CARDAMOM DALEC Model ID 813

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1 Mathematical Dynamics

Let $\vec{C}^{(t)} = \left[C_{\text{lab}}^{(t)}, C_{\text{fol}}^{(t)}, C_{\text{roo}}^{(t)}, C_{\text{woo}}^{(t)}, C_{\text{lit}}^{(t)}, C_{\text{som}}^{(t)} \right]$ denote the labile (lab), foliar (fol), fine root (roo), wood (woo), litter (lit), and soil organic matter (SOM) carbon pools on day t. The dynamics are given by

$$\vec{C}^{(t+1)} = \text{DALEC}_{813}\left(\vec{C}^{(t)} \right)$$
$$= \text{DALEC}_{813}^{\text{fire}}\left(\text{DALEC}_{813}^{\text{pre-fire}}\left(\vec{C}^{(t)} \right) \right),$$

where $\text{DALEC}_{813}^{\text{pre-fire}}(\cdot)$ denotes Carbon dynamics in the absence of fire and $\text{DALEC}_{813}^{\text{fire}}(\cdot)$ denotes Carbon Carbon dynamics due to fires.

1.1 Pre-fire Dynamics

Denote $\left[C_{\text{lab}}^{(t+1)'}, C_{\text{fol}}^{(t+1)'}, C_{\text{roo}}^{(t+1)'}, C_{\text{woo}}^{(t+1)'}, C_{\text{lit}}^{(t+1)'}, C_{\text{som}}^{(t+1)'} \right] = \vec{C}^{(t+1)'} = \text{DALEC}_{813}^{\text{pre-fire}}(\vec{C}^{(t)})$. The pre-fire dynamics are given by

$$C_{\text{lab}}^{(t+1)'} = f_{\text{lab}} F_{\text{gpp}}^{(t)} + \left(1 - \phi_{\text{onset}}^{(t)} \right) C_{\text{lab}}^{(t)}, \tag{1}$$

$$C_{\text{fol}}^{(t+1)'} = \phi_{\text{onset}}^{(t)} C_{\text{lab}}^{(t)} + \left(1 - \phi_{\text{fall}}^{(t)} \right) C_{\text{fol}}^{(t)} + f_{\text{fol}} F_{\text{gpp}}^{(t)}, \tag{2}$$

$$C_{\text{roo}}^{(t+1)'} = (1 - \theta_{\text{roo}}) C_{\text{roo}}^{(t)} + f_{\text{roo}} F_{\text{gpp}}^{(t)}, \tag{3}$$

$$C_{\text{woo}}^{(t+1)'} = (1 - \theta_{\text{woo}}) C_{\text{woo}}^{(t)} + f_{\text{woo}} F_{\text{gpp}}^{(t)}, \tag{4}$$

$$C_{\text{lit}}^{(t+1)'} = \left(1 - (\theta_{\text{lit}} + \theta_{\text{min}}) \rho^{(t)} \right) C_{\text{lit}}^{(t)} + \phi_{\text{fall}}^{(t)} C_{\text{fol}}^{(t)} + \theta_{\text{roo}} C_{\text{roo}}^{(t)}, \tag{5}$$

$$C_{\text{som}}^{(t+1)'} = \left(1 - \theta_{\text{som}} \rho^{(t)} \right) C_{\text{som}}^{(t)} + \theta_{\text{woo}} C_{\text{woo}}^{(t)} + \theta_{\text{min}} \rho^{(t)} C_{\text{lit}}^{(t)}. \tag{6}$$

Here $F_{\text{gpp}}^{(t)}$ denotes the gross primary production (GPP) of Carbon on day t and is a function of meteorology and atmospheric vapor pressure:

$$F_{\text{gpp}}^{(t)} = F_{\text{gpp(max)}}^{(t)} \times \min\left(1, \frac{W^{(t)}}{\omega} \right), \tag{7}$$

where ω is the water stress threshold, $F_{\text{gpp(max)}}^{(t)}$ denotes the maximum GPP on day t, which is given by the Aggregated Canopy Model (ACM), described in another document, and $W^{(t)}$ is the plant-available water pool on day t. $W^{(t)}$ is given dynamically:

$$W^{(t+1)} = W^{(t)} + P^{(t)} - R^{(t)} - ET^{(t)}, \tag{8}$$

where $P^{(t)}$, $R^{(t)}$, and $ET^{(t)}$ are the precipitation, runoff, and evapotranspiration on day t, respectively. Evapotranspiration and runoff are given by

$$ET^{(t)} = F_{\text{gpp}}^{(t)} \frac{VPD^{(t)}}{v_e}, \tag{9}$$

$$R^{(t)} = \begin{cases} \alpha \left(W^{(t)} \right)^2 & \text{if } W^{(t)} \leq \frac{1}{2\alpha}, \\ W^{(t)} - \frac{1}{2\alpha} & \text{if } W^{(t)} > \frac{1}{2\alpha}, \end{cases} \tag{10}$$

where v_e is the inherent water-use efficiency, α is a runoff decay constant, and $VPD^{(t)}$ is the vapor pressure defincit on day t.

θ_{roo} and θ_{woo} are the fine root and stem C turnover rates, respectively. $\theta_{\text{lit}}\rho^{(t)}$ and $\theta_{\text{som}}\rho^{(t)}$ are the litter and SOM C turnover rates, respectively, and $\theta_{\text{min}}\rho^{(t)}$ is the rate of litter mineralization to SOM, where $\rho^{(t)}$ is a function of daily and monthly average temperature and precipitation values:

$$\rho^{(t)} = e^{\Theta\left(T^{(t)} - \overline{T}\right)} \left(\left(\frac{P^{(t)}}{\overline{P}} - 1 \right) s_p + 1 \right) \tag{11}$$

Here Θ and s_p are heterotrophic temperature and precipitation dependence factors, $\overline{T}$ and $\overline{P}$ are long term running averages of temperature and precipitation, and $T^{(t)}$ and $P^{(t)}$ are the average temperature and precipitation on day t.

A constant proportion f_{auto} of GPP is lost due to autotrophic respiration. f_{lab}, f_{fol}, f_{roo}, and f_{woo} are constant proportions of GPP allocated to the labile, foliar, fine root, and wood Carbon pools. The sum of these proportions is equal to 1: $f_{\text{lab}} + f_{\text{fol}} + f_{\text{roo}} + f_{\text{woo}} + f_{\text{auto}} = 1$.

$\phi_{\text{onset}}^{(t)}$ and $\phi_{\text{fall}}^{(t)}$ are phenological functions describing the rates of labile to foliar and foliar to litter pool transfer on day t:

$$\phi_{\text{onset}}^{(t)} = \frac{\sqrt{2}}{\sqrt{\pi}} \times \left(\frac{-\log(1 - c_{\text{lr}})}{c_{\text{ronset}}} \right) \times \exp\left[-\left(\frac{s\sqrt{2}}{c_{\text{ronset}}} \sin\left(\frac{t - d_{\text{onset}} - 0.6245 c_{\text{ronset}}}{s} \right) \right)^2 \right], \tag{12}$$

$$\phi_{\text{fall}}^{(t)} = \frac{\sqrt{2}}{\sqrt{\pi}} \times \left(\frac{-\log(1 - c_{\text{ll}})}{c_{\text{rfall}}} \right) \times \exp\left[-\left(\frac{s\sqrt{2}}{c_{\text{rfall}}} \sin\left(\frac{t - d_{\text{fall}} - \psi_f}{s} \right) \right)^2 \right]. \tag{13}$$

Here c_{ronset} and c_{rfall} are the labile release and leaf fall periods, d_{onset} and d_{fall} are the leaf onset and fall days, c_{lr} and c_{ll} are the annual labile C release and leaf loss fractions, respectively, and $s = \frac{365.25}{\pi}$. ψ_f is the solution to a transcendental equationm discussed in another document.

1.2   Fire Dynamics

Denote $\left[C_{\text{lab}}^{(t+1)}, C_{\text{fol}}^{(t+1)}, C_{\text{roo}}^{(t+1)}, C_{\text{woo}}^{(t+1)}, C_{\text{lit}}^{(t+1)}, C_{\text{som}}^{(t+1)} \right] = \vec{C}^{(t+1)} = \text{DALEC}_{813}^{\text{fire}}(\vec{C}^{(t+1)'})$. The fire dynamics are given by

$$C_i^{(t+1)} = C_i^{(t+1)'} - FE_i^{(t)} - FM_i^{(t)}, \tag{14}$$

for $i = \text{lab}, \text{fol}, \text{roo}, \text{woo}$, and

$$C_{\text{lit}}^{(t+1)} = C_{\text{lit}}^{(t+1)'} - FE_{\text{lit}}^{(t)} + FM_{\text{lab}}^{(t)} + FM_{\text{fol}}^{(t)} + FM_{\text{roo}}^{(t)}, \tag{15}$$

$$C_{\text{som}}^{(t+1)} = C_{\text{som}}^{(t+1)'} - FM_{\text{som}}^{(t)} + FM_{\text{woo}}^{(t)}. \tag{16}$$

Here $FE_i^{(t)}$ and $FM_i^{(t)}$ are fire emission and fire mortality fluxes for pool i on day t:

$$FE_i^{(t)} = C_i^{(t+1)'} BA^{(t)} k_i, \tag{17}$$

$$FM_i^{(t)} = C_i^{(t+1)'} BA^{(t)} (1 - k_i) r, \tag{18}$$

where $BA^{(t)}$ is the burned area on day t, k_i are the combustion factors for pool i, and r is the resilience factor.

1.3   Leaf Area Index

Leaf Area Index (LAI) on day t is proportional to the foliar pool on day t:

$$\text{LAI}^{(t)} = \frac{C_{\text{fol}}^{(t)}}{c_{\text{lma}}}, \tag{19}$$

where c_{lma} is the leaf Carbon mass per unit area (sq. m).

2 Comparison to DALEC_813.c

2.1 Dynamic State Variables

$$\texttt{POOLS[p+0]} = C_{\text{lab}}^{(t)} \qquad\qquad \text{Eqn. (1)} = \text{Line 206}$$

$$\texttt{POOLS[p+1]} = C_{\text{fol}}^{(t)} \qquad\qquad \text{Eqn. (2)} = \text{Line 207}$$

$$\texttt{POOLS[p+2]} = C_{\text{roo}}^{(t)} \qquad\qquad \text{Eqn. (3)} = \text{Line 208}$$

$$\texttt{POOLS[p+3]} = C_{\text{woo}}^{(t)} \qquad\qquad \text{Eqn. (4)} = \text{Line 209}$$

$$\texttt{POOLS[p+4]} = C_{\text{lit}}^{(t)} \qquad\qquad \text{Eqn. (5)} = \text{Line 200}$$

$$\texttt{POOLS[p+5]} = C_{\text{som}}^{(t)} \qquad\qquad \text{Eqn. (6)} = \text{Line 208}$$

$$\texttt{POOLS[p+6]} = W^{(t)} \qquad\qquad \text{Eqn. (8)} = \text{Line 219}$$

$$\texttt{POOLS[nxp+0]} = C_{\text{lab}}^{(t+1)'}, \text{ then } C_{\text{lab}}^{(t+1)} \qquad\qquad \text{Eqn. (14)} = \text{Line 236}$$

$$\texttt{POOLS[nxp+1]} = C_{\text{fol}}^{(t+1)'}, \text{ then } C_{\text{fol}}^{(t+1)}$$

$$\texttt{POOLS[nxp+2]} = C_{\text{roo}}^{(t+1)'}, \text{ then } C_{\text{roo}}^{(t+1)}$$

$$\texttt{POOLS[nxp+3]} = C_{\text{woo}}^{(t+1)'}, \text{ then } C_{\text{woo}}^{(t+1)}$$

$$\texttt{POOLS[nxp+4]} = C_{\text{lit}}^{(t+1)'}, \text{ then } C_{\text{lit}}^{(t+1)} \qquad\qquad \text{Eqn. (15)} = \text{Line 239}$$

$$\texttt{POOLS[nxp+5]} = C_{\text{som}}^{(t+1)'}, \text{ then } C_{\text{som}}^{(t+1)} \qquad\qquad \text{Eqn. (16)} = \text{Line 241}$$

2.2 Fluxes and Supporting Equations

$$\texttt{FLUXES[f+0]} = F_{\text{gpp}}^{(t)} \qquad\qquad \text{Eqn. (7)} = \text{Line 169}$$

$$\texttt{FLUXES[f+3]} = f_{\text{fol}} F_{\text{gpp}}^{(t)}$$

$$\texttt{FLUXES[f+4]} = f_{\text{lab}} F_{\text{gpp}}^{(t)}$$

$$\texttt{FLUXES[f+5]} = f_{\text{roo}} F_{\text{gpp}}^{(t)}$$

$$\texttt{FLUXES[f+6]} = f_{\text{woo}} F_{\text{gpp}}^{(t)}$$

$$\texttt{FLUXES[f+7]} = \phi_{\text{onset}}^{(t)} C_{\text{lab}}^{(t)}$$

$$\texttt{FLUXES[f+9]} = \phi_{\text{fall}}^{(t)} C_{\text{fol}}^{(t)}$$

$$\texttt{FLUXES[f+10]} = \phi_{\text{fall}}^{(t)} C_{\text{fol}}^{(t)}$$

$$\texttt{FLUXES[f+11]} = \theta_{\text{roo}} C_{\text{roo}}^{(t)}$$

$$\texttt{FLUXES[f+12]} = \theta_{\text{lit}} \rho^{(t)} C_{\text{lit}}^{(t)}$$

$$\texttt{FLUXES[f+13]} = \theta_{\text{som}} \rho^{(t)} C_{\text{lit}}^{(t)}$$

$$\texttt{FLUXES[f+14]} = \theta_{\text{min}} \rho^{(t)} C_{\text{lit}}^{(t)}$$

$$\texttt{FLUXES[f+1]} = \rho^{(t)} \qquad\qquad \text{Eqn (11)} = \text{Line 174}$$

$$\texttt{FLUXES[f+8]} = \phi_{\text{fall}}^{(t)} \qquad\qquad \text{Eqn (13)} = \text{Line 186}$$

$$\texttt{FLUXES[f+15]} = \phi_{\text{onset}}^{(t)} \qquad\qquad \text{Eqn (12)} = \text{Line 188}$$

$$\texttt{FLUXES[f+28]} = ET^{(t)} \qquad\qquad \text{Eqn. (9)} = \text{Line 171}$$

$$\texttt{FLUXES[f+29]} = R^{(t)} \qquad\qquad \text{Eqn. (10)} = \text{Lines 215, 217}$$

$$\texttt{FLUXES[f+17+nn]} = FE_i^{(t)} \qquad\qquad \text{Eqn. (17)} = \text{Line 231}$$

$$\texttt{FLUXES[f+13+nn]} = FM_i^{(t)} \qquad\qquad \text{Eqn. (18)} = \text{Line 232}$$

$$\texttt{LAI[n]} = \text{LAI}^{(t)} \qquad\qquad \text{Eqn. (19)} = \text{Line 157}$$

2.3 Parameters

$$\texttt{pars[0]} = \theta_{\text{min}}$$

$$\texttt{pars[1]} = \%\text{GPP lost to autotrophic respiration } (f_{\text{auto}})$$

$$\texttt{pars[2]} = \%\text{NPP allocated to foliar pool}$$

$$f_{\text{fol}} = (1 - \texttt{pars[1]})\texttt{pars[2]}$$

$$\texttt{pars[12]} = \%(\text{NPP not allocated to foliar pool}) \text{ allocated to labile pool}$$

$$f_{\text{lab}} = (1 - \texttt{pars[1]})(1 - \texttt{pars[2]})\texttt{pars[12]}$$

$$\texttt{pars[3]} = \%(\text{NPP not allocated to foliar or labile pools}) \text{ allocated to fine root pool}$$

$$f_{\text{roo}} = (1 - \texttt{pars[1]})(1 - \texttt{pars[2]})(1 - \texttt{pars[12]})\texttt{pars[3]}$$

$$f_{\text{woo}} = (1 - \texttt{pars[1]})(1 - \texttt{pars[2]})(1 - \texttt{pars[12]})(1 - \texttt{pars[3]})$$

$$\texttt{pars[4]} = \frac{1}{c_{\text{ll}}}$$

$$\texttt{pars[5]} = \theta_{\text{woo}}$$

$$\texttt{pars[6]} = \theta_{\text{roo}}$$

$$\texttt{pars[7]} = \theta_{\text{lit}}$$

$$\texttt{pars[8]} = \theta_{\text{som}}$$

$$\texttt{pars[9]} = \Theta$$

$$\texttt{pars[10]} = \text{one of the nitrogen constants in the ACM}$$

$$\texttt{pars[11]} = d_{\text{onset}}$$

$$\texttt{pars[13]} = c_{\text{ronset}}$$

$$\texttt{pars[14]} = d_{\text{fall}}$$

$$\texttt{pars[15]} = c_{\text{rfall}}$$

$$\texttt{pars[16]} = c_{\text{lma}}$$

$$\texttt{pars[17]} \text{ through } \texttt{pars[22]} = \left[C_{\text{lab}}^{(0)}, C_{\text{fol}}^{(0)}, C_{\text{roo}}^{(0)}, C_{\text{woo}}^{(0)}, C_{\text{lit}}^{(0)}, C_{\text{som}}^{(0)} \right] \text{ (initial condition)}$$

$$\texttt{pars[23]} = v_e$$

$$\texttt{pars[24]} = \frac{1}{\alpha}$$

$$\texttt{pars[25]} = \omega$$

$$\texttt{pars[26]} = W^{(0)} \text{ (initial condition)}$$

$$\texttt{pars[27]} = k_{\text{fol}}$$

$$\texttt{pars[28]} = k_{\text{lab}} = k_{\text{roo}} = f_{\text{woo}}$$

$$\texttt{pars[27]/2+pars[28]/2} = k_{\text{lit}}$$

$$\texttt{pars[29]} = k_{\text{som}}$$

$$\texttt{pars[30]} = 1 - r$$

$$\texttt{pars[31]} = \frac{1}{c_{\text{lr}}}$$

$$\texttt{pars[32]} = s_p$$

2.4 Meteorological Data and Other Inputs

$$\texttt{DATA.MET[m+0]} = t \qquad\qquad \text{eqn. (12) and (13)}$$

$$\texttt{DATA.MET[m+1]} = \text{min. temp on day } t \qquad\qquad \text{ACM}$$

$$\texttt{DATA.MET[m+2]} = \text{max. temp on day } t \qquad\qquad \text{ACM}$$

$$\texttt{DATA.MET[m+2]*0.5+DATA.MET[m+1]*0.5} = T^{(t)} \qquad\qquad \text{eqn. (11)}$$

$$\texttt{DATA.MET[m+3]} = \text{radiation} \qquad\qquad \text{ACM}$$

$$\texttt{DATA.MET[m+4]} = CO_2 \qquad\qquad \text{ACM}$$

$$\texttt{DATA.MET[m+5]} = \text{yearday} \qquad\qquad \text{ACM}$$

$$\texttt{DATA.MET[m+6]} = BA^{(t)} \qquad\qquad \text{eqn. (17) and (18)}$$

$$\texttt{DATA.MET[m+7]} = VPD^{(t)} \qquad\qquad \text{eqn. (9)}$$

$$\texttt{DATA.MET[m+8]} = P^{(t)} \qquad\qquad \text{eqn. (8) and (11)}$$

$$\texttt{DATA.meantemp} = \overline{T} \qquad\qquad \text{eqn. (11)}$$

$$\texttt{DATA.meanprec} = \overline{P} \qquad\qquad \text{eqn. (11)}$$