Test case

One reservoir, two-objectives, stationary system:

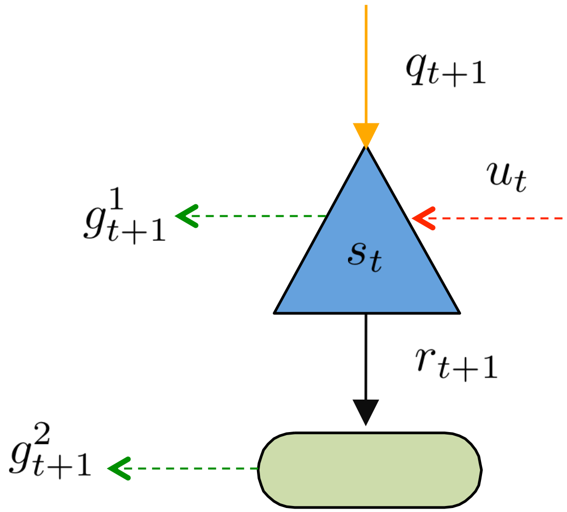
s_t = reservoir storage

u_t = reservoir release

q_{t+1} = reservoir inflow

Objectives:

- in-reservoir flood control
- downstream water supply



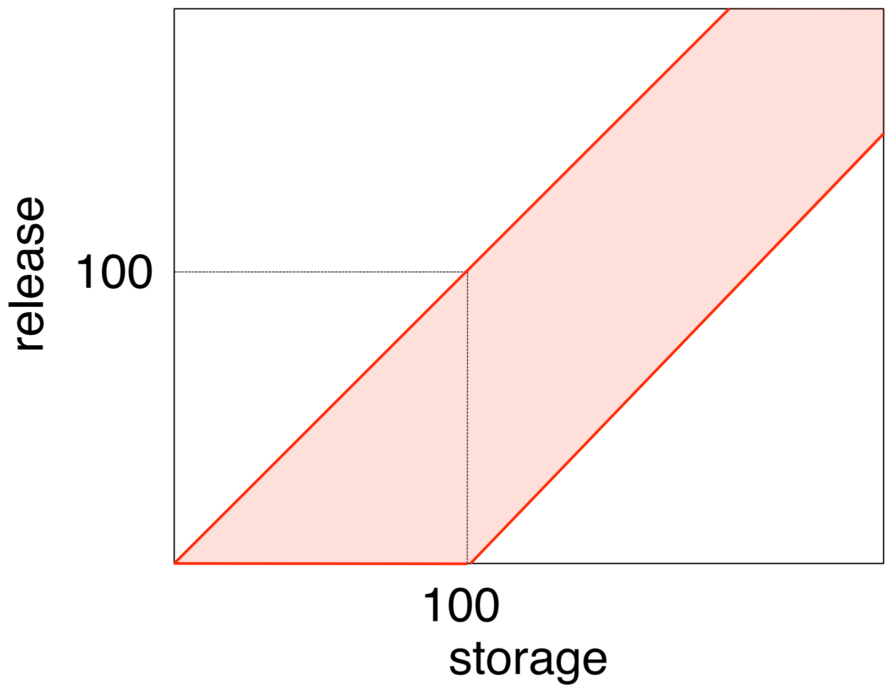
$$
\begin{aligned}
s_{t+1} &= s_t + q_{t+1} - r_{t+1} \\
r_{t+1} &= \max\left(v_t, \min(V_t, u_t)\right) \\
V_t &= s_t \\
v_t &= \max(s_t - 100, 0) \\
g^1_{t+1} &= \max(s_{t+1}/S - \bar{h}, 0) \\
g^2_{t+1} &= \max(\bar{r} - r_{t+1}, 0) \\
q_{t+1} &\sim N(\mu_q, \sigma_q)
\end{aligned}
$$

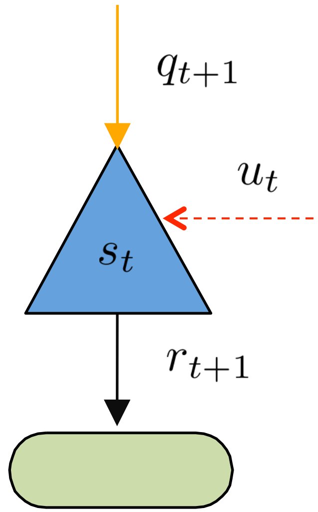
$S = 1$	reservoir surface
$\bar{h} = 50$	flooding threshold
$\bar{r} = 50$	water demand
$\mu_q = 40$	inflow mean
$\sigma_q = 10$	inflow standard deviation

Analysis

The space of feasible controls:

$$V_t = s_t$$
$$v_t = \max(s_t - 100, 0)$$

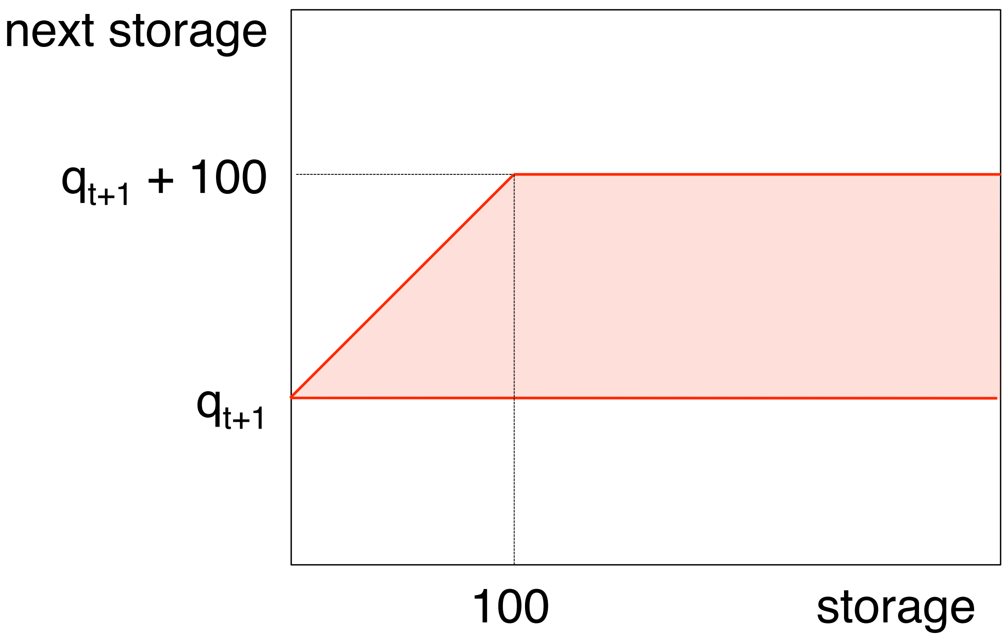


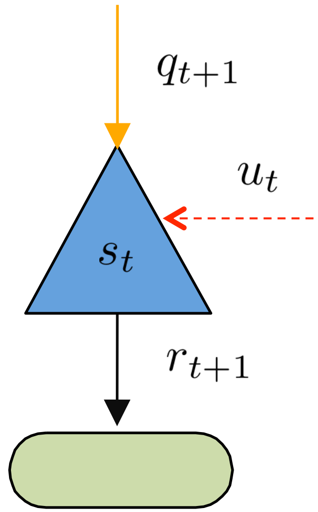


Analysis

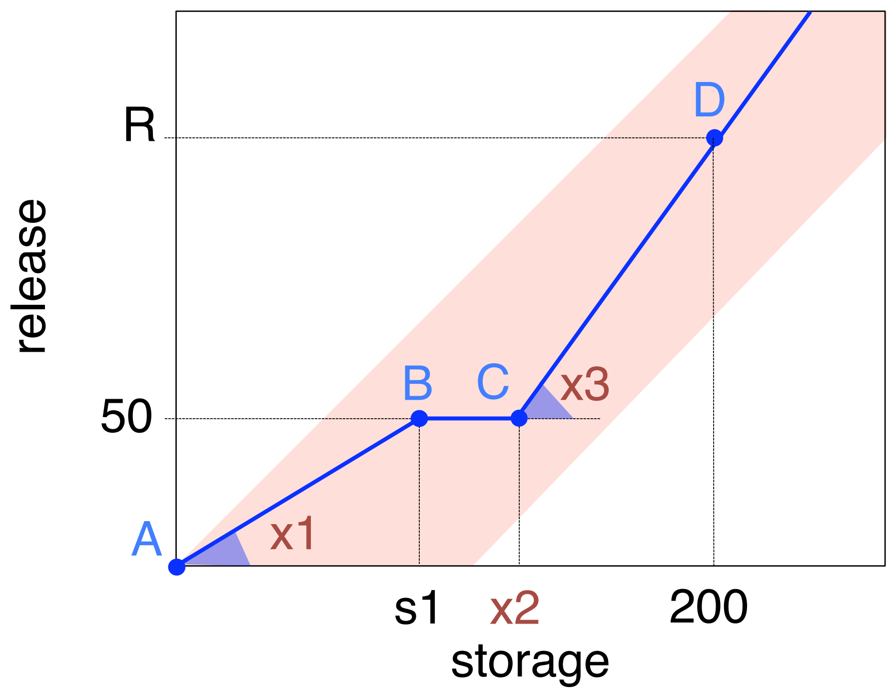
The next storage:

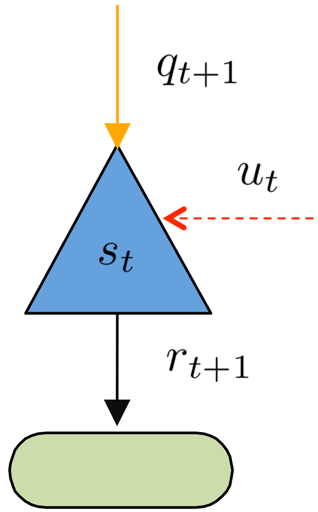
$$
\begin{aligned}
s_{t+1}^{\min} &= s_t + q_{t+1} - V_t \\
&= s_t + q_{t+1} - s_t = q_{t+1} \\
s_{t+1}^{\max} &= s_t + q_{t+1} - v_t \\
&= \begin{cases} s_t + q_{t+1} & \text{if } s_t < 100 \\ s_t + q_{t+1} - s_t + 100 = q_{t+1} + 100 & \text{otherwise} \end{cases}
\end{aligned}
$$





Piecewise linear control law





s1 = 50 / tan(x1)
R = 50 + (200-x2) tan(x3)

A = (0,0)
B = (50 / tan(x1), 50)
C = (x2, 50)
D = (200, 50 + (200-x2) tan(x3))