1. Lipschitz Function

Consider two metric spaces (X, d_X), (Y, d_Y), and a function $f: X \to Y$. If $\forall\, x_1, x_2 \in X$, it holds that

$$d_Y(f(x_1), f(x_2)) \leq K \cdot d_X(x_1, x_2),$$

then f is K-Lipschitz.

2. Bi-Lipschitz Function

If there exists $K \geq 1$ s.t. $\forall\, x_1, x_2 \in X$,

$$\frac{1}{K} \cdot d_X(x_1, x_2) \leq d_Y(f(x_1), f(x_2)) \leq K \cdot d_X(x_1, x_2),$$

then f is a bi-Lipschitz function.

3. Doubling Space

A metric space (X, d) is said to be doubling if there exists some doubling constant M such that $\forall\, x \in X$, $r > 0$, the ball $B(x, r) = \{y \in X : d(x, y) \leq r\}$ could be cover with the union of at most M balls of radius $\frac{r}{2}$.

Remark. The doubling constant of 1-dimensional Euclidean space with the Euclidean metric is 2. (2-dimensional $\Rightarrow$ 7).