Feedback — Week 1 - Problem Set

You submitted this homework on **Fri 2 Jan 2015 3:53 PM CET**. You got a score of **8.45** out of **8.45**.

Question 1

Data compression is often used in data storage and transmission. Suppose you want to use data compression in conjunction with encryption. Does it make more sense to:

Your Answer		Score	Explanation
⃝ Encrypt then compress.			
⃝ The order does not matter -- neither one will compress the data.			
⃝ The order does not matter -- either one is fine.			
⦿ Compress then encrypt.	✔	1.00	Ciphertexts tend to look like random strings and therefore the only opportunity for compression is prior to encryption.
Total		1.00 / 1.00	

Question 2

Let $G : \{0,1\}^s \rightarrow \{0,1\}^n$ be a secure PRG. Which of the following is a secure PRG (there is more than one correct answer):

Your Answer		Score	Explanation
☑ $G'(k) = G(k)[0, \ldots, n-2]$ (i.e., $G'(k)$ drops the last bit of $G(k)$)	✔	0.17	a distinguisher for G' gives a distinguisher for G.
☐ $G'(k) = G(0)$	✔	0.17	A distinguisher will output *not random* whenever its input is equal to $G(0)$.

☐ $G'(k) = G(k) \parallel 0$ (here $\parallel$ denotes concatenation)	✔	0.17	A distinguisher will output *not random* whenever the last bit of its input is 0.
☑ $G'(k) = G(k) \oplus 1^n$	✔	0.17	a distinguisher for G' gives a distinguisher for G.
☑ $G'(k) = \text{reverse}(G(k))$ where reverse(x) reverses the string x so that the first bit of x is the last bit of reverse(x), the second bit of x is the second to last bit of reverse(x), and so on.	✔	0.17	a distinguisher for G' gives a distinguisher for G.
☐ $G'(k) = G(k) \parallel G(k)$ (here $\parallel$ denotes concatenation)	✔	0.17	A distinguisher will output *not random* whenever the first n bits are equal to the last n bits.
Total		1.00 / 1.00	

Question 3

Let $G : K \to \{0, 1\}^n$ be a secure PRG. Define $G'(k_1, k_2) = G(k_1) \bigwedge G(k_2)$ where $\bigwedge$ is the bit-wise AND function. Consider the following statistical test A on $\{0, 1\}^n$:

$A(x)$ outputs $\text{LSB}(x)$, the least significant bit of x.

What is $Adv_{\text{PRG}}[A, G']$?   You may assume that $\text{LSB}(G(k))$ is 0 for exactly half the seeds k in K.

Note: Please enter the advantage as a decimal between 0 and 1 with a leading 0. If the advantage is 3/4, you should enter it as 0.75

You entered:

0.25

Your Answer	Score	Explanation
0.25	✔ 1.00	for a random string x we have $Pr[A(x) = 1] = 1/2$ but for a pseudorandom string $G'(k_1, k_2)$ we have $Pr_{k_1,k_2}[A(G'(k_1, k_2)) = 1] = 1/4$.
Total	1.00 /	

Question 4

Let (E, D) be a (one-time) semantically secure cipher with key space $K = \{0, 1\}^\ell$. A bank wishes to split a decryption key $k \in \{0, 1\}^\ell$ into two pieces p_1 and p_2 so that both are needed for decryption. The piece p_1 can be given to one executive and p_2 to another so that both must contribute their pieces for decryption to proceed.

The bank generates random k_1 in $\{0, 1\}^\ell$ and sets $k'_1 \leftarrow k \oplus k_1$. Note that $k_1 \oplus k'_1 = k$. The bank can give k_1 to one executive and k'_1 to another. Both must be present for decryption to proceed since, by itself, each piece contains no information about the secret key k (note that each piece is a one-time pad encryption of k).

Now, suppose the bank wants to split k into three pieces p_1, p_2, p_3 so that any two of the pieces enable decryption using k. This ensures that even if one executive is out sick, decryption can still succeed. To do so the bank generates two random pairs (k_1, k'_1) and (k_2, k'_2) as in the previous paragraph so that $k_1 \oplus k'_1 = k_2 \oplus k'_2 = k$ How should the bank assign pieces so that any two pieces enable decryption using k, but no single piece can decrypt?

Your Answer	Score	Explanation
○ $p_1 = (k_1, k_2), \quad p_2 = (k'_1, k'_2), \quad p_3 = (k'_2)$		
○ $p_1 = (k_1, k_2), \quad p_2 = (k'_1), \quad p_3 = (k'_2)$		
○ $p_1 = (k_1, k_2), \quad p_2 = (k_1, k_2), \quad p_3 = (k'_2)$		
⦿ $p_1 = (k_1, k_2), \quad p_2 = (k'_1, k_2), \quad p_3 = (k'_2)$	✔ 1.00	executives 1 and 2 can decrypt using k_1, k'_1, executives 1 and 3 can decrypt using k_2, k'_2, and executives 2 and 3 can decrypt using k_2, k'_2. Moreover, a single executive has no information about k.
○ $p_1 = (k_1, k_2), \quad p_2 = (k_2, k'_2), \quad p_3 = (k'_2)$		
Total	1.00 /	

Question 5

Let $M = C = K = \{0, 1, 2, \ldots, 255\}$ and consider the following cipher defined over (K, M, C):

$$E(k, m) = m + k \pmod{256} \qquad ; \qquad D(k, c) = c - k \pmod{256}.$$

Does this cipher have perfect secrecy?

Your Answer		Score	Explanation
○ No, only the One Time Pad has perfect secrecy.			
⊙ Yes.	✔	1.00	as with the one-time pad, there is exactly one key mapping a given message m to a given ciphertext c.
○ No, there is a simple attack on this cipher.			
Total		1.00 / 1.00	

Question 6

Let (E, D) be a (one-time) semantically secure cipher where the message and ciphertext space is $\{0, 1\}^n$. Which of the following encryption schemes are (one-time) semantically secure?

Your Answer		Score	Explanation
☐ $E'(k, m) = E(k, m) \parallel \text{LSB}(m)$	✔	0.17	To break semantic security, an attacker would ask for the encryption of 0^n and $0^{n-1}1$ and can distinguish EXP(0) from EXP(1).
☑ $E'(k, m) = $ compute $c \leftarrow E(k, m)$ and output $c \parallel c$ (i.e., output c twice)	✔	0.17	an attack on E' gives an attack on E.
☑ $E'(k, m) = 0 \parallel E(k, m)$ (i.e. prepend 0 to the	✔	0.17	an attack on E'

ciphertext)		gives an attack on E.
☑ $E'((k, k'), m) = E(k, m) \parallel E(k', m)$	✔ 0.17	an attack on E' gives an attack on E.
☐ $E'(k, m) = E(k, m) \parallel k$	✔ 0.17	To break semantic security, an attacker would read the secret key from the challenge ciphertext and use it to decrypt the challenge ciphertext. Basically, any ciphertext reveals the secret key.
☐ $E'(k, m) = E(0^n, m)$	✔ 0.17	To break semantic security, an attacker would ask for the encryption of 0^n and 1^n and can easily distinguish EXP(0) from EXP(1) because it knows the secret key, namely 0^n.
Total	1.00 / 1.00	

Question 7

Suppose you are told that the one time pad encryption of the message "attack at dawn" is

6c73d5240a948c86981bc294814d (the plaintext letters are encoded as 8-bit ASCII and the given ciphertext is written in hex). What would be the one time pad encryption of the message "attack at dusk" under the same OTP key?

You entered:

6c73d5240a948c86981bc2808548

Your Answer	Score	Explanation
6c73d5240a948c86981bc2808548	✔ 1.00	

Question 8

The movie industry wants to protect digital content distributed on DVD's. We develop a variant of a method used to protect Blu-ray disks called AACS.

Suppose there are at most a total of n DVD players in the world (e.g. $n = 2^{32}$). We view these n players as the leaves of a binary tree of height $\log_2 n$. Each node in this binary tree contains an AES key k_i. These keys are kept secret from consumers and are fixed for all time. At manufacturing time each DVD player is assigned a serial number $i \in [0, n-1]$ Consider the set of nodes S_i along the path from the root to leaf number i in the binary tree. The manufacturer of the DVD player embeds in player number i the keys associated with the nodes in the set S_i. A DVD movie m is encrypted as

$$E(k_{\mathrm{root}}, k) \big\| E(k, m)$$

where k is a random AES key called a content-key and k_{root} is the key associated with the root of the tree. Since all DVD players have the key k_{root} all players can decrypt the movie m. We refer to $E(k_{\mathrm{root}}, k)$ as the header and $E(k, m)$ as the body. In what follows the DVD header may contain multiple ciphertexts where each ciphertext is the encryption of the content-key k under some key k_i in the binary tree.

Suppose the keys embedded in DVD player number r are exposed by hackers and published on the Internet. In this problem we show that when the movie industry distributes a new DVD movie, they can encrypt the contents of the DVD using a slightly larger header (containing about $\log_2 n$ keys) so that all DVD players, except for player number r, can decrypt the movie. In effect, the movie industry disables player number r without affecting other players.

As shown below, consider a tree with $n = 16$ leaves. Suppose the leaf node labeled 25 corresponds to an exposed DVD player key. Check the set of keys below under which to encrypt the key k so that *every player* other than player 25 can decrypt the DVD. Only four keys are needed.
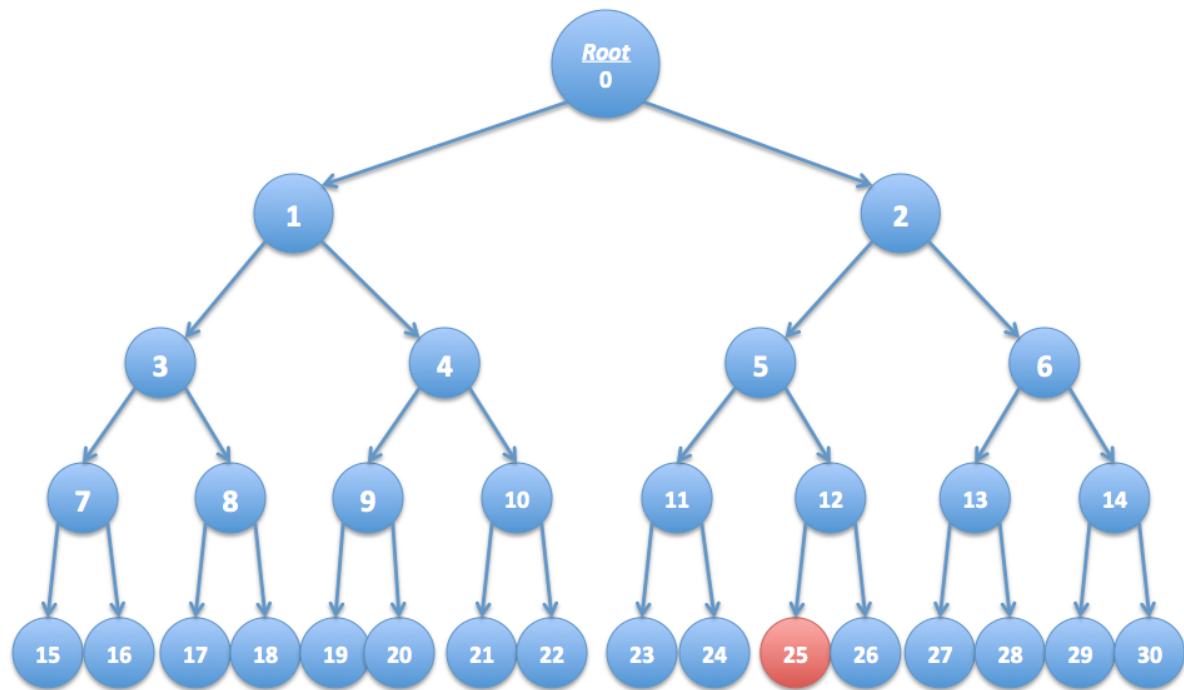
Your Answer		Score	Explanation
☐ 0	✔	0.03	No, this will let node 25 decrypt the DVD.
☑ 26	✔	0.03	You cannot encrypt k under any key on the path from the root to node 25. Therefore 26 can only decrypt if you encrypt k under key k_{26}.
☐ 21	✔	0.03	There is a better solution that does not require encrypting on the key of this node.
☑ 6	✔	0.03	You cannot encrypt k under 2, but 6's children must be able to decrypt k.
☑ 1	✔	0.03	You cannot encrypt k under the root, but 1's children must be able to decrypt k.
☑ 11	✔	0.03	You cannot encrypt k under key 5, but 11's children must be able to decrypt k.
☐ 28	✔	0.03	There is a better solution that does not require encrypting on the key of this node.
☐ 25	✔	0.03	No, this will let node 25 decrypt the DVD.
Total		0.25 / 0.25	

Question Explanation

Question explanation

Question 9

Continuing with the previous question, if there are n DVD players, what is the number of keys under which the content key k must be encrypted if exactly one DVD player's key needs to be revoked?

Your Answer	Score	Explanation
$\bigcirc$ $n/2$		
$\odot$ $\log_2 n$	✔ 1.00	That's right. The key will need to be encrypted under one key for each node on the path from the root to the revoked leaf. There are $\log_2 n$ nodes on the path.
$\bigcirc$ 2		
$\bigcirc$ $\sqrt{n}$		
$\bigcirc$ $n-1$		
Total	1.00 / 1.00	

Question 10

Continuing with question 8, suppose the leaf nodes labeled 16, 18, and 25 correspond to exposed DVD player keys. Check the smallest set of keys under which to encrypt the key k so that every player other than players 16,18,25 can decrypt the DVD. Only six keys are needed.

Your Answer	Score	Explanation
☑ 4	✔ 0.02	Yes, this will let players 19-22 decrypt.
☐ 12	✔ 0.02	
☑ 17	✔ 0.02	Yes, this will let player 17 decrypt.
☑ 15	✔ 0.02	Yes, this will let player 15 decrypt.
☑ 26	✔ 0.02	Yes, this will let player 26 decrypt.

☑ 6	✔	0.02	Yes, this will let players 27-30 decrypt.
☐ 13	✔	0.02	
☑ 11	✔	0.02	Yes, this will let players 23,24 decrypt.
☐ 24	✔	0.02	
☐ 27	✔	0.02	
Total		0.20 / 0.20	