Theorem 3.9 [Supermartingale Maximal Inequality]

Let $(X_t)_{t=0}^{+\infty}$ be a supermartingale with $X_t \geqslant 0$ a.s. for all t. Then for any $\varepsilon > 0$,

$$\mathbb{P}\left(\sup_{t \in \mathbb{N}} X_t \geqslant \varepsilon \right) \leq \frac{\mathbb{E}[X_0]}{\varepsilon}.$$

Proof. TBD. 2022.09.07