



# Collision resistance

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## Introduction

# Recap: message integrity

So far, four MAC constructions:

PRFs { **ECBC-MAC, CMAC** : commonly used with AES (e.g. 802.11i)  
**NMAC** : basis of HMAC (this segment)  
**PMAC**: a parallel MAC

randomized  
MAC { **Carter-Wegman MAC**: built from a fast one-time MAC

This module: MACs from collision resistance.

# Collision Resistance

Let  $H: M \rightarrow T$  be a hash function  $(|M| \gg |T|)$

A **collision** for  $H$  is a pair  $m_0, m_1 \in M$  such that:

$$H(m_0) = H(m_1) \quad \text{and} \quad m_0 \neq m_1$$

A function  $H$  is **collision resistant** if for all (explicit) “eff” algs.  $A$ :

$$\text{Adv}_{\text{CR}}[A, H] = \Pr[ A \text{ outputs collision for } H]$$

is “neg”.

Example: SHA-256 (outputs 256 bits)

# MACs from Collision Resistance

Let  $I = (S, V)$  be a MAC for short messages over  $(K, M, T)$  (e.g. AES)

Let  $H: M^{\text{big}} \rightarrow M$

Def:  $I^{\text{big}} = (S^{\text{big}}, V^{\text{big}})$  over  $(K, M^{\text{big}}, T)$  as:

$$S^{\text{big}}(k, m) = S(k, H(m)) \quad ; \quad V^{\text{big}}(k, m, t) = V(k, H(m), t)$$

**Thm**: If  $I$  is a secure MAC and  $H$  is collision resistant  
then  $I^{\text{big}}$  is a secure MAC.

Example:  $S(k, m) = \text{AES}_{2\text{-block-cbc}}(k, \text{SHA-256}(m))$  is a secure MAC.

# MACs from Collision Resistance

$$S^{\text{big}}(k, m) = S(k, H(m)) \quad ; \quad V^{\text{big}}(k, m, t) = V(k, H(m), t)$$

Collision resistance is necessary for security:

Suppose adversary can find  $m_0 \neq m_1$  s.t.  $H(m_0) = H(m_1)$ .

Then:  $S^{\text{big}}$  is insecure under a 1-chosen msg attack

step 1: adversary asks for  $t \leftarrow S(k, m_0)$

step 2: output  $(m_1, t)$  as forgery

# Protecting file integrity using C.R. hash

Software packages:



When user downloads package, can verify that contents are valid

H collision resistant  $\Rightarrow$

attacker cannot modify package without detection

no key needed (public verifiability), but requires read-only space

End of Segment