The total variation distance w.r.t. two probability distributions μ and ν defined on Ω is

$$\|\mu-\nu\|_{TV} := \sup_{A\subseteq\Omega} |\mu(A)-\nu(A)| .$$

Lemma [TV distance & L_1-norm]

Assume Ω is finite or countable. Then

$$\|\mu-\nu\|_{TV} = \frac{1}{2}\sum_{a\in\Omega} |\mu(a)-\nu(a)| . \qquad (1)$$

$$= \sum_{a\in\Omega:\, \mu(a)\geq\nu(a)} (\mu(a)-\nu(a)) \qquad (2)$$

$$= 1-\sum_{a\in\Omega}(\mu(a)\wedge\nu(a)) \qquad (3)$$

Proof. Let $B := \{a\in\Omega : \mu(a)\geq\nu(a)\}$.

$$\|\mu-\nu\|_{TV} = \sup_{A\subseteq\Omega} |\mu(A)-\nu(A)|$$

$$= \sup_{A\subseteq\Omega} \left(\mu(A\cap B)-\nu(A\cap B)-(\mu(A\setminus B)-\nu(A\setminus B))\right)$$

$$= \mu(B)-\nu(B).$$

Besides, $\sum_{a\in B^c}(\nu(a)-\mu(a)) = \sum_{a\in B^c}(\nu(a)-\mu(a)) + \sum_{a\in B}(\nu(a)-\nu(a))$

$$= 1-\sum_{a\in\Omega}(\mu(a)\wedge\nu(a))$$

$$= \sum_{a \in B} (\mu(a) - \nu(a)) + \sum_{a \in B^c} (\mu(a) - \mu(a))$$

$$= \sum_{a \in B} (\mu(a) - \nu(a))$$

Hence $\mu(B) - \nu(B) = \nu(B^c) - \mu(B^c)$ and $\|\mu - \nu\|_{TV} = \frac{1}{2} \sum_{a \in \Omega} |\mu(a) - \nu(a)|$, which concludes (1) (2) (3).

$\blacksquare$