ARV95], which has the disadvantage of needing to store a large array of random bits.)

Computerized methods for random number generation go back to von Neumann [Neu51]. Many designers of PRNGs used clever techniques to control correlations between adjacent outputs of their algorithms, but few generators needed it as badly as LFSR-based algorithms [Golo67]. Indeed, since the Berlekamp-Massey algorithm [Mas69] efficiently determines the state of an LFSR of length n given only $2n$ bits, all LFSR-based constructions necessarily must hide the LFSR's exact output sequence.

Historically, the first method to hedge LFSR's from the Berlekamp-Massey attack was due to Geffe [Gef73]. It combines outputs of three synchronously clocked LFSR's to produce one stream of output bits, using one of them as a multiplexer. This is a lossy combiner in the sense it outputs only one of three bits generated by the LFSR's. It was broken by Siegenthaler [Sie84], who also broke another 3-way non-linear combiner of Bruer [Bru84]. Another attempt by Pless [Ple77] — to make use of non-linear J-K flip-flops to combine eight LFSR's into one key stream — was broken shortly thereafter [Rub79].

A recommended approach to designing LFSR-based ciphers is the shrinking generator [CKM93]. It outputs only one quarter of its generated bits, but has proved to be secure after 10 years of wide use and extensive scrutiny.

Most combiners considered in the LFSR literature are constructed from two building blocks: a (non)linear function that mixes inputs of several generators (this function may either be memoryless or stateful, though usually of very small memory), and a clocking rule that controls the clock of some LFSR's. None of them uses deep buffers or tries to space LFSR's outputs using schemes with guaranteed properties. For attacks on combiners with small memory (up to 4 bits) see [Cou04].

A different approach for combining generators' outputs is called *randomization by shuffling* [Knu97, Ch. 3.2.2]. Two algorithms popularized by Knuth are often used in modern generators: the "algorithm M" or MacLaren-Marsaglia algorithm [MM65], and the "algorithm B" or Bays-Durham algorithm [BD76]. Both are analogous to our proposal in the sense that they store the generator's

output in a buffer and output the stored elements out of order. The fundamental difference — and source of weakness — of both algorithms M and B is that they only reorder elements without modifying them. We omit the details. For example, the Bays-Durham algorithm operators as follows:

Bays-Durham Algorithm.
Y is an auxiliary variable, T is the size of the buffer V, m is the range of the generator $\langle X_n \rangle$. Initially V is filled with T elements $X_0, \ldots, X_{T-1}$. Iterate the following:

1. set $j \leftarrow \lfloor TY/m \rfloor$.
2. set $Y \leftarrow V[j]$.
3. output Y.
4. set $V[j] \leftarrow$ next element of $\langle X_n \rangle$.

Since the position of the output element is completely determined by the previous element, the construction does not improve cryptographic properties of the cipher. If Y is chosen by an independent process (as in the algorithm M), there is still a $1/T$ chance that two elements X_i and X_{i+1} will end up next to each other in the output sequence. More generally, the distance between X_i and X_{i+1} is distributed according to a geometric distribution and has average T. Depending on the generator, this property may be exploitable.

Klimov and Shamir [KS04] proposed a class of invertible mappings $\{0,1\}^n \rightarrow \{0,1\}^n$ called T-functions that allow introduction of non-linearity using elementary register operations $(\vee, \wedge, \oplus, *, +, -, x \mapsto \overline{x}, x \mapsto -x, \ll)$. The T-functions are particularly well suited for fast software implementations. An example of such a function is $f(x) = x + (x^2 \vee 5) \pmod{2^n}$, for which the sequence $x_{i+1} = f(x_i)$ spans the entire domain in one cycle. Each iteration requires only 3 cycles. Nevertheless, by choosing $n = 64$ and outputting the top half of x_i (i.e. $H(x_i) = \text{MSB}_{32}(x_i)$), they discovered that the resulting pseudo-random sequence passed the statistical test suite for AES candidates with significance level $\alpha = 0.01$, which is better than some of the AES candidates. Surprisingly, the best known cryptanalytic attacks take time 2^{cn}, where c is a constant. These attacks depend on using the structure of the iterated output: this structure is important for proving the properties of these functions, and slightly altering the construction would destroy the properties. These functions allow some of their parameters be chosen at random subject to certain constraints.

The methods in this paper allow us to resist such attacks better, with minimal overhead, and extend the length of the underlying key for the stream cipher. We do not know how to extend the known attacks in this new model.