恭喜！您通过了！

通过条件 80% 或更高

坚持学习

成绩

100%

Week 3 - Problem Set

最新提交作业的评分

100%

1. Suppose a MAC system (S, V) is used to protect files in a file system by appending a MAC tag to each file. The MAC signing algorithm S is applied to the file contents and nothing else. What tampering attacks are not prevented by this system?

1/1 分

- (●) Changing the name of a file.
- ( ) Changing the first byte of the file contents.
- ( ) Appending data to a file.
- ( ) Replacing the contents of a file with the concatenation of two files on the file system.

正确

The MAC signing algorithm is only applied to the file contents and does not protect the file name.

2. Let (S, V) be a secure MAC defined over (K, M, T) where $M = \{0,1\}^n$ and $T = \{0,1\}^{128}$. That is, the key space is K, message space is $\{0,1\}^n$, and tag space is $\{0,1\}^{128}$.

1/1 分

Which of the following is a secure MAC: (as usual, we use $\|$ to denote string concatenation)

- [ ] $S'(k, m) = S(k, m \oplus m)$ and

$$V'(k, m, t) = V(k,\ m \oplus m,\ t)$$

- [ ] $S'(k, m) = S(k, m)$ and

$$V'(k, m, t) = \begin{cases} V(k, m, t) & \text{if } m \neq 0^n \\ \text{“1”} & \text{otherwise} \end{cases}$$

- [x] $S'(k, m) = S(k, m \oplus 1^n)$ and

$$V'(k, m, t) = V(k, m \oplus 1^n, t).$$

正确

a forger for (S', V') gives a forger for (S, V).

- [ ] $S'(k, m) = \begin{cases} S(k, 1^n) & \text{if } m = 0^n \\ S(k, m) & \text{otherwise} \end{cases}$ and

$$V'(k, m) = \begin{cases} V(k, 1^n, t) & \text{if } m = 0^n \\ V(k, m, t) & \text{otherwise} \end{cases}$$

- [x] $S'(k, m) = S(k, m)[0, \ldots, 126]$ and

$$V'(k, m, t) = [V(k,\ m,\ t\|0) \text{ or } V(k,\ m,\ t\|1)]$$

(i.e., $V'(k, m, t)$ outputs ``1" if either $t\|0$ or $t\|1$ is a valid tag for m)

✓ **正确**

a forger for (S', V') gives a forger for (S, V).

☑ $S'(k,\ m) = [t \leftarrow S(k,m),\ \text{output } (t,t)\ )$ and

$$V'(k, m, (t_1, t_2)) = \begin{cases} V(k, m, t_1) & \text{if } t_1 = t_2 \\ \text{"0"} & \text{otherwise} \end{cases}$$

(i.e., $V'(k, m, (t_1, t_2))$ only outputs "1"

if t_1 and t_2 are equal and valid)

✓ **正确**

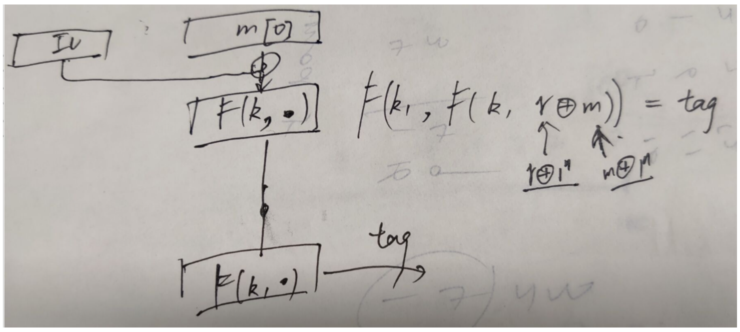
a forger for (S', V') gives a forger for (S, V).

3. Recall that the ECBC-MAC uses a fixed IV (in the lecture we simply set the IV to 0). Suppose instead we chose a random IV for every message being signed and include the IV in the tag.

1/1 分

In other words, $S(k,m) := (r,\ \ \mathrm{ECBC}_r(k,m))$

where $\mathrm{ECBC}_r(k,m)$ refers to the ECBC function using r as the IV. The verification algorithm V given key k, message m, and tag (r,t) outputs ``1'' if $t = \mathrm{ECBC}_r(k,m)$ and outputs ``0'' otherwise.



The resulting MAC system is insecure.

An attacker can query for the tag of the 1-block message m and obtain the tag (r,t). He can then generate the following existential forgery: (we assume that the underlying block cipher operates on n-bit blocks)

◉ The tag $(r \oplus 1^n,\ t)$ is a valid tag for the 1-block message $m \oplus 1^n$.

○ The tag $(r,\ t \oplus r)$ is a valid tag for the 1-block message 0^n.

○ The tag $(r \oplus t,\ m)$ is a valid tag for the 1-block message 0^n.

○ The tag $(m \oplus t,\ r)$ is a valid tag for the 1-block message 0^n.

✓ **正确**

The CBC chain initiated with the IV $r \oplus m$ and applied to the message 0^n will produce exactly the same output as the CBC chain initiated with the IV r and applied to the message m. Therefore, the tag $(r \oplus 1^n,\ t)$ is a valid existential forgery for the message $m \oplus 1^n$.

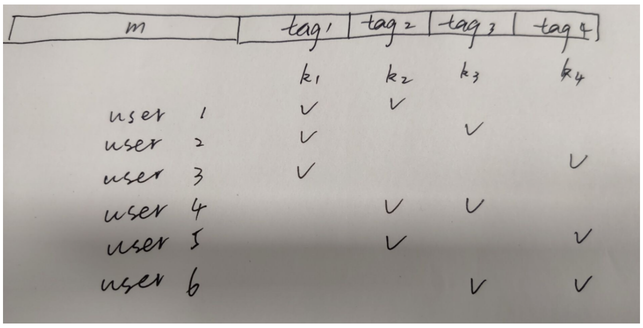
4. Suppose Alice is broadcasting packets to 6 recipients $B_1, \ldots, B_6$. Privacy is not important but integrity is.

1/1 分

In other words, each of $B_1, \ldots, B_6$ should be assured that the packets he is receiving were sent by Alice.

Alice decides to use a MAC. Suppose Alice and $B_1, \ldots, B_6$ all share a secret key k. Alice computes a tag for every packet she sends using key k. Each user B_i verifies the tag when receiving the packet and drops the packet if the tag is invalid.

Alice notices that this scheme is insecure because user B_1 can use the key k to send packets with a valid tag to users $B_2, \dots, B_6$ and they will all be fooled into thinking that these packets are from Alice.



Instead, Alice sets up a set of 4 secret keys $S = \{k_1, \dots, k_4\}$.

She gives each user B_i some subset $S_i \subseteq S$

of the keys. When Alice transmits a packet she appends 4 tags to it

by computing the tag with each of her 4 keys. When user B_i receives

a packet he accepts it as valid only if all tags corresponding

to his keys in S_i are valid. For example, if user B_1 is given keys $\{k_1, k_2\}$ he will accept an incoming packet only if the first and second tags are valid. Note that B_1 cannot validate the 3rd and 4th tags because he does not have k_3 or k_4.

How should Alice assign keys to the 6 users so that no single user

can forge packets on behalf of Alice and fool some other user?

- [ ] $S_1 = \{k_1, k_2\},\ S_2 = \{k_2, k_3\},\ S_3 = \{k_3, k_4\},\ S_4 = \{k_1, k_3\},\ S_5 = \{k_1, k_2\},\ S_6 = \{k_1, k_4\}$

- [x] $S_1 = \{k_1, k_2\},\ S_2 = \{k_1, k_3\},\ S_3 = \{k_1, k_4\},\ S_4 = \{k_2, k_3\},\ S_5 = \{k_2, k_4\},\ S_6 = \{k_3, k_4\}$

  ✓ **正确**
  Every user can only generate tags with the two keys he has.

  Since no set S_i is contained in another set S_j, no user i

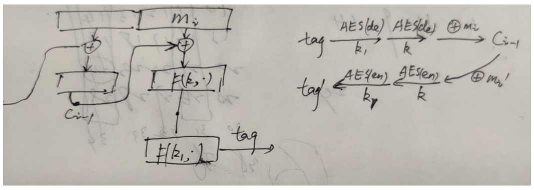
  can fool a user j into accepting a message sent by i.

- [ ] $S_1 = \{k_1, k_2\},\ S_2 = \{k_1, k_3\},\ S_3 = \{k_1, k_4\},\ S_4 = \{k_2, k_3, k_4\},\ S_5 = \{k_2, k_3\},\ S_6 = \{k_3, k_4\}$

- [ ] $S_1 = \{k_1, k_2\},\ S_2 = \{k_1, k_3, k_4\},\ S_3 = \{k_1, k_4\},\ S_4 = \{k_2, k_3\},\ S_5 = \{k_2, k_3, k_4\},\ S_6 = \{k_3, k_4\}$

5. Consider the encrypted CBC MAC built from AES. Suppose we **1/1 分**

compute the tag for a long message m comprising of n AES blocks.



Let m' be the n-block message obtained from m by flipping the

last bit of m (i.e. if the last bit of m is b then the last bit

of m' is $b \oplus 1$). How many calls to AES would it take

to compute the tag for m' from the tag for m and the MAC key? (in this question please ignore message padding and simply assume that the message length is always a multiple of the AES block size)

- ( ) 5
- ( ) n
- ( ) 6
- (•) 4

  ✓ **正确**
  You would decrypt the final CBC MAC encryption step done using k_2,

  the decrypt the last CBC MAC encryption step done using k_1,

  flip the last bit of the result, and re-apply the two encryptions.

6. Let $H : M \to T$ be a collision resistant hash function. **1/1 分**

Which of the following is collision resistant:

(as usual, we use $\|$ to denote string concatenation)

- [x] $H'(m) = H(m) \| H(m)$

  ✓ **正确**
  a collision finder for H' gives a collision finder for H.

- [ ] $H'(m) = H(m) \bigoplus H(m \oplus 1^{|m|})$

(where $m \oplus 1^{|m|}$ is the complement of m)

- [x] $H'(m) = H(m\|m)$

✓ **正确**
a collision finder for H' gives a collision finder for H.

- [ ] $H'(m) = H(0)$
- [x] $H'(m) = H(H(m))$

✓ **正确**
a collision finder for H' gives a collision finder for H.

- [ ] $H'(m) = H(|m|)$

(i.e. hash the length of m)

- [ ] $H'(m) = H(m) \oplus H(m)$

7. Suppose H_1 and H_2 are collision resistant **1/1 分**

hash functions mapping inputs in a set M to $\{0,1\}^{256}$.

Our goal is to show that the function $H_2(H_1(m))$ is also

collision resistant. We prove the contra-positive:

suppose $H_2(H_1(\cdot))$ is not collision resistant, that is, we are

given $x \neq y$ such that $H_2(H_1(x)) = H_2(H_1(y))$.

We build a collision for either H_1 or for H_2.

This will prove that if H_1 and H_2 are collision resistant

then so is $H_2(H_1(\cdot))$. Which of the following must be true:

- ○ Either $H_2(x), H_2(y)$ are a collision for H_1 or

  x, y are a collision for H_2.
- ◉ Either x, y are a collision for H_1 or

  $H_1(x), H_1(y)$ are a collision for H_2.
- ○ Either $x, H_1(y)$ are a collision for H_2 or

  $H_2(x), y$ are a collision for H_1.
- ○ Either x, y are a collision for H_2 or

  $H_1(x), H_1(y)$ are a collision for H_1.

✓ **正确**
If $H_2(H_1(x)) = H_2(H_1(y))$ then

1 either $H_1(x) = H_1(y)$ and $x \neq y$, thereby giving us

a collision on H_1. 2 Or $H_1(x) \neq H_1(y)$ but

$H_2(H_1(x)) = H_2(H_1(y))$ giving us a collision on H_2.

8. In this question you are asked to find a collision for the compression function: 1/1 分

$f_1(x, y) = \text{AES}(y, x) \bigoplus y,$

where $\text{AES}(x, y)$ is the AES-128 encryption of y under key x.

Your goal is to find two distinct pairs (x_1, y_1) and (x_2, y_2) such that $f_1(x_1, y_1) = f_1(x_2, y_2)$.

Which of the following methods finds the required (x_1, y_1) and (x_2, y_2)?

$AES(y_1, x_1) \oplus y_1 = AES(y_2, x_2) \oplus y_2$

$x_2 = AES^{-1}(\underbrace{y_2}_{\text{key}}, \; y_2 \oplus y_1 \oplus AES(y_1, x_1))$

$x_1 = AES^{-1}(y_1, \; y_1 \oplus y_2 \oplus AES(y_2, x_2))$

- ○ Choose x_1, y_1, x_2 arbitrarily (with $x_1 \neq x_2$) and let $v := AES(y_1, x_1)$.

  Set $y_2 = AES^{-1}(x_2, \; v \oplus y_1 \oplus x_2)$

- ○ Choose x_1, y_1, y_2 arbitrarily (with $y_1 \neq y_2$) and let $v := AES(y_1, x_1)$.

  Set $x_2 = AES^{-1}(y_2, \; v \oplus y_1)$

- ◉ Choose x_1, y_1, y_2 arbitrarily (with $y_1 \neq y_2$) and let $v := AES(y_1, x_1)$.

  Set $x_2 = AES^{-1}(y_2, \; v \oplus y_1 \oplus y_2)$

- ○ Choose x_1, y_1, y_2 arbitrarily (with $y_1 \neq y_2$) and let $v := AES(y_1, x_1)$.

  Set $x_2 = AES^{-1}(y_2, \; v \oplus y_2)$

✓ **正确**

You got it !

9. Repeat the previous question, but now to find a collision for the compression function $f_2(x, y) = \text{AES}(x, x) \bigoplus y$. 1/1 分

Which of the following methods finds the required (x_1, y_1) and (x_2, y_2)?

○ Choose x_1, x_2, y_1 arbitrarily (with $x_1 \neq x_2$) and set

$y_2 = AES(x_1, x_1) \oplus AES(x_2, x_2)$

◉ Choose x_1, x_2, y_1 arbitrarily (with $x_1 \neq x_2$) and set

$y_2 = y_1 \oplus AES(x_1, x_1) \oplus AES(x_2, x_2)$

$AES(x_1, x_1) \oplus y_1 = AES(x_2, \cancel{x_2}) \oplus y_2$

○ Choose x_1, x_2, y_1 arbitrarily (with $x_1 \neq x_2$) and set

$y_2 = y_1 \oplus AES(x_1, x_1)$

○ Choose x_1, x_2, y_1 arbitrarily (with $x_1 \neq x_2$) and set

$y_2 = y_1 \oplus x_1 \oplus AES(x_2, x_2)$

✓ **正确**

Awesome!

10. Let $H : M \to T$ be a random hash function where $|M| \gg |T|$ (i.e. the size of M is much larger than the size of T). **1/1 分**

In lecture we showed

that finding a collision on H can be done with $O\big(|T|^{1/2}\big)$

random samples of H. How many random samples would it take

until we obtain a three way collision, namely distinct strings x, y, z

in M such that $H(x) = H(y) = H(z)$?

◉ $O\big(|T|^{2/3}\big)$

○ $O\big(|T|^{1/2}\big)$

○ $O\big(|T|\big)$

○ $O\big(|T|^{1/3}\big)$

✓ **正确**

An informal argument for this is as follows: suppose we collect n random samples. The number of triples among the n samples is n choose 3 which is $O(n^3)$. For a particular triple x, y, z to be a 3-way collision we need $H(x) = H(y)$ and $H(x) = H(z)$. Since each one of these two events happens with probability $1/|T|$ (assuming H behaves like a random function) the probability that a particular triple is a 3-way collision is $O(1/|T|^2)$. Using the union bound, the probability that some triple is a 3-way collision is $O(n^3/|T|^2)$ and since we want this probability to be close to 1, the bound on n follows.