Given $v_n = (1-\alpha_n)\cdot v_{n-1} + \alpha x_n$

and $\lim\limits_{n\to+\infty} \alpha_n = 0$, $\{\alpha_n\}$ decreasing and $\begin{cases} \{x_n\} \text{ bounded.} \\ v_0 \text{ bounded.} \end{cases}$

prove $\{v_n\}$ converges.

① v_0 有界. $|x_n| \le C_1$. $|v_{n-1}| \le C_2$.

② 假设 v_{n-1} 有界

$$
\begin{aligned}
|v_n| &= |(1-\alpha_n)\cdot v_{n-1} + x_n \cdot \alpha_n| \\
&\le (1-\alpha_n)\cdot C_2 + \alpha_n \cdot C_1 \\
&\le (1-\alpha_n)\cdot \max(C_1, C_2) + \alpha_n \cdot \max(C_1, C_2) \\
&= \max \cdot (C_1, C_2)
\end{aligned}
$$