CARDAMOM DALEC Model ID 813

January 7, 2021

1 Mathematical Dynamics

Let $\vec{C}^{(t)} = \left[C_{\text{lab}}^{(t)}, C_{\text{fol}}^{(t)}, C_{\text{roo}}^{(t)}, C_{\text{woo}}^{(t)}, C_{\text{lit}}^{(t)}, C_{\text{som}}^{(t)}\right]$ denote the labile (lab), foliar (fol), fine root (roo), wood (woo), litter (lit), and soil organic matter (SOM) carbon pools on day t. The dynamics are given by

$$\begin{aligned}\vec{C}^{(t+1)} &= \text{DALEC}_{813}\left(\vec{C}^{(t)}\right) \\ &= \text{DALEC}_{813}^{\text{fire}}\left(\text{DALEC}_{813}^{\text{pre-fire}}\left(\vec{C}^{(t)}\right)\right),\end{aligned}$$

where $\text{DALEC}_{813}^{\text{pre-fire}}(\cdot)$ denotes Carbon dynamics in the absence of fire and $\text{DALEC}_{813}^{\text{fire}}(\cdot)$ denotes Carbon Carbon dynamics due to fires.

1.1 Pre-fire Dynamics

Denote $\left[C_{\text{lab}}^{(t+1)'}, C_{\text{fol}}^{(t+1)'}, C_{\text{roo}}^{(t+1)'}, C_{\text{woo}}^{(t+1)'}, C_{\text{lit}}^{(t+1)'}, C_{\text{som}}^{(t+1)'}\right] = \vec{C}^{(t+1)'} = \text{DALEC}_{813}^{\text{pre-fire}}(\vec{C}^{(t)})$. The pre-fire dynamics are given by

$$\begin{aligned}
C_{\text{lab}}^{(t+1)'} &= f_{\text{lab}} F_{\text{gpp}}^{(t)} + \left(1 - \phi_{\text{onset}}^{(t)}\right) C_{\text{lab}}^{(t)}, && (1)\\
C_{\text{fol}}^{(t+1)'} &= \phi_{\text{onset}}^{(t)} C_{\text{lab}}^{(t)} + \left(1 - \phi_{\text{fall}}^{(t)}\right) C_{\text{fol}}^{(t)} + f_{\text{fol}} F_{\text{gpp}}^{(t)}, && (2)\\
C_{\text{roo}}^{(t+1)'} &= (1 - \theta_{\text{roo}}) C_{\text{roo}}^{(t)} + f_{\text{roo}} F_{\text{gpp}}^{(t)}, && (3)\\
C_{\text{woo}}^{(t+1)'} &= (1 - \theta_{\text{woo}}) C_{\text{woo}}^{(t)} + f_{\text{woo}} F_{\text{gpp}}^{(t)}, && (4)\\
C_{\text{lit}}^{(t+1)'} &= \left(1 - (\theta_{\text{lit}} + \theta_{\text{min}})\, \rho^{(t)}\right) C_{\text{lit}}^{(t)} + \phi_{\text{fall}}^{(t)} C_{\text{fol}}^{(t)} + \theta_{\text{roo}} C_{\text{roo}}^{(t)}, && (5)\\
C_{\text{som}}^{(t+1)'} &= \left(1 - \theta_{\text{som}} \rho^{(t)}\right) C_{\text{som}}^{(t)} + \theta_{\text{woo}} C_{\text{woo}}^{(t)} + \theta_{\text{min}} \rho^{(t)} C_{\text{lit}}^{(t)}. && (6)
\end{aligned}$$

Here $F_{\text{gpp}}^{(t)}$ denotes the gross primary production (GPP) of Carbon on day t and is a function of meteorology and atmospheric vapor pressure:

$$F_{\text{gpp}}^{(t)} = F_{\text{gpp(max)}}^{(t)} \times \min\left(1, \frac{W^{(t)}}{\omega}\right), \tag{7}$$

where ω is the water stress threshold, $F_{\text{gpp(max)}}^{(t)}$ denotes the maximum GPP on day t, which is given by the Aggregated Canopy Model (ACM), described in another document, and $W^{(t)}$ is the plant-available water pool on day t. $W^{(t)}$ is given dynamically:

$$W^{(t+1)} = W^{(t)} + P^{(t)} - R^{(t)} - ET^{(t)}, \tag{8}$$

where $P^{(t)}$, $R^{(t)}$, and $ET^{(t)}$ are the precipitation, runoff, and evapotranspiration on day t, respectively. Evapotranspiration and runoff are given by

$$ET^{(t)} = F_{\text{gpp}}^{(t)} \frac{VPD^{(t)}}{v_e}, \tag{9}$$

$$R^{(t)} = \begin{cases} \alpha\left(W^{(t)}\right)^2 & \text{if } W^{(t)} \leq \frac{1}{2\alpha}, \\ W^{(t)} - \frac{1}{2\alpha} & \text{if } W^{(t)} > \frac{1}{2\alpha}, \end{cases} \tag{10}$$

where v_e is the inherent water-use efficiency, α is a runoff decay constant, and $VPD^{(t)}$ is the vapor pressure defincit on day t.

θ_{roo} and θ_{woo} are the fine root and stem C turnover rates, respectively. $\theta_{\text{lit}}\rho^{(t)}$ and $\theta_{\text{som}}\rho^{(t)}$ are the litter and SOM C turnover rates, respectively, and $\theta_{\text{min}}\rho^{(t)}$ is the rate of litter mineralization to SOM, where $\rho^{(t)}$ is a function of daily and monthly average temperature and precipitation values:

$$\rho^{(t)} = e^{\Theta\left(T^{(t)}-\overline{T}\right)}\left(\left(\frac{P^{(t)}}{\overline{P}}-1\right)s_p+1\right) \tag{11}$$

Here Θ and s_p are heterotrophic temperature and precipitation dependence factors, $\overline{T}$ and $\overline{P}$ are long term running averages of temperature and precipitation, and $T^{(t)}$ and $P^{(t)}$ are the average temperature and precipitation on day t.

A constant proportion f_{auto} of GPP is lost due to autotrophic respiration. f_{lab}, f_{fol}, f_{roo}, and f_{woo} are constant proportions of GPP allocated to the labile, foliar, fine root, and wood Carbon pools. The sum of these proportions is equal to 1: $f_{\text{lab}}+f_{\text{fol}}+f_{\text{roo}}+f_{\text{woo}}+f_{\text{auto}}=1$.

$\phi_{\text{onset}}^{(t)}$ and $\phi_{\text{fall}}^{(t)}$ are phenological functions describing the rates of labile to foliar and foliar to litter pool transfer on day t:

$$\phi_{\text{onset}}^{(t)} = \frac{\sqrt{2}}{\sqrt{\pi}} \times \left(\frac{-\log(1-c_{\text{lr}})}{c_{\text{ronset}}}\right) \times \exp\left[-\left(\frac{s\sqrt{2}}{c_{\text{ronset}}}\sin\left(\frac{t-d_{\text{onset}}-0.6245c_{\text{ronset}}}{s}\right)\right)^2\right], \tag{12}$$

$$\phi_{\text{fall}}^{(t)} = \frac{\sqrt{2}}{\sqrt{\pi}} \times \left(\frac{-\log(1-c_{\text{ll}})}{c_{\text{rfall}}}\right) \times \exp\left[-\left(\frac{s\sqrt{2}}{c_{\text{rfall}}}\sin\left(\frac{t-c_{\text{rfall}}-\psi_f}{s}\right)\right)^2\right]. \tag{13}$$

Here c_{ronset} and c_{rfall} are the labile release and leaf fall periods, d_{onset} and d_{fall} are the leaf onset and fall days, c_{lr} and c_{ll} are the annual labile C release and leaf loss fractions, respectively, and $s=\frac{365.25}{\pi}$.

1.2 Fire Dynamics

Denote $\left[C_{\text{lab}}^{(t+1)}, C_{\text{fol}}^{(t+1)}, C_{\text{roo}}^{(t+1)}, C_{\text{woo}}^{(t+1)}, C_{\text{lit}}^{(t+1)}, C_{\text{som}}^{(t+1)}\right] = \vec{C}^{(t+1)} = \text{DALEC}_{813}^{\text{fire}}(\vec{C}^{(t+1)'})$. The fire dynamics are given by

$$C_i^{(t+1)} = C_i^{(t+1)'} - FE_i^{(t)} - FM_i^{(t)}, \tag{14}$$

for $i=\text{lab},\text{fol},\text{roo},\text{woo}$, and

$$C_{\text{lit}}^{(t+1)} = C_{\text{lit}}^{(t+1)'} - FE_{\text{lit}}^{(t)} + FM_{\text{lab}}^{(t)} + FM_{\text{fol}}^{(t)} + FM_{\text{roo}}^{(t)}, \tag{15}$$

$$C_{\text{som}}^{(t+1)} = C_{\text{som}}^{(t+1)'} - FM_{\text{som}}^{(t)} + FM_{\text{woo}}^{(t)}. \tag{16}$$

Here $FE_i^{(t)}$ and $FM_i^{(t)}$ are fire emission and fire mortality fluxes for pool i on day t:

$$FE_i^{(t)} = C_i^{(t+1)'}BA^{(t)}k_i, \tag{17}$$

$$FM_i^{(t)} = C_i^{(t+1)'}BA^{(t)}(1-k_i)r, \tag{18}$$

where $BA^{(t)}$ is the burned area on day t, k_i are the combustion factors for pool i, and r is the resilience factor.

1.3 Leaf Area Index

Leaf Area Index (LAI) on day t is proportional to the foliar pool on day t:

$$\text{LAI}^{(t)} = \frac{C_{\text{fol}}^{(t)}}{c_{\text{lma}}}, \tag{19}$$

where c_{lma} is the leaf Carbon mass per unit area (sq. m).

2 Comparison to DALEC_813.c

2.1 Dynamic State Variables

`POOLS[p+0]`	$= C_{\text{lab}}^{(t)}$	Eqn. (1) = Line 206
`POOLS[p+1]`	$= C_{\text{fol}}^{(t)}$	Eqn. (2) = Line 207
`POOLS[p+2]`	$= C_{\text{roo}}^{(t)}$	Eqn. (3) = Line 208
`POOLS[p+3]`	$= C_{\text{woo}}^{(t)}$	Eqn. (4) = Line 209
`POOLS[p+4]`	$= C_{\text{lit}}^{(t)}$	Eqn. (5) = Line 200
`POOLS[p+5]`	$= C_{\text{som}}^{(t)}$	Eqn. (6) = Line 208
`POOLS[p+6]`	$= W^{(t)}$	Eqn. (8) = Line 219
`POOLS[nxp+0]`	$= C_{\text{lab}}^{(t+1)'}$, then $C_{\text{lab}}^{(t+1)}$	Eqn. (14) = Line 236
`POOLS[nxp+1]`	$= C_{\text{fol}}^{(t+1)'}$, then $C_{\text{fol}}^{(t+1)}$	
`POOLS[nxp+2]`	$= C_{\text{roo}}^{(t+1)'}$, then $C_{\text{roo}}^{(t+1)}$	
`POOLS[nxp+3]`	$= C_{\text{woo}}^{(t+1)'}$, then $C_{\text{woo}}^{(t+1)}$	
`POOLS[nxp+4]`	$= C_{\text{lit}}^{(t+1)'}$, then $C_{\text{lit}}^{(t+1)}$	Eqn. (15) = Line 239
`POOLS[nxp+5]`	$= C_{\text{som}}^{(t+1)'}$, then $C_{\text{som}}^{(t+1)}$	Eqn. (16) = Line 241

2.2 Fluxes and Supporting Equations

`FLUXES[f+0]`	$= F_{\text{gpp}}^{(t)}$	Eqn. (7) = Line 169
`FLUXES[f+3]`	$= f_{\text{fol}} F_{\text{gpp}}^{(t)}$	
`FLUXES[f+4]`	$= f_{\text{lab}} F_{\text{gpp}}^{(t)}$	
`FLUXES[f+5]`	$= f_{\text{roo}} F_{\text{gpp}}^{(t)}$	
`FLUXES[f+6]`	$= f_{\text{woo}} F_{\text{gpp}}^{(t)}$	
`FLUXES[f+7]`	$= \phi_{\text{onset}}^{(t)} C_{\text{lab}}^{(t)}$	
`FLUXES[f+9]`	$= \phi_{\text{fall}}^{(t)} C_{\text{fol}}^{(t)}$	
`FLUXES[f+10]`	$= \phi_{\text{fall}}^{(t)} C_{\text{fol}}^{(t)}$	
`FLUXES[f+11]`	$= \theta_{\text{roo}} C_{\text{roo}}^{(t)}$	
`FLUXES[f+12]`	$= \theta_{\text{lit}} \rho^{(t)} C_{\text{lit}}^{(t)}$	
`FLUXES[f+13]`	$= \theta_{\text{som}} \rho^{(t)} C_{\text{lit}}^{(t)}$	
`FLUXES[f+14]`	$= \theta_{\text{min}} \rho^{(t)} C_{\text{lit}}^{(t)}$	
`FLUXES[f+1]`	$= \rho^{(t)}$	Eqn (11) = Line 174
`FLUXES[f+8]`	$= \phi_{\text{fall}}^{(t)}$	Eqn (13) = Line 186
`FLUXES[f+15]`	$= \phi_{\text{onset}}^{(t)}$	Eqn (12) = Line 188
`FLUXES[f+28]`	$= ET^{(t)}$	Eqn. (9) = Line 171
`FLUXES[f+29]`	$= R^{(t)}$	Eqn. (10) = Lines 215, 217
`FLUXES[f+17+nn]`	$= FE_i^{(t)}$	Eqn. (17) = Line 231
`FLUXES[f+13+nn]`	$= FM_i^{(t)}$	Eqn. (18) = Line 232
`LAI[n]`	$= \text{LAI}^{(t)}$	Eqn. (19) = Line 157

2.3 Parameters

$$\begin{aligned}
\texttt{pars[0]} &= \theta_{\min} \\
\texttt{pars[1]} &= \%\text{GPP lost to autotrophic respiration } (f_{\text{auto}}) \\
\texttt{pars[2]} &= \%\text{NPP allocated to foliar pool} \\
&\quad f_{\text{fol}} = (1-\texttt{pars[1]})\texttt{pars[2]} \\
\texttt{pars[12]} &= \%\text{(NPP not allocated to foliar pool) allocated to labile pool} \\
&\quad f_{\text{lab}} = (1-\texttt{pars[1]})(1-\texttt{pars[2]})\texttt{pars[12]} \\
\texttt{pars[3]} &= \%\text{(NPP not allocated to foliar or labile pools) allocated to fine root pool} \\
&\quad f_{\text{roo}} = (1-\texttt{pars[1]})(1-\texttt{pars[2]})(1-\texttt{pars[12]})\texttt{pars[3]} \\
&\quad f_{\text{woo}} = (1-\texttt{pars[1]})(1-\texttt{pars[2]})(1-\texttt{pars[12]})(1-\texttt{pars[3]}) \\
\texttt{pars[4]} &= \frac{1}{c_{\text{ll}}} \\
\texttt{pars[5]} &= \theta_{\text{woo}} \\
\texttt{pars[6]} &= \theta_{\text{roo}} \\
\texttt{pars[7]} &= \theta_{\text{lit}} \\
\texttt{pars[8]} &= \theta_{\text{som}} \\
\texttt{pars[9]} &= \Theta \\
\texttt{pars[10]} &= \text{one of the nitrogen constants in the ACM} \\
\texttt{pars[11]} &= d_{\text{onset}} \\
\texttt{pars[13]} &= c_{\text{ronset}} \\
\texttt{pars[14]} &= d_{\text{fall}} \\
\texttt{pars[15]} &= c_{\text{rfall}} \\
\texttt{pars[16]} &= c_{\text{lma}} \\
\texttt{pars[17]} \text{ through } \texttt{pars[22]} &= \left[C_{\text{lab}}^{(0)}, C_{\text{fol}}^{(0)}, C_{\text{roo}}^{(0)}, C_{\text{woo}}^{(0)}, C_{\text{lit}}^{(0)}, C_{\text{som}}^{(0)}\right] \text{ (initial condition)} \\
\texttt{pars[23]} &= v_e \\
\texttt{pars[24]} &= \frac{1}{\alpha} \\
\texttt{pars[25]} &= \omega \\
\texttt{pars[26]} &= W^{(0)} \text{ (initial condition)} \\
\texttt{pars[27]} &= k_{\text{fol}} \\
\texttt{pars[28]} &= k_{\text{lab}} = k_{\text{roo}} = f_{\text{woo}} \\
\texttt{pars[27]/2+pars[28]/2} &= k_{\text{lit}} \\
\texttt{pars[29]} &= k_{\text{som}} \\
\texttt{pars[30]} &= 1-r \\
\texttt{pars[31]} &= \frac{1}{c_{\text{lr}}} \\
\texttt{pars[32]} &= s_p
\end{aligned}$$

2.4 Meteorological Data and Other Inputs

`DATA.MET[m+0]`	$= t$	eqn. (12) and (13)
`DATA.MET[m+1]`	= min. temp on day t	ACM
`DATA.MET[m+2]`	= max. temp on day t	ACM
`DATA.MET[m+2]*0.5+DATA.MET[m+1]*0.5`	$= T^{(t)}$	eqn. (11)
`DATA.MET[m+3]`	= radiation	ACM
`DATA.MET[m+4]`	$= CO_2$	ACM
`DATA.MET[m+5]`	= yearday	ACM
`DATA.MET[m+6]`	$= BA^{(t)}$	eqn. (17) and (18)
`DATA.MET[m+7]`	$= VPD^{(t)}$	eqn. (9)
`DATA.MET[m+8]`	$= P^{(t)}$	eqn. (8) and (11)
`DATA.meantemp`	$= \overline{T}$	eqn. (11)
`DATA.meanprec`	$= \overline{P}$	eqn. (11)